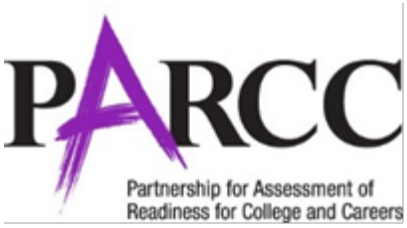

Math
Released Item 2016

Grade 8

Four Rational Numbers
5062-M25380

Four rational numbers are represented by j, k, m, and p.

Part A

State all possible conditions that must be true for j and k so that the product of this expression is negative.

$$-2jk$$

Justify your response.

Enter your response and your justification in the space provided.

Part B

What must be true for $(m+p)$ so that the value of this expression is negative?

$$-2+(m+p)$$

Justify your response.

Enter your response and your justification in the space provided.

Rubric Part A	
Score	**Description**
2	Student response includes each of the following 2 elements. • **Reasoning component** = 1 point o The student provides all possible conditions for j and k under which $-2jk$ is negative. • **Reasoning component** = 1 point o The student provides a justification for his or her response. Sample Student Response: The rational numbers j and k must both be positive or both be negative. In other words, they must have the same sign. The expression $-2jk$ is the product of the three rational numbers -2, j, and k. For the product of three rational numbers to be negative, exactly one of the numbers must be negative or all three of the numbers must be negative. This is because there must be an odd quantity of negative numbers in a product for the product itself to be negative. Since -2 is negative, one of the following must be true: j and k are both positive, so that only one of the three numbers -2, j, and k is negative, or j and k are both negative, so that all three of the numbers are negative. Therefore, j and k must have the same sign. or other valid response
1	Student response includes 1 of the above elements. OR Student response states one condition and provides a concrete example to justify the condition.
0	Student response is incorrect or irrelevant.

Rubric Part B	
Score	**Description**
2	Student response includes each of the following 2 elements. • **Reasoning component** = 1 point o The student provides the conditions for $(m + p)$ under which the sum $-2 + (m + p)$ is negative. • **Reasoning component** = 1 point o The student provides a justification for his or her response. Sample Student Response: In order for $-2 + (m + p)$ to be negative, the rational numbers m and p must satisfy the inequality $(m + p) < 2$. In other words, their sum must be less than 2. The sum $-2 + (m + p)$ can be considered the sum of -2 and the number $(m + p)$. Since 0 is 2 units to the right of -2, when any value greater than or equal to 2 is added to -2, the sum will not be negative. Therefore, in

	order for $-2 + (m + p)$ to be negative, the value of $(m + p)$ must be less than 2, which is the same as $(m + p) < 2$. or other valid response
1	Student response includes 1 of the above elements.
0	Student response is incorrect or irrelevant.

Anchor Set
A1 – A10

With Annotations

A1

Part A: Score Point 2
Part B: Score Point 2

Four rational numbers are represented by *j*, *k*, *m*, and *p*.

Part A

State all possible conditions that must be true for *j* and *k* so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

The variables j and k must both be positive, since a positive times a positve is a positive, but when multiples by a negative, the answer equals a negative.
Another condition is that both variables must be negative, since two negative numbers multiplied equals a positive, but when multiplied with a negative, the number becomes a negative.

Part B

What must be true for (*m*+*p*) so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

The sum of m and p must be less than 2 so the value of the expression is negative. Any integer greater than or equal to 2 will make the expression untrue since if the sum of m and p is 2 the expression would be 0, which is not negative.

Annotation

Anchor Paper 1

Part A: Score Point 2

This response receives full credit. It includes each of the two required elements:

- The response provides all possible conditions for j and k under which -2jk is negative (The variables j and k must both be positive; Another condition is that both variables must be negative).

- The response provides justification for the response (since a positive times a positive is a positive, but when multiples by a negative, the answer equals a negative; since two negative numbers multiplied equals a positive, but when multiplied with a negative, the number becomes a negative).

Part B: Score Point 2

This response receives full credit. It includes each of the two required elements:

- The response provides the conditions for (m + p) under which the sum -2 + (m + p) is negative (The sum of m and p must be less than 2).

- The response provides justification that the sum of m + p must be less than 2 (Any integer greater than or equal to 2 will make the expression untrue since if the sum of m and p is 2 the expression would be 0, which is not negative).

Part A: Score Point 2
Part B: Score Point 2

Four rational numbers are represented by *j*, *k*, *m*, and *p*.

Part A

State all possible conditions that must be true for *j* and *k* so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

The variables j and k must both be negative or they both must be positive. There must be a uneven total of negative numbers for the expression to remain negative.

Part B

What must be true for (*m*+*p*) so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

The variables m and p must have a sum that is less than two. If it is equal to 2, then the expression will be equal to 0 and if the sum is higher than two, it will be a positive number. In order for the expression to remain a negative number, $m + p < 2$.

Annotation

Anchor Paper 2

Part A: Score Point 2

This response receives full credit. It includes each of the two required elements:

- The response provides all possible conditions for j and k under which -2jk is negative (The variables j and k must both be negative or they both must be positive).

- The response provides justification for the response (There must be a uneven total of negative numbers for the expression to remain negative).

Part B: Score Point 2

This response receives full credit. It includes each of the two required elements:

- The response provides the conditions for (m + p) under which the sum -2 + (m + p) is negative (The variables m and p must have a sum that is less than two).

- The response provides justification that the sum of m + p must be less than 2 (If it is equal to 2, then the expression will be equal to 0 and if the sum is higher than two, it will a positive number. In order for the expression to remain a negative number, $m + p < 2$).

A3

Part A: Score Point 1
Part B: Score Point 2

Four rational numbers are represented by j, k, m, and p.

Part A

State all possible conditions that must be true for j and k so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

Both j and k must be negative. When you put numbers to the variables you put parentheses around j and k.
For example, if you had $^{-}2jk$ and $j = {}^{-}3$ and $k = -4$your equation would be $^{-}2\left({}^{-}3 \cdot {}^{-}4\right)$
You would multiply $^{-}3 \cdot {}^{-}4$ and get 12.
Then you would multiply $^{-}2 \cdot 12$ and get -24

Part B

What must be true for $(m+p)$ so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

The sum of m and p would have to be below 2.
For example, if $m = 1$ and $p = 1$ it wouldnot work because $1 + 1 = 2$ and $^{-}2 + 2 = 0$ which is not a negative number. If you had $m = .99$ and $p = 1$ then it would work because $.99 + 1 = 1.99$ and that is below two.
$^{-}2 + 1.99 = -.1$

Annotation

Anchor Paper 3

Part A: Score Point 1

This response receives partial credit. It includes one of the two required elements:

- The response provides one possible condition for j and k under which -2jk is negative (Both j and k must be negative) and provides a concrete example to justify the condition (j = -3 and k = -4your equation would be -2 (-3 • -4) You would multiply -3 • -4 and get 12. Then you would multiply -2 • 12 and get -24).

Part B: Score Point 2

This response receives full credit. It includes each of the two required elements:

- The response provides the conditions for (m + p) under which the sum -2 + (m + p) is negative (The sum of m and p would have to be below 2).

- The response provides justification that the sum of m + p must be less than 2 (if m = 1 and p = 1 it would not work because 1 + 1 = 2 and -2 + 2 = 0 which is not a negative number. If you had m = .99 and p = 1 then it would work because .99 + 1 = 1.99 and that is below two. -2 + 1.99 = -.1). The sum should be - .01, however the calculation error does not detract from the demonstrated understanding.

A4

Part A: Score Point 1
Part B: Score Point 2

Four rational numbers are represented by *j*, *k*, *m*, and *p*.

Part A

State all possible conditions that must be true for *j* and *k* so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

J and K must both be negative because a negative times a negative times a negative is negative.
$^{-}2 \times {}^{-}2 \times {}^{-}2 = {}^{-}8$

Part B

What must be true for (*m*+*p*) so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

M and P combined must be less than 2. $^{-}2 + (.5 + 1) = {}^{-}.5$ but $^{-}2 + (1 + 1) = 0$

Annotation

Anchor Paper 4

Part A: Score Point 1

This response receives partial credit. It includes one of the two required elements:

- The response provides one possible condition for j and k under which -2jk is negative (J and K must both be negative) and provides a concrete example to justify the condition (a negative times a negative times a negative is negative, -2 x -2 x -2 = -8).

Part B: Score Point 2

This response receives full credit. It includes each of the two required elements:

- The response provides the conditions for (m + p) under which the sum -2 + (m + p) is negative (M and P combined must be less than 2).

- The response provides justification that the sum of m + p must be less than 2 [-2 + (.5 + 1) = -.5 but -2 + (1 + 1) = 0]. For the justification to be valid using examples, the student must provide 1 example showing the condition does not work and one example showing the condition does work.

A5

Part A: Score Point 1
Part B: Score Point 1

Four rational numbers are represented by *j*, *k*, *m*, and *p*.

Part A

State all possible conditions that must be true for *j* and *k* so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

j and k can either both be positive or both be negative to have the product come out as a negative number.

$j = \pm 2$
$k = \pm 2$

Part B

What must be true for (*m*+*p*) so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

The value of $(m + p)$ must be less than 2 for the value of $^{-}2 + (m + p)$ to be a negative

Annotation

Anchor Paper 5

Part A: Score Point 1

This response receives partial credit. It includes one of the two required elements:

- The response provides all possible conditions for j and k under which -2jk is negative (j and k can either be positive or both be negative).

The response does not provide justification for the response.

Part B: Score Point 1

This response receives partial credit. It includes one of the two required elements:

- The response provides conditions for (m + p) under which the sum -2 + (m + p) is negative (The value of (m + p) must be less than 2).

The response does not provide justification that the sum of m + p must be less than 2.

A6

Part A: Score Point 0
Part B: Score Point 2

Four rational numbers are represented by j, k, m, and p.

Part A

State all possible conditions that must be true for j and k so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

$^{-}2 \times j \times k$
j and k multiply with negative 2.
j or k would have to be a negative number for the product to be negative.
if it was a positve number it wouldnt be a negative number anymore.

Part B

What must be true for $(m+p)$ so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

m and p would have be negative.
Ex. $^{-}2 + (^{-}1 + ^{-}2)$
$^{-}2 + ^{-}3 = ^{-}5$
Ex. $^{-}2 + (2 + 3)$
$^{-}2 + 5 = 3$
it could also be a positive but it would need to be under 2.

Annotation

Anchor Paper 6

Part A: Score Point 0

This response receives no credit. It does not include either of the two required elements:

The response does not provide any conditions for j and k under which -2jk is negative (j or k would have to be a negative number for the product to be negative). If j or k were negative the product would be positive.

The response does not provide justification for the response.

Part B: Score Point 2

This response receives full credit. It includes each of the two required elements:

- The response provides the conditions for (m + p) under which the sum -2 + (m + p) is negative (it could also be a positive but it would need to be under 2).

- The response provides justification that the sum of m + p must be less than 2 [-2 + (-1 + -2), -2 + -3 = -5; -2 + (2 + 3), -2 + 5 = 3]. For the justification to be valid using examples, the student must provide 1 example showing the condition does not work and one example showing the condition does work.

A7

Part A: Score Point 0
Part B: Score Point 1

Four rational numbers are represented by j, k, m, and p.

Part A

State all possible conditions that must be true for j and k so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

j and k must not be negative because a negative times a negative is a positive

Part B

What must be true for $(m+p)$ so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

their compined value must be less than two

Annotation

Anchor Paper 7

Part A: Score Point 0

This response receives no credit. It does not include either of the two required elements:

The response does not provide conditions for j and k under which -2jk is negative (j and k must not be a negative).

The response does not provide complete justification for the response (a negative times a negative is a positive).

Part B: Score Point 1

This response receives partial credit. It includes one of the two required elements:

- The response provides conditions for (m + p) under which the sum -2 + (m + p) is negative (their compined value must be less than two).

The response does not provide justification that the sum of m + p must be less than 2.

A8

Part A: Score Point 1
Part B: Score Point 0

Four rational numbers are represented by *j*, *k*, *m*, and *p*.

Part A

State all possible conditions that must be true for *j* and *k* so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

The product can be a negative if both j and k is a negative because two negatives make a positive but if there is a another negative it will be a negative.
ex. $-2 \times (-5 \times (-3)) = -30$

Part B

What must be true for (*m*+*p*) so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

To make it a a negative m can be 6 and p can be -6. Then both of the sixes can cancel each other.

Annotation

Anchor Paper 8

Part A: Score Point 1

This response receives partial credit. It includes one of the two required elements:

- The response provides one possible condition for j and k under which -2jk is negative (both j and k is a negative) and provides a concrete example to justify the condition (two negatives make a positive but if there is another negative it will be a negative).

Part B: Score Point 0

This response receives no credit. It includes none of the two required elements:

The response does not provide conditions for (m + p) under which the sum -2 + (m + p) is negative.

The response does not provide justification that the sum of m + p must be less than 2. The response just provides an example.

A9

Part A: Score Point 0
Part B: Score Point 0

Four rational numbers are represented by j, k, m, and p.

Part A

State all possible conditions that must be true for j and k so that the product of this expression is negative.

$$-2jk$$

Justify your response.

Enter your response and your justification in the space provided.

J and K must both equal a negative number when multiplied together. One of those letters must be a positive number, and the other letter must equal a negative number. If you multiply a negative and a negative, it will equal a pausitive. So if you multiply a negative and a positive, it will equal a negative.

Part B

What must be true for $(m+p)$ so that the value of this expression is negative?

$$-2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

M and P have the same rules as J and k. One of them have to be a positive, and the other letter must be a negative to equal a negative.

Annotation

Anchor Paper 9

Part A: Score Point 0

This response receives no credit. It does not include either of the two required elements:

The response does not provide conditions for j and k under which -2jk is negative. The response provides incorrect information (One of those letters must be a positive number, and the other must equal a negative number)

The response does not provide valid justification for the response (If you multiply a negative and a negative, it will equal a pausitive. So if you multiply a negative and a positive, it will equal a negative). If one of the letters is positive and one letter is negative, the result is a positive number. The justification does not match the conditions.

Part B: Score Point 0

This response receives no credit. It does not include either of the two required elements:

The response does not provide correct conditions for (m + p) under which the sum -2 + (m + p) is negative (One of them have to be a positive, and the other must be a negative to equal a negative).

The response does not provide justification that the sum of m + p must be less than 2.

A10

Part A: Score Point 0
Part B: Score Point 0

Four rational numbers are represented by j, k, m, and p.

Part A

State all possible conditions that must be true for j and k so that the product of this expression is negative.

$$-2jk$$

Justify your response.

Enter your response and your justification in the space provided.

j or k is negative while the other variable is positive. That is how they would get a negative number.

Part B

What must be true for $(m+p)$ so that the value of this expression is negative?

$$-2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

m and p are both negative.

Annotation

Anchor Paper 10

Part A: Score Point 0

This response receives no credit. It does not include either of the two required elements:

The response does not provide conditions for j and k under which -2jk is negative (j or k is negative while the other variable is positive). If one of the variables is positive and the other is negative, the product of the expression would be positive.

The response does not provide valid justification for the response.

Part B: Score Point 0

This response receives no credit. It does not include either of the two required elements:

The response provides a condition for (m + p) under which the sum -2 + (m + p) is negative. By itself, (m and p are both negative) is insufficient for credit.

The response does not provide justification that the sum of m + p must be less than 2.

Practice Set
P101 - P105

No Annotations Included

P101

Four rational numbers are represented by *j*, *k*, *m*, and *p*.

Part A

State all possible conditions that must be true for *j* and *k* so that the product of this expression is negative.

$$-2jk$$

Justify your response.

Enter your response and your justification in the space provided.

In order for the product of the expression $-2jk$ to be negative, j and k must both be postive, or both be negative. The reason this works is because if they are both negative, then three negatives equal a negative. If they were both positive, then two postives and a negative equals a negative.

Part B

What must be true for (*m*+*p*) so that the value of this expression is negative?

$$-2+(m+p)$$

Justify your response.

Enter your response and your justification in the space provided.

The value of $m+p$ must be less than two. If $m+p$ equaled two or more then the equation would equal 0 or a positive number. As long as $m+p$ is less than two, when you subtract two from the sum it will be a negative number.

Four rational numbers are represented by *j*, *k*, *m*, and *p*.

Part A

State all possible conditions that must be true for *j* and *k* so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

First you multiply j and k then you multiply by -2 and u will get a negative expression.

Part B

What must be true for (*m*+*p*) so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

When u add the total from $m + p$ then you will add it to the -2 which will give you a negative answer.

Four rational numbers are represented by *j*, *k*, *m*, and *p*.

Part A

State all possible conditions that must be true for *j* and *k* so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

1. they can both be positive
2.both negative

1. negative times positive is negative.
2. negative times negative is positive times negative is negative.

Part B

What must be true for (*m*+*p*) so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

it needs to be a number LESS than two.

P104

Four rational numbers are represented by j, k, m, and p.

Part A

State all possible conditions that must be true for j and k so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

Both j and k would either have to be neagative or positive to keep the answer a negative.

Part B

What must be true for $(m+p)$ so that the value of this expression is negative?

$$^{-}2 + (m + p)$$

Justify your response.

Enter your response and your justification in the space provided.

If m was the negative than it would have to be a lower number than p

Four rational numbers are represented by *j*, *k*, *m*, and *p*.

Part A

State all possible conditions that must be true for *j* and *k* so that the product of this expression is negative.

$$^{-}2jk$$

Justify your response.

Enter your response and your justification in the space provided.

Either:

j and k must both be positive because a positive number times a negative number equals a negative number.

$^{-}2\left(+j\cdot+k\right)=^{-}2\left(+jk\right)=-2jk$

or

j and k must both be negative because a negative number times a negative enumber quals a positive numer. Then a positive number times a negative number equals a negative number.

$^{-}2\left(^{-}j\cdot^{-}k\right)=^{-}2\left(+jk\right)=-2jk$

Part B

What must be true for (*m*+*p*) so that the value of this expression is negative?

$$^{-}2+(m+p)$$

Justify your response.

Enter your response and your justification in the space provided.

The value of $m+p$ must be less than or equal to 1 because anything above 1 added to -2 would be a positive number.

Practice Set

Paper	Score
P101	2,2
P102	0,0
P103	2,1
P104	1,0
P105	2,0