Georgia Standards of Excellence Curriculum Frameworks

Mathematics

GSE Fifth Grade

Unit 4: Adding, Subtracting, Multiplying, and Dividing Fractions

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

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Unit 4: Adding, Subtracting, Multiplying, and Dividing Fractions

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*****Please note that all changes made will appear in green**

IF YOU HAVE NOT READ THE 5th GRADE CURRICULUM OVERVIEW IN ITS ENTIRETY PRIOR TO USE OF THIS UNIT, PLEASE **STOP** AND CLICK HERE: https://www.georgiastandards.org/Georgia-Standards/Frameworks/5th-Math-Grade-Level-Overview.pdf Return to the use of this unit once you've completed reading the Curriculum Overview. Thank you.

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OVERVIEW

Use equivalent fractions as a strategy to add and subtract fractions.

Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, equivalent, addition/ add, sum, subtraction/subtract, difference, unlike denominator, numerator, benchmark fraction, estimate, reasonableness, and mixed numbers.**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.) Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional side lengths, scaling, comparing.**

Represent and interpret data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **line plot, length, mass, liquid volume**

It is important that students are eventually able use an algorithm to compute with fractions. However, building understanding through the use of manipulatives, mathematical representations, and student discourse while students develop these algorithms through problem solving tasks is research-based best practice. (Huinker, 1998)

The following guidelines should be kept in mind when developing computational strategies with children (Elementary and Middle School Mathematics, Teaching Developmentally, Van de Walle, John A., Karp, Karen S., and Bay-Williams, Jennifer M. 2010, Pearson Ed. Inc., pg 310).

1. *Begin with simple, contextual tasks.* What you want is a context for both the meaning of the operation and the fractions involved.

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2. *Connect the meaning of fraction computation with whole-number computation.* To consider what 2 ½ x ¾ might mean, we should ask, "What does 2 x 3 mean?" Follow this with "What does 2 x 3 ½ mean?" slowly moving to a fraction times a fraction.
3. *Let estimation and informal methods play a big role in the development of strategies.* "Should 2 ½ x ¼ be more or less than 1? More or less than 2?" Estimation keeps the focus on the meanings of the numbers and the operations, encourages reflective thinking, and helps build informal number sense with fractions.
4. *Explore each of the operations using models.* Use a variety of models. Have students defend their solutions using the models, including simple student drawings. Sometimes it may happen that you get answers with models that do not seem to help with pencil and paper methods. This is fine! The ideas will help students learn to think about fractions and the operations, contribute to mental methods, and provide a useful background when you do get to the standard algorithms.

Mentor texts that may be useful for teaching this unit are listed below.

My Half Day by Doris Fisher
Ed Emberly's Picture Pie by Ed Emberley
Two Ways to Count to ten by Ruby Dee
My Even Day by Doris Fisher
The Wishing Club: A story about fractions by Donna Jo Naoli

For more detailed information about unpacking the content standards, unpacking a task, math routines and rituals, maintenance activities and more, please refer to the Grade Level Overview.

STANDARDS FOR MATHEMATICAL PRACTICE

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.

1. **Make sense of problems and persevere in solving them.** Students make sense of the meaning of addition, subtraction, multiplication and division of fractions with whole-number multiplication and division.

2. **Reason abstractly and quantitatively.** Students demonstrate abstract reasoning to create and display area models of multiplication and both sharing and measuring models for division. They extend this understanding from whole numbers to their work with fractions.

3. **Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding their understanding of fractions greater than, equal to, and less than one whole.

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4. **Model with mathematics.** Students draw representations of their mathematical thinking as well as use words and numbers to explain their thinking

5. **Use appropriate tools strategically.** Students select and use tools such as candy bars, measuring sticks, and manipulatives of different fraction sizes to represent situations involving the relationship between fractions.

6. **Attend to precision.** Students attend to the precision when comparing and contrasting fractions and whether or not they are equivalent. Students use appropriate terminology when referring to fractions.

7. **Look for and make use of structure.** Students develop the concept of addition with fractions using common and unlike denominators through the use of various manipulatives.

8. **Look for and express regularity in repeated reasoning.** Students relate new experiences to experiences with similar contexts when allowing students to develop relationships for fluency and understanding of fractional computation. Students explore operations with fractions with visual models and begin to formulate generalizations.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson*****

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

MGSE5.NF.2 Solve word problems involving addition and subtraction of fractions, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < ½.

MGSE5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. Example: $\frac{3}{5}$ can be interpreted as "3 divided by 5 and as 3 shared by 5".

MGSE5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Apply and use understanding of multiplication to multiply a fraction or whole number by a fraction.

Examples: $\frac{a}{b} \times q$ as $\frac{a}{b} \times \frac{q}{1}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

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b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

MGSE5.NF.5 Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. Example 4 x 10 is twice as large as 2 x 10.

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

MGSE5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

MGSE5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many 1/3-cup servings are 2 cups of raisins

[1] *Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.*

MGSE5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were

COMMON MISCONCEPTIONS

MGSE5.NF.1, MGSE5.NF.2 – Students often mix up models when adding, subtracting or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when

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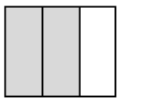
comparing fractions with thirds and fourths. Remind students that the representations need to be from the same whole models with the same shape and size.



These models of fractions are difficult to compare because the size of the whole is not the same for all representations

These models of fractions use the same size rectangle to represent the whole unit and are therefore much easier to compare fractions.

BIG IDEAS

- A fraction is another representation for division.
- Fractions are relations – the size or amount of the whole matters.
- Fractions may represent division with a quotient less than one.
- Equivalent fractions represent the same value.
- With unit fractions, the greater the denominator, the smaller the equal share.
- Shares don't have to be congruent to be equivalent.
- Fractions and decimals are different representations for the same amounts and can be used interchangeably.

ESSENTIAL QUESTIONS (Choose one or two that are appropriate to meet the needs of your students.)

- How are equivalent fractions helpful when solving problems?
- How can a fraction be greater than 1?
- How can a fraction model help us make sense of a problem?
- How can comparing factor size to 1 help us predict what will happen to the product?
- How can decomposing fractions or mixed numbers help us model fraction multiplication?
- How can decomposing fractions or mixed numbers help us multiply fractions?
- How can fractions be used to describe fair shares?
- How can fractions with different denominators be added together?
- How can looking at patterns help us find equivalent fractions?
- How can making equivalent fractions and using models help us solve problems?
- How can modeling an area help us with multiplying fractions?
- How can we describe how much someone gets in a fair-share situation if the fair share is less than 1?

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- How can we describe how much someone gets in a fair-share situation if the fair share is between two whole numbers?
- How can we model an area with fractional pieces?
- How can we model dividing a unit fraction by a whole number with manipulatives and diagrams?
- How can we tell if a fraction is greater than, less than, or equal to one whole?
- How does the size of the whole determine the size of the fraction?
- What connections can we make between the models and equations with fractions?
- What do equivalent fractions have to do with adding and subtracting fractions?
- What does dividing a unit fraction by a whole number look like?
- What does dividing a whole number by a unit fraction look like?
- What does it mean to decompose fractions or mixed numbers?
- What models can we use to help us add and subtract fractions with different denominators?
- What strategies can we use for adding and subtracting fractions with different denominators?
- When should we use models to solve problems with fractions?
- How can I use a number line to compare relative sizes of fractions?
- How can I use a line plot to compare fractions?

CONCEPTS AND SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Add/Subtract fractions with like denominators
- Add/Subtract mixed numbers
- Convert mixed numbers to improper fractions
- Convert improper fractions to mixed numbers
- Compare fractions using >, <, =
- Plot fractions on a number line
- Use visual models to compare and find equivalent fractions
- Multiply a fraction by a whole number
- Convert fractions to decimals with powers of ten

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

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Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/nctm-timed-tests.pdf

STRATEGIES FOR TEACHING AND LEARNING *Adapted from the Ohio DOE*

USE EQUIVALENT FRACTIONS AS A STRATEGY TO ADD AND SUBTRACT FRACTIONS.

MGSE5.NF.1

This standard builds on the work in 4th grade where students add fractions with like denominators. In 5th grade, the example provided in the standard has students find a common denominator by finding the product of both denominators. For 1/3 + 1/6, a common denominator is 18, which is the product of 3 and 6. This process should be introduced using visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm.

Students should apply their understanding of equivalent fractions and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the least common denominator.

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Example 1:

$$\frac{2}{5}+\frac{7}{8}=\frac{16}{40}+\frac{35}{40}=\frac{51}{40}$$

Example 2:

Present students with the problem 1/3 + 1/6. Encourage students to use the clock face as a model for solving the problem. Have students share their approaches with the class and demonstrate their thinking using the clock model.

MGSE5.NF.2

This standard refers to number sense, which means students' understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as 7/8 is greater than 3/4 because 7/8 is missing only 1/8 and 3/4 is missing ¼, so7/8 is closer to a whole Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers. An example of using a benchmark fraction is illustrated with comparing 5/8 and 6/10. Students should recognize that 5/8 is 1/8 larger than 1/2 (since 1/2 = 4/8) and 6/10 is 1/10 1/2 (since 1/2 = 5/10).

Example:

Your teacher gave you 1/7 of the bag of candy. She also gave your friend 1/3 of the bag of candy. If you and your friend combined your candy, what fraction of the bag would you have? Estimate your answer and then calculate. How reasonable was your estimate?

USING DIVISION TO MULTIPLY AND DIVIDE FRACTIONS.

MGSE5.NF.3

This standard calls for students to extend their work of partitioning a number line from third and fourth grade. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities. Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read 3/5 as "three fifths" and after many experiences with sharing problems, learn that 3/5 can also be interpreted as "3 divided by 5."

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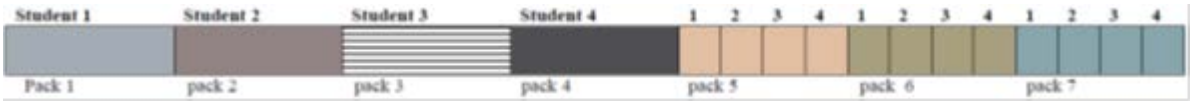
Example 1
Ten team members are sharing 3 boxes of cookies. How much of a box will each student get? When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, 10 □ n = amount is 3 boxes) which can also be written as n = 3 ÷ 10. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting 3/10 of a box

Example 2
Your teacher gives 7 packs of paper to your group of 4 students. If you share the paper equally, how much paper does each student get?

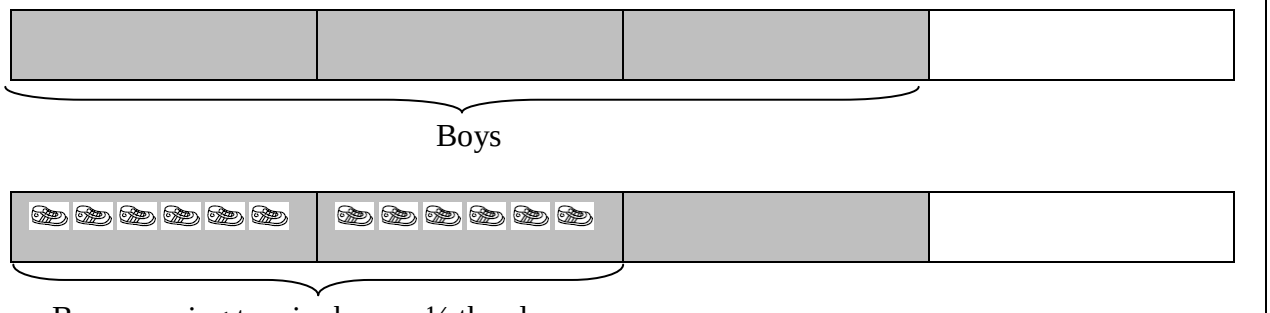


Each student receives 1 whole pack of paper and 1/4 of the each of the 3 packs of paper. So each student gets 13/4 packs of paper.

MGSE5.NF.4 **a.**
This standard references both the multiplication of a fraction by a whole number and the multiplication of two fractions.

Example 1
Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys wearing tennis shoes?

This question is asking what is 2/3 of 3/4 what is 2/3 □ 3/4. (A way to think about it in terms of the language for whole numbers is by using an example such as 4 □ 5, which means you have 4 groups of size 5.)



Boys wearing tennis shoes = ½ the class

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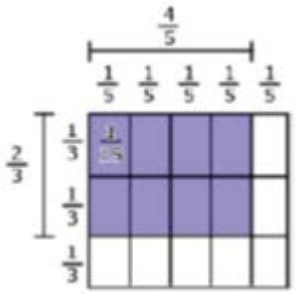
MGSE5.NF.4b
This standard extends students' work with area. In third grade students determine the area of rectangles and composite rectangles. In fourth grade students continue this work. The fifth grade standard calls students to continue the process of covering (with tiles). Grids (see picture) below can be used to support this work.

Example 1
In solving the problem 2/3 x 4/5, students use an area model to visualize it as a 2 by 4 array of small rectangles each of which has side lengths 1/3 and 1/5. They reason that 1/3 x 1/5 = 1/(3 x 5) by counting squares in the entire rectangle, so the area of the shaded area is (2 x 4) x 1/(3 x 5) = (2 x 5)/(3 x 5). They can explain that the product is less than 4/5 because they are finding 2/3 of 4/5. They can further estimate that the answer must be between 2/5 and 4/5 because of is more than 1/2 of 4/5 and less than one group of 4/5.



The area model and the line segments show that the area is the same quantity as the product of the side lengths.

Example 1:
Mrs. Jones teaches in a room that is 60 feet wide and 40 feet long. Mr. Thomas teaches in a room that is half as wide, but has the same length. How do the dimensions and area of Mr. Thomas' classroom compare to Mrs. Jones' room? Draw a picture to prove your answer.

MGSE5.NF.5**b**
This standard asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard:
a) when multiplying by a fraction greater than 1, the number increases and
b) when multiplying by a fraction less the one, the number decreases. This standard should be explored and discussed while students are working with MGSE5.NF.4, and should not be taught in isolation.

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Example 1:
Mrs. Bennett is planting two flower beds. The first flower bed is 5 meters long and 6/5 meters wide. The second flower bed is 5 meters long and 5/6 meters wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square meters? Draw pictures to prove your answer.

Example 2:
22/3 x 8 must be more than 8 because 2 groups of 8 is 16 and 22/3 is almost 3 groups of 8. So the answer must be close to, but less than 24.
3/4 = (5 x 3)/(5 x 4) because multiplying 3/4 by 5/5 is the same as multiplying by 1

MGSE5.NF.6
This standard builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number.

Example 1
There are 21/2 bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. 2/5 of the students on each bus are girls. How many busses would it take to carry only the girls?

Student 2

$2^1/_2 \times {}^2/_5 = ?$

I split the $2^1/_2$ 2 and $^1/_2$. $2^1/_2 \times {}^2/_5 = {}^4/_5$, and $^1/_2 \times {}^2/_5 = {}^2/_{10}$. Then I added $^4/_5$ and $^2/_{10}$. Because $^2/_{10} = {}^1/_5$, $^4/_5 + {}^2/_{10} = {}^4/_5 + {}^1/_5 = 1$. So there is 1 whole bus load of just girls.

Example 1
You have 1/8 of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?

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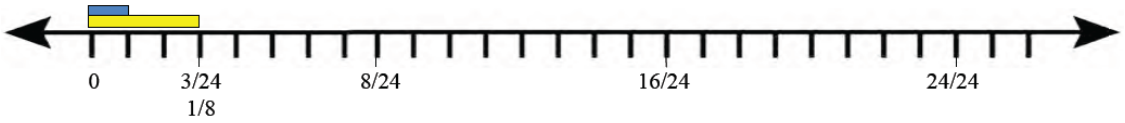
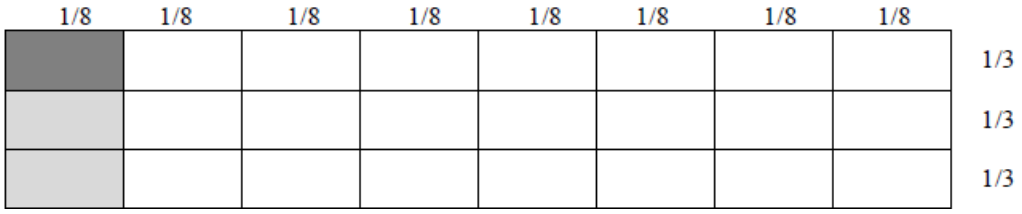
Student 1

I know I need to find the value of the expression $^1/_8 \div 3$, and I want to use a number line.



Student 2

I drew a rectangle and divided it into 8 columns to represent my $^1/_8$. I shaded the first column. I then needed to divide the shaded region into 3 parts to represent sharing among 3 people. I shaded one-third of the first column even darker. The dark shade is $^1/_{24}$ of the grid or $^1/_{24}$ of the bag of pens.



Student 3

$^1/_8$ of a bag of pens divided by 3 people. I know that my answer will be less than $^1/_8$ since I'm sharing $^1/_8$ into 3 groups. I multiplied 8 by 3 and got 24, so my answer is $^1/_{24}$ of the bag of pens. I know that my answer is correct because $(^1/_{24}) \times 3 = {}^3/_{24}$ which equals $^1/_8$.

MGSE5.NF.7**b**
This standard calls for students to create story contexts and visual fraction models for division situations where a whole number is being divided by a unit fraction.

Example 1
Create a story context for 5 ÷ 1/6. Find your answer and then draw a picture to prove your answer and use multiplication to reason about whether your answer makes sense. How many 1/6 are there in 5?

Student

The bowl holds 5 Liters of water. If we use a scoop that holds $^1/_6$ of a Liter, how many scoops will we need in order to fill the entire bowl?

I created 5 boxes. Each box represents 1 Liter of water. I then divided each box into sixths to represent the size of the scoop. My answer is the number of small boxes, which is 30.

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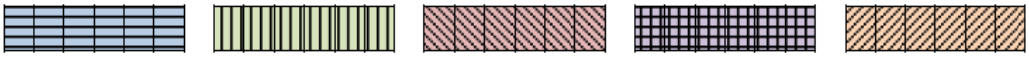
That makes sense since $6 \times 5 = 30$.



$1 = {}^{1}/_{6} + {}^{1}/_{6} + {}^{1}/_{6} + {}^{1}/_{6} + {}^{1}/_{6}$ a whole has ${}^{6}/_{6}$ so five wholes would be ${}^{6}/_{6} + {}^{6}/_{6} + {}^{6}/_{6} + {}^{6}/_{6} + {}^{6}/_{6} = {}^{30}/_{6}$.

MGSE5.NF.7**c**
This standard extends students' work from other standards in MGSE5.NF.7. Student should continue to use visual fraction models and reasoning to solve these real-world problems.

Example 1
How many 1/3-cup servings are in 2 cups of raisins?

Student
I know that there are three 1/3 cup servings in 1 cup of raisins. Therefore, there are 6 servings in 2 cups of raisins. I can also show this since 2 divided by 1/3 = 2 *□ 3 = 6 se*

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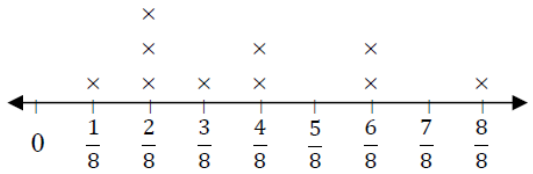
REPRESENT AND INTERPRET DATA.

MGSE5. MD.2

This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

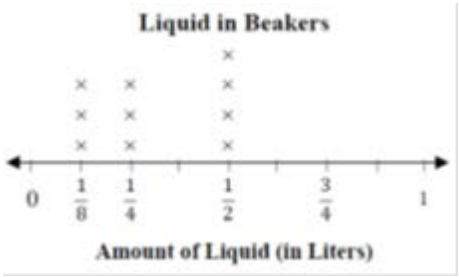
Example 1

Students measured objects in their desk to the nearest 1/2, 1/4, or 1/8 of an inch then displayed data collected on a line plot. How many objects measured 1/4? 1/2? If you put all the objects together end to end what would be the total length of all the objects?



Example:

Ten beakers, measured in liters, are filled with a liquid.



The line plot above shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.)

Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The terms below are for teacher reference only and are not to be memorized by students. Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.

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- **simplify**
- **common denominator**
- **unlike denominator**
- **numerator**
- **improper fraction**
- **mixed number**
- **unit fraction**
- **equivalent**
- **reasonableness**
- **estimate**
- **benchmark fraction**
- **addition/add**
- **subtraction/subtract**
- **difference**

TASKS

Scaffolding Task	Tasks that build up to the learning task.
Constructing Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Intervention Table	The Intervention Table provides links to interventions specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from New Zealand's Numeracy Project.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
CTE Classroom Tasks	Designed to demonstrate how the Common Core and Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.
3-Act Task	A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org and the K-5 MGSE Mathematics Wiki.

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Task Name	**Task Type** *Grouping Strategy*	**Content Addressed**	**Standard(s)**	**Task Description**
Arrays, Number Puzzles, and Factor Trees	Formative Assessment Lesson *Individual/Small Group*	Understand differences between factors, multiples, prime & composite	Skill to maintain-use task at your discretion MGSE.4.OA.4	Formative assessment
Equal to One Whole, More or Less	Scaffolding Task *Small Group/Partner Task*	Determining whether a fraction is Greater, Less, or Equal to 1	Skill to maintain-use task at your discretion MGSE4.NF.2	Determining if the fractional part is less than, greater than, or equal to one whole
Sharing Candy Bars	Constructing Task *Small Group/Partner Task*	Fractions as Division	MGSE5.NF.3 MGSE5.NF.4 MGSE5.NF.6	Splitting candy bars amongst groups of students
Sharing Candy Bars Differently	Constructing Task *Small Group/PartnerTask*	Fractions as Division	MGSE5.NF.3 MGSE5.NF.4a MGSE5.NF.6	Part 2 of splitting candy bars amongst groups of students
Hiking Trail	Constructing Task *Individual/Partner Task*	Fractions of whole numbers, introducing operations with fractions	MGSE5.NF.3 MGSE5.NF.4a	Determining equivalent fractions to add and subtract with unlike denominators
The Black Box	Learning Task *Partner/Small Group Task*	Fraction addition using models	MGSE5.NF.1	Wondering what a black box does with fraction models
The Wishing Club	Constructing Task *Partner/Small Group Task*	Fraction Addition	MGSE5.NF.1 MGSE5.NF.2	Determining equivalent fractions to add unlike denominators
Fraction Addition and Subtraction	Constructing Task *Individual/Partner Task*	Fraction Addition and Subtraction	MGSE5.NF.1 MGSE5.NF.2	Using manipulatives to add and subtract fractions
Flip it Over	Practice Task *Partner/Small Group Task*	Building Fluency with Addition of Fractions	MGSE5.NF.1	Understanding fractional computation

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Up and Down the Number Line	Practice Task *Small Group/Partner Task*	Building Fluency with Addition and Subtraction of Fractions	MGSE5.NF.1	Adding fractions on a number line to determine who can get closest to one whole
Create Three	Practice Task *Small Group/Partner Task*	Building Fluency with Addition of Fractions	MGSE5.NF.1	Playing a game to create 3 wholes on a number line
Comparing MP3s	Constructing Task *Partner Task*	Multiplication of Fractions as an area model	MGSE5.NF.1 MGSE5.NF.2 MGSE5.NF.3 MGSE5.NF.4 MGSE5.NF.5 MGSE5.NF.6	Multiplying fractions using arrays and the distributive property
Measuring for a Pillow	Performance Task *Individual/Partner Task*	Using an area model to multiply and compare products based on factors	MGSE5.NF.4 MGSE5.NF.5 MGSE5.NF.6	Determining what happens to products when one factor remains the same and the other changes
Reasoning with Fractions	Constructing Task *Individual/Partner Task*	Determine the effect on a product, of multiplying a number by a factor greater than 1 and less than 1.	MGSE5.NF.4 MGSE5.NF.5	Using manipulatives and grid paper to determine patterns of multiplication of fractions
Sweet Tart Hearts	Performance Task *Partner/Small Group Task*	Problem solving by adding and multiplying fractions	MGSE5.NF.1 MGSE5.NF.4 MGSE5.NF.5 MGSE5.NF.6	Wondering how many sweet tarts are placed in a glass
Where are the cookies?	Formative Assessment Lesson *Individual/Small Group*	Conceptualizing fractional parts of different wholes	MGSE5.NF.3 MGSE5.NF.4 MGSE5.NF.6 MGSE5.NF.7	Formative Assessment

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Dividing with Unit Fractions	Constructing Task *Partner Task*	Investigate dividing whole numbers by unit fractions and unit fractions by whole numbers	MGSE5.NF.7	Use reasoning to solve fraction division word problems
Adjusting a Recipe	Culminating Task	Multiply, divide, add, and subtract unit fractions	MGSE5.NF.1 MGSE5.NF.2 MGSE5.NF.3 MGSE5.NF.4 MGSE5.NF.5 MGSE5.NF.6 MGSE5.NF.7	Fractional Computations

If you need further information about this unit visit the GaDOE website and reference the unit webinars.
https://www.georgiastandards.org/Archives/Pages/default.aspx

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*INTERVENTION TABLE

The Intervention Table below provides links to interventions specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from New Zealand's Numeracy Project.

Cluster of Standards	Name of Intervention	Snapshot of summary or Student I can statement. . .	Materials Master
Use equivalent fractions as a strategy to add and subtract fractions. **MGSE5.NF.1 MGSE5.NF.2**	Fractions	Generate fractions of a given magnitude using fraction number sense.	
	Equivalent Fractions	Compare fractions by generating like units and constructing a visual model.	MM 8-8 MM 8-9
	Non-Unit Fractions	Determine "closeness to 1" or distance from 1 for a variety of fractions.	MM 4-19 MM 4-22
	Estimating with Fractions	Estimate fraction sums by benchmarking to ½.	MM 8-7
	Comparing Apples with Apples	Add and subtract fractions with like denominators and extend the concept of like units to adding and subtracting unlike denominators.	MM 7-9
Apply and extend previous understandings of multiplication and division. **MGSE5.NF.3 MGSE5.NF.4 MGSE5.NF.5 MGSE5.NF.6 MGSE5.NF.7**	Trains	Rename improper fractions as mixed numbers and position improper fractions on a number line.	
	Linking Money and Decimal Fractions	Relate the quotient of a simple division problem to the context of money.	
	To Turn or Not to Turn	Explain the relationship between a/b and b/a.	
	Fractions Times Whole Numbers	Generalize a rule for multiplying a fraction by a whole number.	
	A Fraction Times a Fraction	Generalize a rule for multiplying a fraction by a fraction.	
	Fractional Blocks	Find fractional parts of a region (fraction of set).	
	The Unit Fraction Game	Compare/anchor fraction sums and products to 1.	Game Mat

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Arrays, Number Puzzles, and Factor Trees

Formative Assessments Lessons (FALs)

Back to Task Table

What is a Formative Assessment Lesson (FAL)? The Formative Assessment Lesson is designed to be part of an instructional unit typically implemented approximately two-thirds of the way through the instructional unit. The results of the tasks should then be used to **inform** the instruction that will take place for the remainder of the unit. Formative Assessment Lessons are intended to support teachers in formative assessment. They both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

What does a Formative Assessment Lesson look like in action? Videos of Georgia Teachers implementing FALs can be accessed HERE and a sample of a FAL lesson may be seen HERE

Where can I find more information on FALs? More information on types of Formative Assessment Lessons, their use, and their implementation may be found on the Math Assessment Project's guide for teachers.

Where can I find samples of FALs?

Formative Assessment Lessons can also be found at the following sites:

Mathematics Assessment Project
Kenton County Math Design Collaborative
MARS Tasks by grade level

A **sample FAL** with extensive dialog and suggestions for teachers may be found HERE. This resource will help teachers understand the flow and purpose of a FAL.

Where can I find more training on the use of FALs?

All of our Georgia RESAs have had a math specialist trained to provide instruction on the use of formative assessment lessons in the classroom. The request should be made through the teacher's local RESA and can be referenced by asking for more information on the Mathematics Design Collaborative (MDC).
Also, if done properly, these lessons should take about 120-150 minutes, 2-3 classroom periods.
Sources of Information: Vicki Mixon, Former MDC (Math Design Collaborative) trainer, http://www.reneeyates2math.com/ and from The Mathematics Assessment Project and http://melissatabor.wikispaces.com/Formative+Assessment+Lessons+%28FALs%29 Division and Interpreting Remainders (FAL)

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Scaffolding Task: Equal to One Whole, More, or Less?

Back to Task Table

Students are given a collection of fractional parts (all the same type) and they need to indicate the kind of fractional part they have. The task is to decide if the collection of fractional parts is less than, greater than, or equal to one whole. Use at your discretion. This is review for those who need it.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.2 Solve word problems involving addition and subtraction of fractions, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < ½.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should understand how to compare fractions. They should be able to use a previously learned strategy (number line, cross multiplication, etc.) to compare fractions to another fraction or a whole number. They must also have an understanding of mixed numbers and their relationship to one whole.

COMMON MISCONCEPTIONS

Students often mix models when adding, subtracting or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when comparing fractions with thirds and fourths. Remind students that the representations need to be from the same whole models with the same shape and size.

ESSENTIAL QUESTIONS

- How can we tell if a fraction is greater than, less than, or equal to one whole?
- How can a fraction be greater than 1?

MATERIALS

- Paper
- Pencil

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- Accessible manipulatives

GROUPING
Whole/Individual/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:
Students need to be confident in their understanding of fractions greater than, equal to, and less than one whole. This task builds on understandings from Fourth Grade M3 NF 3. This task was adapted from Teaching Student Centered Mathematics, Grades 3-5, Van de Walle, John A. and Lovin, LouAnn H., pg. 138.

Teacher Notes:
Make sure there are a variety of math manipulatives in the collections students are using. Several templates are included at the end of this task, but any familiar manipulative may be used. For example, place three blue rhombi from a set of pattern blocks in a baggie with a note card that says, "These are fifths." Students need to decide whether these three fifths are more, less, or equal to one whole. Likewise, place 12 green triangles in a baggie with a card that says, "These are eighths."

Resist the temptation to only use one type of manipulative for this task, such as pattern blocks. Make some sets with pattern blocks as the fractional parts, some with color tiles, some with Cuisenaire Rods, some with (unlabeled) fraction strips, etc. Stay away from fraction circle pieces, as they are too much of a giveaway.

Task:
Students are given a collection of fractional parts (all the same type) and they need to indicate the kind of fractional part they have. The task is to decide if the collection of fractional parts is less than, greater than, or equal to one whole.

FORMATIVE ASSESSMENT QUESTIONS

- Justify and explain why your collection is less than one whole, equal to one whole or more than one whole.
- What if the pieces were halves instead of eighths? How would your answer change?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!

DIFFERENTIATION

Extension
- Allow students to tell how close the set is to a complete whole.

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- Allow students, who are ready, to create baggies. Students who are ready for this extension consistently identify the fractional parts in a general task above, and justify their thinking in using multiple strategies.

Intervention

- Allow students to work through a task that identifies the pieces as halves, thirds, or fourths. Students may need multiple experiences using different manipulatives. It is important for students to draw diagrams to represent the physical models before moving to a completely abstract algorithm to determine whether a fraction is greater, less, or equal to one whole.

*Intervention Table

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Scaffolding Task: Equal to One Whole, More, or Less?

Using the baggie with fractional parts with the card inside, decide if the collection is less than one whole, equal to one whole, or more than one whole. Do this for at least three different collections of fractional parts. Draw pictures, and/or use numbers to explain your answer.

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These are halves	These are thirds	These are fourths	These are fifths
These are sixths	These are sevenths	These are eighths	These are ninths
These are tenths	These are twelfths	These are sixteenths	These are eighteenths
These are twentieths	These are twenty-fifths	These are fiftieths	These are hundredths

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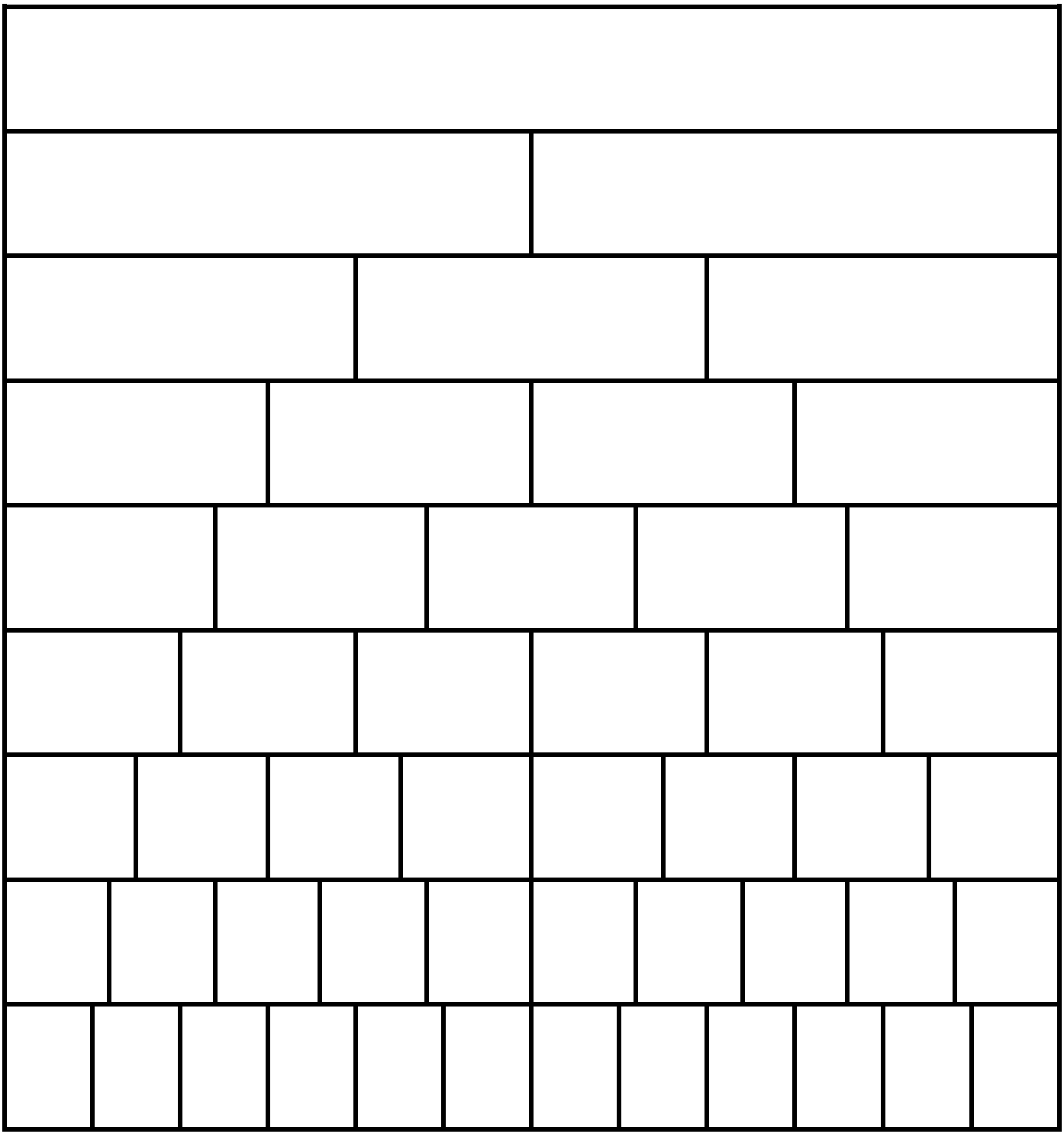

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Constructing Task: Sharing Candy Bars

Back to Task Table

Adapted from Contexts for Learning Mathematics Fractions, Decimals, and Percents by Fosnot, Catherine Twomey et.al.

Children learn mathematics by using what they know to make sense of new mathematical ideas. Equal sharing problems offer students the opportunity to use what they know about partitioning and division to learn fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. Example: $\frac{3}{5}$ can be interpreted as "3 divided by 5 and as 3 shared by 5".

MGSE5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Apply and use understanding of multiplication to multiply a fraction or whole number by a fraction.
   Examples: $\frac{a}{b} \times q$ as $\frac{a}{b} \times \frac{q}{1}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

MGSE5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students engaging in this task have an understanding of fair shares. If students lack this understanding, they will benefit from the previous task, as well as activities from Teaching Student Centered Mathematics, by John Van de Walle, pg. 136.

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COMMON MISCONCEPTIONS

Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to see that he results will be smaller. Additionally, students may believe that division always results in a smaller number. Using models when dividing with fractions will enable students to see that the results will be larger. **Students need to make sense of the numbers in the problem to decide which number is the numerator and which is the denominator, regardless of the size of the numbers. For example, the larger number is not always the dividend.**

ESSENTIAL QUESTIONS

- How can we determine how much someone gets in a fair-share situation if the fair share is less than 1?
- How can we determine how much someone gets in a fair-share situation if the fair share is between two whole numbers?
- Explain how fractions be used to illustrate fair shares?

MATERIALS

- Copy of Sharing Candy Bars task (1 per pair or small group),
- Pencil
- Accessible manipulatives and blank index cards to represent the candy bars (students will need these once they realize they must use the same sized whole to represent the candy bars.

GROUPING

Pair/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:

This task was developed from *Contexts for Learning Mathematics*, by Fosnot and Jacob. A recording sheet is provided, but is not necessary for this task, especially if students are using a math journal or learning log.

Teacher Notes:

Introduce the problem and be sure everyone is clear with the context. You may wish to use the pictures included at the end of this task to help develop this context.
Facilitate a preliminary discussion with the class, before students begin working on the problem. Allow students to share their initial thoughts, then ask them to work in pairs to investigate the following:
Was the distribution of candy bars fair – did everyone in the class get the same amount?
How much of a candy bar did each person get, assuming the pieces were cut equally?
Possible struggles students may have can be turned into wonderful inquiries! As students cut up the candy bars, you may notice them:

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Cutting each candy bar into a familiar fraction first, such as halves or thirds, then cut the leftovers into slivers. This strategy may cause struggles with what to name the pieces (what is 1/5 of 1/3? for example)

Cutting each candy bar into a number of pieces that is the same as the number of people in the group. For example, if 4 candy bars are shared among 5 people, each of the 4 candy bars is cut into 5 pieces. So, 1/5 of each candy bar goes to each of the 5 people. This may cause students to struggle with the idea that the size of the whole matters. Everyone gets 4/20 of the pieces, but this is also 4/5 of one candy bar.

Using the long division algorithm to find a decimal quotient ($4 \div 5 = 0.8$). This strategy may promote discussion, so please allow students the freedom to make sense of this in the closing part of the lesson.

FORMATIVE ASSESSMENT QUESTIONS

- Justify and explain why your collection is less than one whole, equal to one whole or more than one whole.
- How far away from a whole is your fraction? How do you know?
- What if the pieces were halves instead of eighths? How would your answer change?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!

This task is not enough for children to have a full understanding of fractions as division, nor is it enough for students to gain understandings of multiplication, division, or comparing fractions. It is an introduction to the idea of fractions in an equal sharing context, which opens the door for students to grapple with understanding fractions as division. It also allows for students to make sense of a real life situation that may promote inquiry into other mathematical ideas with fractions.

Questions for Teacher Reflection

- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- One positive thing about today's lesson and one thing you will change.

The following are instructional guidelines for creating more, perhaps similar equal sharing problems for fifth grade students.

Instructional guidelines for Equal Sharing Problems and Introducing Fractions (from Extending Children's Mathematics, Fractions and Decimals, Empson, Susan B., and Levi, Linda)

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- Equal sharing problems with answers that are mixed numbers and fractions less than 1. Focus on problems with 4, 8, 3, 6, 10, and 12 sharers, but include other numbers of sharers as well, such as 15, 20, and 100.
- Represent children's solutions with equations, with an emphasis on linking addition and multiplication and on equations that reflect a multiplicative understanding of fractions. For example, if students solved a problem about 8 children sharing 5 burgers you might write the following equations:
  - 1/8 + 1/8 + 1/8 + 1/8 + 1/8 = 5/8 ("Lura drew 5 hamburgers and gave each person an eighth of each hamburger. She put the pieces together and said that 1/8 plus 1/8 plus 1/8 plus 1/8 plus 1/8 is 5/8. Does this equation show what Lura did?)
  - 5 x 1/8 = 5/8 ("Shelly drew 1 hamburger and split it into 8 pieces. She said that each person would get 1/8 of this hamburger. The other hamburgers would look the same as this and she said 5 groups of 1/8 is the same as 5/8.")
  - 5 divided by 8 is 5/8 ("Krystal said that she knows that when 5 things are shared by 8 people, each person gets 5/8.")
- Represent the word problem situation using equations.
- 8 children are sharing 5 hamburgers equally. How much hamburger does one child get?
  - 5 ÷ 8 = ☐
  - 8 x ☐ = 5

DIFFERENTIATION

Extension

- Allow students to investigate other shares and sharers as identified above. To challenge students, especially with large numbers of sharers, insist that students represent their fractions in multiple ways. For example, our team of 100 5th grade students is sharing the challenge of running a 40 mile race for charity. How many miles is each student's responsibility? Students could shade in a 10x10 grid, show the fraction as 40/100 and (0.40), then show it again as 4/10 (0.4) and again as 2/5.
- After enough time has been devoted to the task, ask pairs of students to make posters to prepare for the closing of the lesson. Posters should be clear for others in the class to understand their thinking, but should not just be the figuring that was initially done copied over again. The posters should be clear and concise presentations of any important ideas and strategies students wish to present.
  Some ideas to encourage discussion about in the presentations of student work:
  - The size or amount of the whole matters
  - With unit fractions, the greater the denominator, the smaller the piece is
  - When naming the piece, the whole matters

Intervention

- Use smaller numbers of sharers. For example, give students one or two candy bars that have 2-3 sharers. The use of student created or commercial manipulatives, with teacher guidance and questioning will help students develop the concept of fractions as division.

*Intervention Table

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TECHNOLOGY CONNECTION

http://calculationnation.nctm.org/Games/ this site, from NCTM, has engaging and sometimes addictive games for practicing calculations based on strategy.

http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

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Sharing Candy Bars

A fifth grade class is split into four groups. Students in the class brought in candy bars for a fraction celebration. When it was time for the celebration, the candy bars were shared as follows:

The first group had 4 people and shared 3 candy bars equally.
The second group had 5 people and shared 4 candy bars equally.
The third group had 8 people and shared 7 candy bars equally.
The fourth group had 5 people and shared 3 candy bars equally.

When the celebration was over the children began to argue that the distribution of candy bars was unfair, that some children got more to eat than others. Were they right? Or, did everyone get the same amount?

Four people share these three candy bars.

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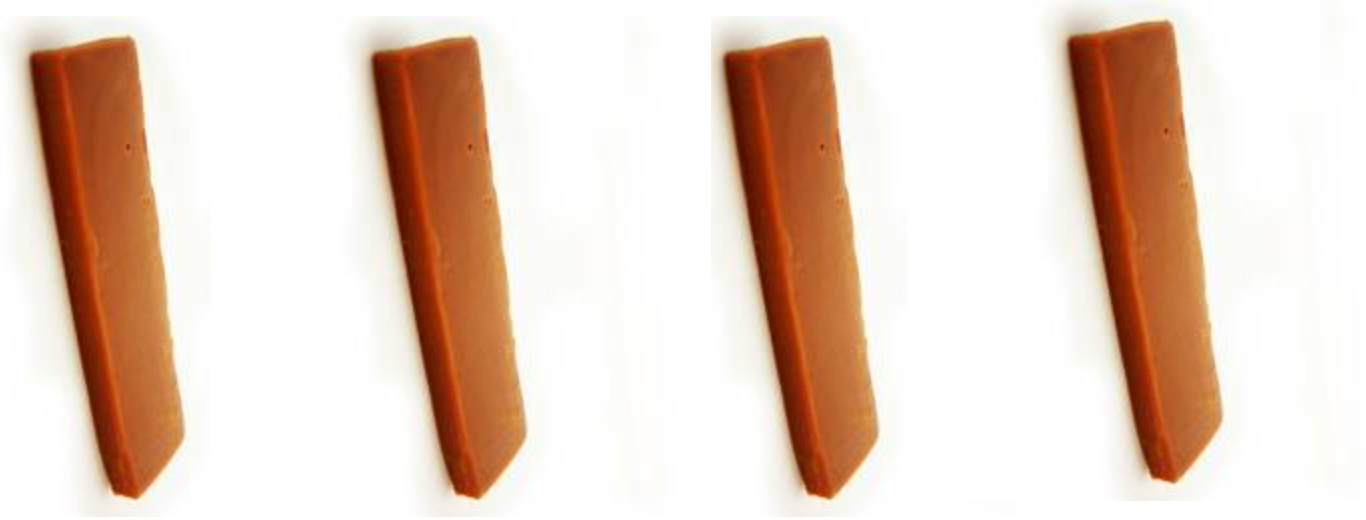
Five people share these four candy bars.



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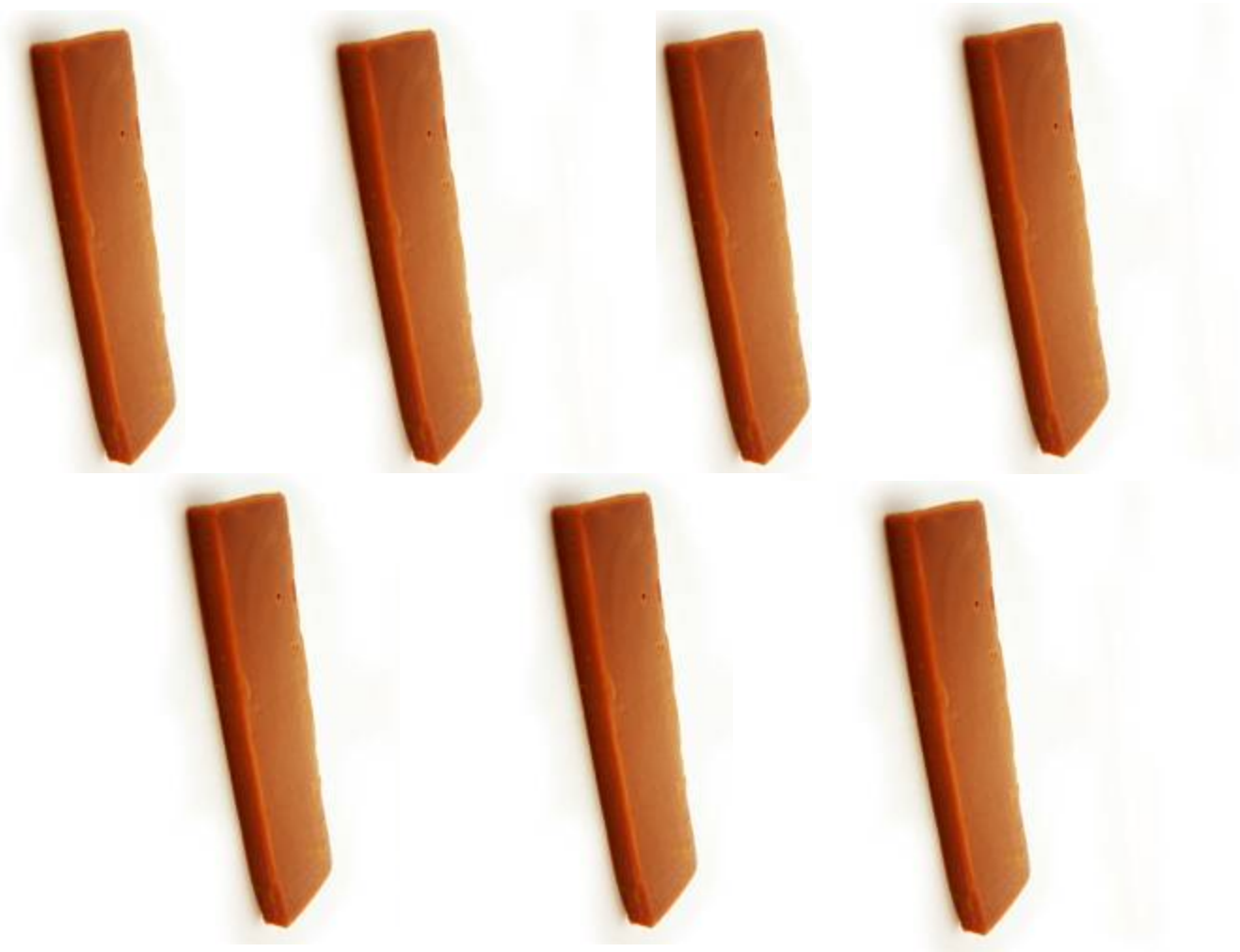
Eight people share these seven candy bars.



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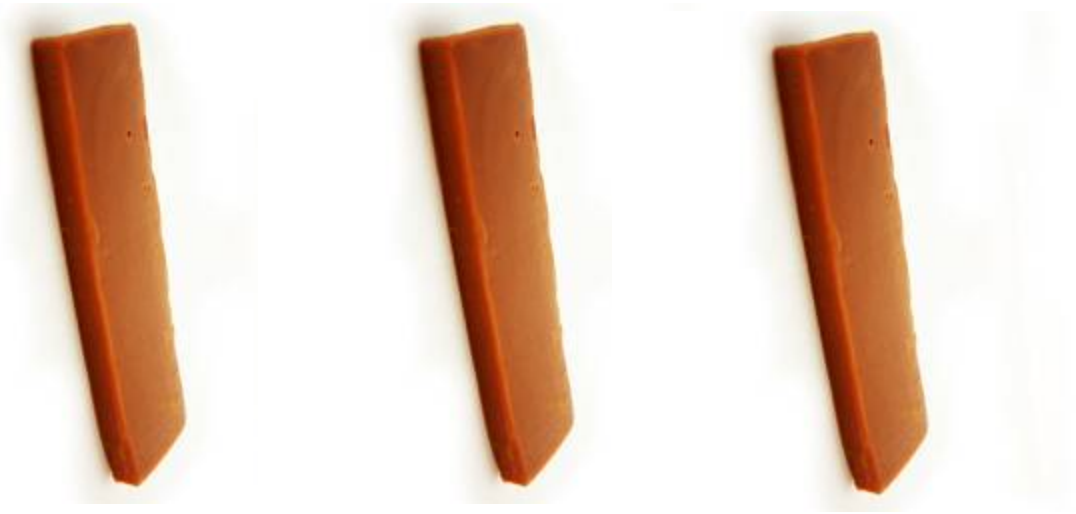
Five people share these three candy bars.



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Constructing Task: Sharing Candy Bars Differently Back to Task Table

Adapted from Contexts for Learning Mathematics Fractions, Decimals, and Percents by Fosnot, Catherine Twomey et.al.

This task continues where the previous task left off. Many students may be curious about whether or not there is a way to share the candy bars equally before you even introduce this part of the lesson. If this is the case, let them think that they're driving the lesson. It will probably promote more inquiry in later tasks. The lesson begins with a quick series of computation problems that promote partial products with whole numbers and fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. Example: $\frac{3}{5}$ can be interpreted as "3 divided by 5 and as 3 shared by 5".

MGSE5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Apply and use understanding of multiplication to multiply a fraction or whole number by a fraction.
   Examples: $\frac{a}{b} \times q$ as $\frac{a}{b} \times \frac{q}{1}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

MGSE5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
(for descriptors of standard cluster, see beginning of unit)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students engaging in this task have an understanding of fair shares. If students lack this understanding, they will benefit from the previous task, as well as activities from Teaching Student Centered Mathematics, by John Van de Walle, pg. 136.

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COMMON MISCONCEPTIONS

Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to see that he results will be smaller. Additionally, students may believe that division always results in a smaller number. Using models when dividing with fractions will enable students to see that the results will be larger. **Students need to make sense of the numbers in the problem to decide which number is the numerator and which is the denominator, regardless of the size of the numbers. For example, the larger number is not always the dividend.**

ESSENTIAL QUESTIONS

- How can we determine how much someone gets in a fair-share situation if the fair share is less than 1?
- How can we determine how much someone gets in a fair-share situation if the fair share is between two whole numbers?

MATERIALS

- Copy of Sharing Candy Bars task (1 set of four pages per pair or small group)
- Pencil
- Accessible manipulatives and index cards to represent candy bars.

GROUPING

Pair/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task was developed from Contexts for Learning Mathematics, by Fosnot and Jacob. A recording sheet is provided, but is not necessary for this task, especially if students are using a math journal or learning log.

Teacher Notes:

Have the computation discussion with students and draw empty arrays to show their thinking. See below.

10 x 128
2 x 128
128 x 12
33 x 10
33 x 9
3 x 1/5
7 x 1/8
3 x ¼
4 x 1/5

These four problems were chosen because they are the same numbers used in the previous task.

For the problem 128 x 2, if a student said, “I did 125 x 2 and got 150, then did 3 x 2 and got 6. 150 + 6 = 156.” You would draw something similar to the open array below.

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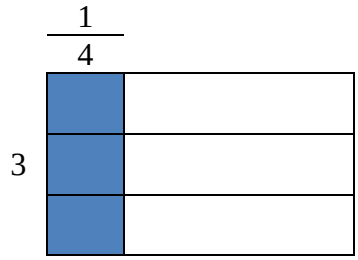
	125	3
2	250	6

The same thing would be true for the computation discussions involving fractions:
For the problem 3 x ¼, if a student said, "I did ¼ three times and got ¾, you might draw something like the following:



Review what was learned from the previous task. Give time for students to make sense and comments on others' work. Introduce the problem and be sure everyone is clear with the context. Use the pictures from the last task to include at the end of this task to help develop this context. Facilitate a preliminary discussion with the class, before students begin to work on the problem. Allow students to share their initial thoughts, then ask them to work in pairs to investigate the following:

Was the distribution of candy bars fair – did everyone in the class get the same amount?

How much of a candy bar did each person get, assuming the pieces were cut equally?

Ask students to investigate if it would be fairer if groups the first and third groups combined and shared their candy bars, and if the second and fourth groups combined and shared their candy bars. Place the papers from the last task together as stated above.

Encourage student struggles to become exciting challenges! As students cut up the candy bars, you may notice them:

1. Cutting halves and using landmark fractions.
   For groups 2 and 4, there are 7 candy bars for 10 people. Students might cut 5 candy bars in half and cut the remaining candy bars into fifths, resulting in ½ + 1/5 per person.
   For groups 1 and 3, there are 10 candy bars for 12 people. Similarly, students may cut 6 candy bars in halves and cut the remaining candy bars into thirds. This would give each person ½ + 1/3 of a candy bar to each person.

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2. Using the idea of division represented as a fraction. Today they see that they have 7/10 and 10/12 to compare. They may not be able to use common denominators to compare, but they will have some of their own ways to compare and they should be encouraged to do this. For example, they may take the information from yesterday's lesson and decide to compare 3/5 and 4/5 to 7/10. Using connecting cubes or other manipulatives, students should be given the chance to make sense of the mathematics within the context of this problem. They may notice that 4/5 is the same as 8/10, and 3/5 is the same as 6/10, so 7/10 is right in the middle.

3. Using the long division algorithm to find a decimal quotient and then comparing. This strategy promotes nice equivalents for students to compare. It may be necessary to remind students who use this strategy that any decimals they find represent tenths.

4. It's possible that some students may begin to think of ratios such as $\frac{3\frac{1}{2}}{5}$

Do not discourage this, since it is mathematically true and is an equivalent relation. This kind of thinking should be supported, since it shows an ability to think flexibly and will help develop fraction sense.

- As students share ideas, listen for important ideas that may be worth spending extra time discussing, such as:
  - How to compare fractions
  - Interesting ways to redistribute candy bars
  - How they know that when combining groups of candy bars (numerators) and groups of people (denominators), you get a fraction in between.
- Look for:
  - Evidence of halves and unit fractions, and an understanding that the redistributing produces an amount in between.
  - Comparing fractions and realizing that redistributing produces a fraction in between the original two fractions.
  - Using division to produce decimals to compare the quantities.

- Final thoughts:
  - This investigation is fairer than the first, but is still not fair. Some students still get more candy than others.
  - The investigation does not need to end here. In fact, students may wonder if it would be fairer to share the 17 candy bars together with all of the 22 students. It is worth the time to do so.
  - Emphasize to students that it would not be efficient to cut the candy bars into 22 little pieces. Ask students if 17/22 is about ½, ¾, or 2/3? Where could one cut be made that would be a nice approximation?

Instructional Guidelines for Equal Sharing

from Extending Children's Mathematics, Fractions and Decimals, Empson, Susan B., and Levi, Linda

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- Equal sharing problems with answers that are mixed numbers and fractions less than 1. Focus on problems with 4, 8, 3, 6, 10, and 12 sharers, but include other numbers of sharers as well, such as 15, 20, and 100.
- Represent children's solutions with equations, with an emphasis on linking addition and multiplication and on equations that reflect a multiplicative understanding of fractions. For example, if students solved a problem about 8 children sharing 5 burgers you might write the following equations:
- 1/8 + 1/8 + 1/8 + 1/8 + 1/8 = 5/8 ("Lura drew 5 hamburgers and gave each person and eighth of each hamburger. She put the pieces together and said that 1/8 plus 1/8 plus 1/8 plus 1/8 plus 1/8 is 5/8. Does this equation show what Lura did?)
- 5 x 1/8 = 5/8 ("Shelly drew 1 hamburger and split it into 8 pieces. She said that each person would get 1/8 of this hamburger. The other hamburgers would look the same as this and she said 5 groups of 1/8 is the same as 5/8.")
    - 5 divided by 8 is 5/8 ("Krystal said that she knows that when 5 things are shared by 8 people, each person gets 5/8.")
- Represent the word problem situation using equations.
    - 8 children are sharing 5 hamburgers equally. How much hamburger does one child get?
        - 5 ÷ 8 = ☐
        - 8 x ☐ = 5

FORMATIVE ASSESSMENT QUESTIONS

- Justify and explain why your collection is less than one whole, equal to one whole or more than one whole.
- How far away from a whole is your fraction? How do you know?
- What if the pieces were halves instead of eighths? How would your answer change?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!

DIFFERENTIATION

Extension

- Allow students to investigate other shares and sharers as identified above. To challenge students, especially with large numbers of sharers, insist that students represent their fractions in multiple ways. For example, our team of 100 5th grade students is sharing the challenge of running a 40 mile race for charity. How many miles is each student's responsibility? Students could shade in a 10x10 grid, show the fraction as 40/100 and (0.40), then show it again as 4/10 (0.4) and again as 2/5.
- After enough time has been devoted to the task, ask pairs of students to make posters to prepare for the closing of the lesson. Posters should be clear for others in the class to understand their thinking, but should not just be the figuring that was initially done copied over again. The posters should be clear and concise presentations of any

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important ideas and strategies students wish to present. The method they used to compare the situations and their justifications for why they think the combined group sharing is fairer (or not) should be included in their poster.

Some ideas to encourage discussion about in the presentations of student work:

- The size or amount of the whole matters
- With unit fractions, the greater the denominator, the smaller the piece is
- When naming the piece, the whole matters

Intervention

- Use smaller numbers of sharers. For example, give students one or two candy bars that have 2-3 sharers. The use of student created or commercial manipulatives, with teacher guidance and questioning, will help students develop the concept of fractions as division.

*Intervention Table

TECHNOLOGY CONNECTION

http://calculationnation.nctm.org/Games/ this site, from NCTM, has engaging and sometimes addictive games for practicing calculations based on strategy.

http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

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Sharing Candy Bars Differently

After looking at your first investigation, perhaps there is a way to make this sharing of candy bars more fair. Do you think it would be fairer if groups 1 and 3 combined and shared, and groups 2 and 4 combined and shared?

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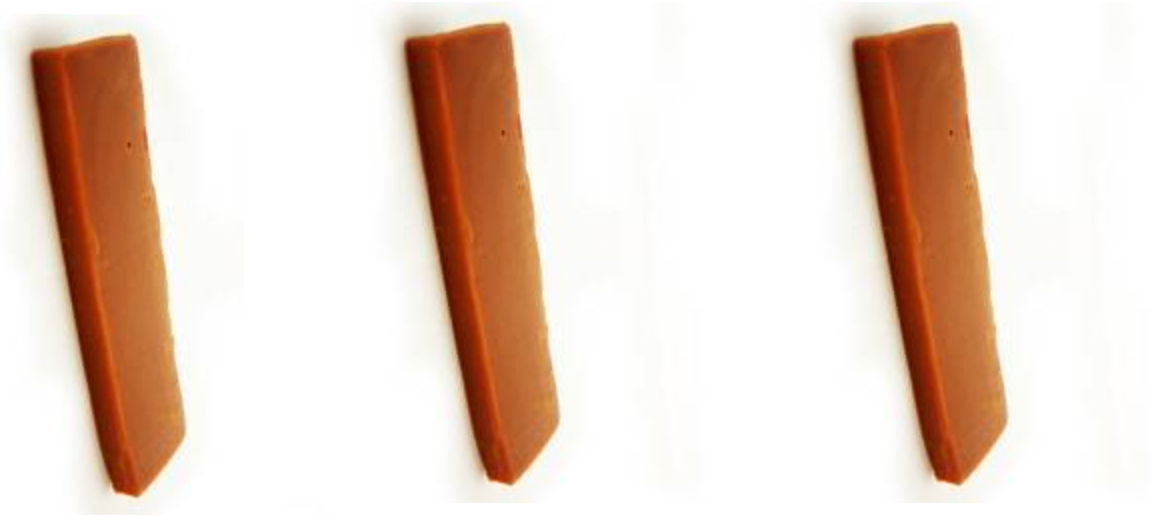
Group 1. Four people share these three candy bars.



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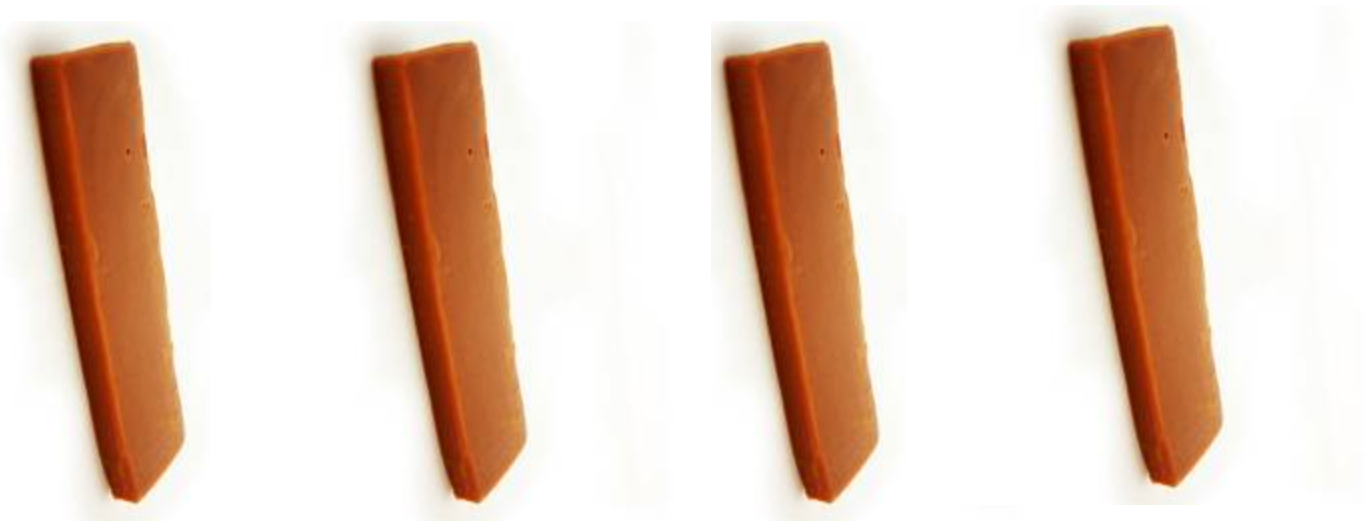
Group 2. Five people share these four candy bars.



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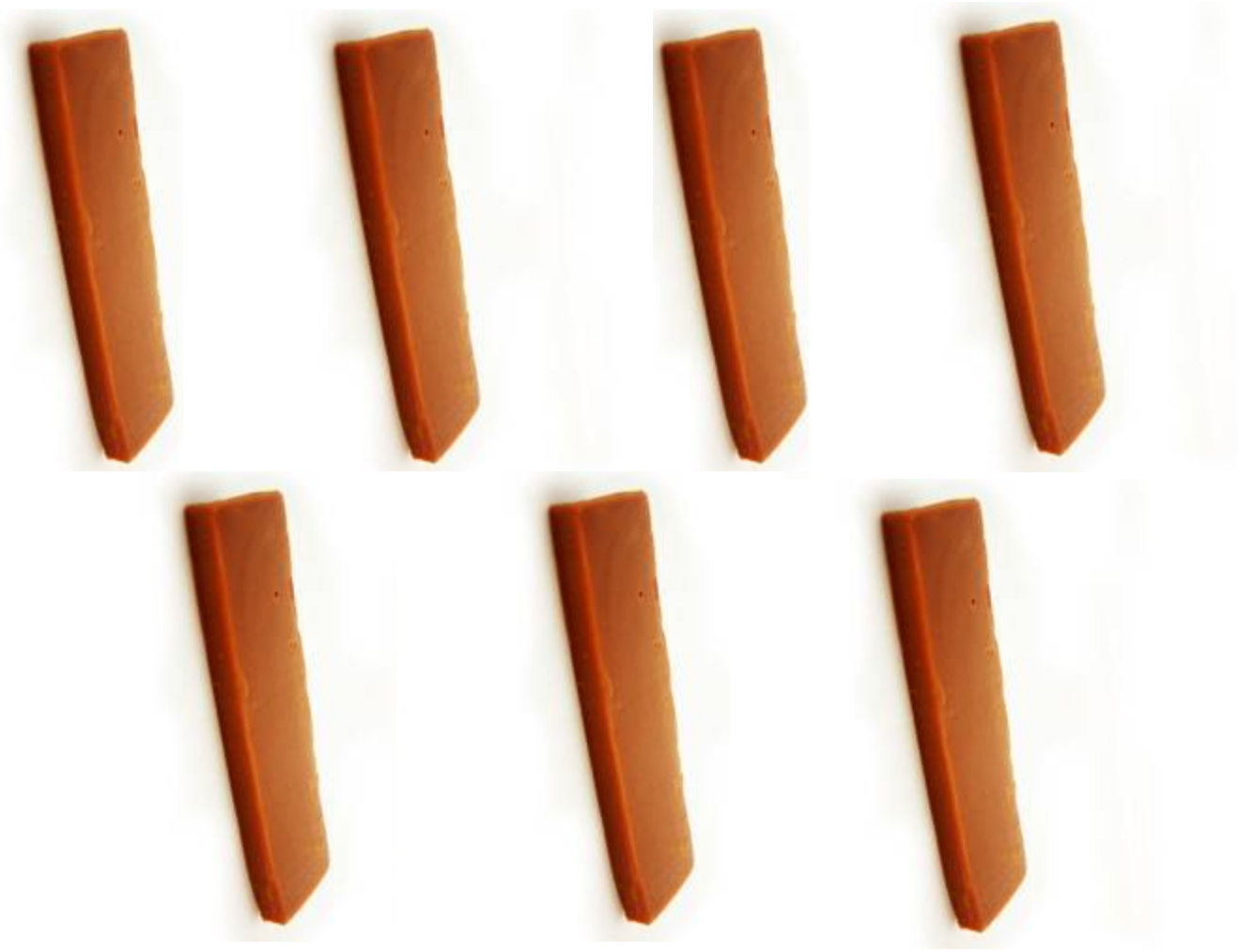
Group 3. Eight people share these seven candy bars.



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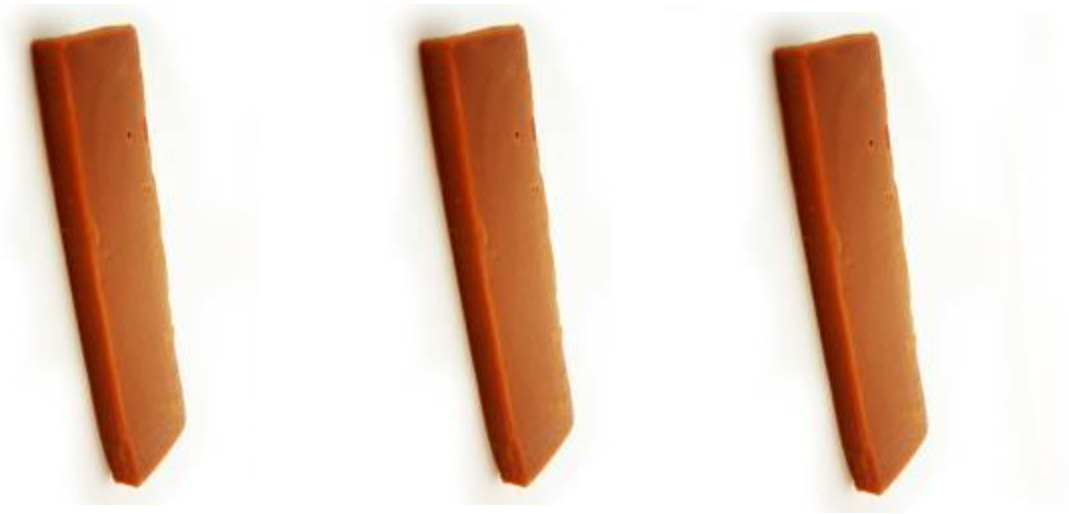
Group 4. Five people share these three candy bars.



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Performance Task: The Hiking Trail

Back to Task Table

Adapted from Contexts for Learning Mathematics Fractions, Decimals, and Percents by Fosnot, Catherine Twomey et.al.

This task develops the concept of equivalent fractions students will need to add fractions with unlike denominators later in the unit. The purpose of this task is to encourage student development of strategies to find equivalent fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. Example: $\frac{3}{5}$ can be interpreted as "3 divided by 5 and as 3 shared by 5".

MGSE5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Apply and use understanding of multiplication to multiply a fraction or whole number by a fraction.
   Examples: $\frac{a}{b} \times q$ as $\frac{a}{b} \times \frac{q}{1}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

This task was developed from *Contexts for Learning Mathematics*, by Fosnot and Jacob. Students engaging in this task have a deep understanding of fractions and the beginnings of fraction sense fostered in previous tasks. If students need additional support in developing this fraction sense, support students with activities from *Teaching Student- Centered Mathematics*, by John A Van de Walle and LouAnn Lovin., pgs. 144 – 146 (activities 5.6 – 5.10).

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COMMON MISCONCEPTIONS

Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to see that the results will be smaller. Additionally, students may believe that division always results in a smaller number. Using models when dividing with fractions will enable students to see that the results will be larger.

ESSENTIAL QUESTIONS

- How can a number line help us to compare fractions?
- How are a ruler and a number line related?

MATERIALS

- Copy of the Task The Hiking Trail (1 per pair of students or small group)
- Pencil
- Ruler (60 inch tape measure or yardstick)
- Accessible manipulatives

GROUPING

Pair/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task was adapted from *Contexts for Learning Mathematics Fractions, Decimals, and Percents* by Fosnot, Catherine Twomey et.al.

Teacher Notes:

Present one computation at a time and facilitate a discussion with students, asking them to find and explain the strategy they used to find:

100/2
100/4
200/4
200/8
400/16

Introduce the problem and be sure everyone is clear with the context.

Let students know that they will be designing a hiking trail for a four day Hike-a-thon. The trail is 6.0 km and is in the GA Mountains. The committee has decided what kind of informational markers and how often they should be placed. Your task is to figure out where to put informational markers along the way.

- A Camping area and Food Wagons should be at each fourth of the trail.
- Resting Points should be at every eighth of the trail.
- Water Stations should be at every tenth of the trail.

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- Juice and Snack Tables should be at every fifth of the trail.
- Recycling and Trash Bins should be placed at every marker
- Kilometer Markers should be placed along the trail, so that hikers know how much of the course they've completed. These markers should be placed at every twelfth, sixth, and half of the course, as well as at all of the other locations above. These markers should show how many kilometers have been completed.

Give pairs of students sixty inch measuring tapes or yard sticks. Have the pairs draw a sixty inch hiking trail on some butcher paper.

Students may use a variety of strategies including, but not limited to:

- Halving. They may take half of the halves to find fourths, and take half of the fourths to find eighths.
- Dividing by the denominator. Students may think of 1/5 of 6.0 = 6.0/5
- Adding parts. Students may think about 3/8 as 1/8 more than 2/8.
- Use equivalence ideas developed in the ratio table task earlier. Students may say 6/8 = ¾ since 3 ***x 2*** = 6 and 4 ***x 2*** = 8

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your map is accurately drawn to scale? Explain.
- Justify why your map has the best marker placement for the hiking trail.
- How do you know that marker goes there? Show me your thinking.
- How can you tell that your markers are in the correct place? Is there another way to think about this?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain what you have found!

After enough time has been devoted to the task, hang the work around the room and have students taken some time to view and make comments on others' work. Students may ask questions, or make mathematical commentary on post-it notes and stick them to the work. Pay attention to students' talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share.

When students have finished the tour, come back to the large group and begin the closing of the lesson. The goal of this closing is to help students make generalizations about equivalent fractions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work.

DIFFERENTIATION

Extension

- Students who are ready for an extension of this lesson can connect it to geography by using a map of the GA Mountains and using map/math skills to highlight a 6.0 km trail.

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Intervention

- Students requiring intervention should have access to manipulatives and, like all other students, share their thinking. All misconceptions are potential learning points for all students. In addition, students requiring intervention should be given a distance of 60 km, rather than 6.0 km, to build their understanding of fractions of whole numbers without the potential decimal as a response. In the closing, the connection between 60 km and 6.0 km (as well as the solutions to the problem) should be made by students through careful teacher questioning.

*Intervention Table

TECHNOLOGY CONNECTION

http://www.bbc.co.uk/schools/ks2bitesize/maths/number/ a lot of games and activities for students to use to practice working with fractions and decimals as well as whole numbers.

http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

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The Hiking Trail

You will be designing a hiking trail for a four day Hike-a-thon. The trail is 6.0 km and is in the GA Mountains. The committee has decided what kind of informational markers and stations are needed and how often they should be placed. Your task is to figure out where to put informational markers and stations along the way. Students must draw a map of the hiking trail, and make it to scale. *(1 kilometer = 10 inches on the map scale)*

- A Camping area and Food Wagons should be at each fourth of the trail.
- Resting Points should be at every eighth of the trail.
- Water Stations should be at every tenth of the trail.
- Juice and Snack Tables should be at every fifth of the trail.
- Recycling and Trash Bins should be placed at every marker
- Kilometer Markers should be placed along the trail, so that hikers know how much of the course they've completed. These markers should be placed at every twelfth, sixth, and half of the course, as well as at all of the other locations above. These markers should show how many kilometers have been completed.

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3 ACT TASK: The Black Box

Back to Task Table

Task adapted from Andrew Stadel (http://www.101qs.com/1772)

APPROXIMATE TIME: 1 class period

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will watch a video of two fraction models entering a black box and one fraction model exiting the black box. After watching two examples entering and exiting the black box, students will see a third pair of model entering the black box, but the video fades to black before they can see what fraction exits the black box. Students will discuss with a partner or small group what the black box is doing to the two fraction models each time. Information for the problem will be inferred by the students so it is necessary that they persist in determining the information that is known after watching the video, what is unknown and what tools will help them in finding a solution to the problem.

3. Construct viable arguments and critique the reasoning of others. Using the models provided in Act 1, students will explain what is happening to the fraction models while they were in the black box. Students will listen to one another and talk about their thinking, as well as ask questions of one another as they solidify their understanding of the idea that the fractions are being combined, or joined after they enter the black box.

4. Model with mathematics. Students will create numerical representations for the pictorial fraction representations shown in Act 1. As they find solutions, students check for reasonableness of results. They also make modifications to the models as needed as they try determine what the model will look like when $\frac{1}{2}$ and $\frac{1}{3}$ are combined. In Act 4, students can create their own pictorial fraction representations to make a given sum.

6. Attend to precision. As they construct arguments when interpreting the pictorial fraction representations, students will need to use precise language and vocabulary as they communicate with others.

7. Look for and make use of structure. When viewing the video during Act 1 and the images in Act 2, students will compare and contrast the representations and the results when they exit the black box in order to find a pattern or structure that helps make sense of what is happening to the fractions. Students will use that structure in Act 4 to help create their own equation for the black box.

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ESSENTIAL QUESTIONS

During Act 1, students view a video of pictorial fraction representations that enter a black box and exit the black box as a combination of the two fractions. Students must infer that the fractions are being combined because the video doesn't give this information. It is imperative that teachers allow students to ask questions of each other and participate in discussion that will lead the students to infer that information during Act 2. The essential questions below can be shared at the beginning of Act 2 to define the emphasis of the problem solving opportunity being presented.

- What connections can be made between pictorial fraction models and equations that use fractions?
- What role do equivalent fractions play in addition and subtraction of fractions?
- What strategies are most effective for adding and subtracting fractions with unlike denominators?

MATERIALS

Act 1 video "The Black Box 2" http://www.101qs.com/1772
Student recording sheet
Act 2 images (Use the link above to locate images.)
Act 3 video (Use the link above to locate the video.)

GROUPING

Whole group, partners or small groups

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Students will watch a video of two fraction models entering a black box and one fraction model exiting the black box. After watching two examples entering and exiting the black box, students will see a third pair of model entering the black box, but the video fades to black before they can see what fraction exits the black box. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer one of the questions generated on the chart. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

BACKGROUND KNOWLEDGE:

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion

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and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org and the K-5 MGSE Mathematics Wiki.

Through this task, students will apply fraction knowledge learned in third and fourth grades. In third grade, students began their fraction journey by learning about unit fractions, which are used in the video. Fourth grade students learn about adding and subtracting fractions with like denominators, which is also present in the Act 1 video. $\frac{1}{2}$ is added to $\frac{1}{2}$ to get $\frac{2}{2}$. $\frac{1}{3}$ is added to $\frac{1}{3}$ to get the sum $\frac{2}{3}$. When learning to add and subtract fractions in fourth grade, an emphasis is placed on making sure student understand that the denominators are not added because the size of the pieces are not changing. It is important for students in fifth grade to build on this prior knowledge as they embark on learning how fractions with different sized pieces can be combined or separated by making the pieces the same size.

COMMON MISCONCEPTIONS:

Students might think that a fraction is two separate numbers, especially if the terms numerator and denominator are introduced at an early age. Students need to understand that a fraction is a quantity and represents an amount.

Additionally, students apply generalizations about whole number addition and subtraction to fraction addition and subtraction. That is oftentimes why students add and subtract both the numerator and denominator when adding or subtracting fractions. An underlying problem exists with the aforementioned idea of students understanding fractions as quantities instead of two separate numbers. For example, when adding $\frac{3}{8}$ and $\frac{1}{2}$, the pieces in each fraction are different sizes. This makes the two fractions difficult to compare. Instead of immediately try to find a solution to the problem, fifth graders need to be stop and make meaning of the equation by thinking, "The fractions do not have the same size pieces. I know that half of eight is four, so I can make the pieces in the problem the same size by changing the halves to eighths." Students can also estimate to make meaning of the equation. If a student understands $\frac{3}{8}$, she will think, "$\frac{3}{8}$ is a little less than half, so when I add $\frac{3}{8}$ to $\frac{1}{2}$, I should get a little less than one. Thinking about the meaning and quantity of the fractions will steer students away from looking at the fraction as a set of two numbers.

Task Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I video or picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

- Show the Act 1 video "Black Box 2" to students. http://www.101qs.com/1772
- Ask students what they noticed in the video, what they wonder about, and what questions they have about what they saw in the video.

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- Give each student a copy of the Student Recording Sheet. Have students record their questions and curiosities in the Act 1 section that asks "What questions come to your mind?" Consider doing a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group. Students may need to watch the video several times.
- Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.
- Share the main question that will be investigated during today's lesson. In the list below it is denoted with an asterisk. (*) Students will record the main question on their recording sheet.
- Ask students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on a number line. Space is available on the recording sheet for students to record an open number line with all three estimates.

Anticipated questions students may ask and wish to answer:

- How much of the box will be filled in?
- *What happens to the fractions when they enter the black box?
- What fraction will come out of the black box?

**Main question(s) to be investigated*

Act 2 – **Student Exploration** - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

- During Act 2, students review the main question from Act 1 and decide on the facts, tools, and other information needed to answer the question. The main question for this task is "What happens to the fractions as they enter the black box?" When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem solving process that students decide what is needed without being given the information up front. Images of the fractions from the video are available for student viewing. Copies of the images can be given to the students at their request. The images can be found under "Act 2" using the link to the video.
- Students can record information that they need to solve the problem, given information, estimates and work on the student recording sheet under Act 2.
- The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
    - What is the problem you are trying to solve?
    - What do you think affects the situation?
    - Can you explain what you've done so far?

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- What strategies are you using?
- What assumptions are you making?
- What tools or models may help you?
- Why is that true?
- Does that make sense?

Additional Information for Act 2

Three images from Act 1 video http://www.101qs.com/1772

Important note: Although students will only investigate the main question for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 3 – Whole Group – Share solutions and strategies.

- Students present their solutions and strategies and compare them.
- Reveal the solution by showing the Act 3 video.
- Lead discussion to compare these, asking questions such as:
  - How reasonable was your estimate?
  - Which strategy was most efficient?
  - Can you think of another method that might have worked?
  - What might you do differently next time?

Act 4, The Sequel - "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

Give students a fraction, such as $\frac{7}{10}$ and have them come up with two fractions that enter the black box that will result in that fraction when it exits the black box. Students can design an image like the ones in Act 2 to share their solutions.

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?
- How is the sum of two fractions with unlike denominators determined?
- How are equivalent fractions used when adding fractions with unlike denominators?

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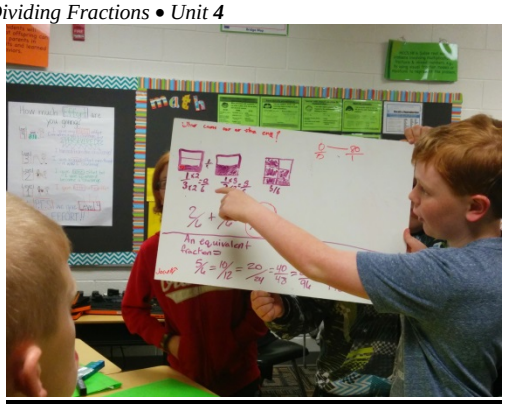

DIFFERENTIATION

Extension
Students can create a Black Box that finds the difference of the two fractions entering the box. Students can design slides like the images in Act 2. Allow students to pair with another student in class to share their Black Box with. The student's partner can determine what is happening to the fractions and what the result will be when the fractions exit the box.

Intervention
Students can create additional examples with like denominators that will review the content of learning how to add and subtract fractions with like denominators. After creating a few more examples with like denominators, students should engage in a discussion about the same size pieces in the visual models with like denominators. Students can use fraction bars to manipulate two unit fractions with unlike denominators and find the same sized pieces by create equivalent fractions. Then, after practicing two or three examples, students can find the sum of $\frac{1}{3}$ and $\frac{1}{2}$ and using same sized pieces.

*Intervention Table

TECHNOLOGY CONNECTIONS

- http://www.sheppardsoftware.com/mathgames/fractions/mathman_fractions_add_uncommon.htm Mathman: Adding Fractions with Unlike Denominators is a game similar to Pacman. Students find the sum of two fractions with unlike denominators and eat ghosts that contain the equivalent sum to earn points in the game.
- *http://www.visualfractions.com/add.htm A set of interactive tools that allows students to represent fraction addition with measurement (lines) and area (circles) models.
- *https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators Highly interactive tool that allows students/teachers to represent fraction addition with number lines and area models, and also allows the use to input a customized word problem.

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Three Act Student Recording Sheet Name _____

ACT 1

What questions come to your mind?

Main Question: _____

What is your first estimate and why? Record an estimate that is too low and an estimate that is too high. On an empty number line, record all three estimates made above.

ACT 2

What information would you like to know or need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimation with this information: _____

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Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

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Performance Task: The Wishing Club

Back to Task Table

Adapted from K-5 Math Teaching Resources

This task develops the concept addition of fractions using equivalent fractions with a variety of manipulatives.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

MGSE5.NF.2 Solve word problems involving addition and subtraction of fractions, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < ½.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students can use any representations they wish, but they will always use representations that are familiar to them. Pattern blocks make great fraction manipulatives, but they can be limiting since the imply part to whole more often than part to set (or group). It is necessary for students to have multiple models for fractions in order to facilitate flexibility in representations and computation.

Teacher Notes:

Part I

Read the story The Wishing Club, to your students, or if the book is not available, set up the problem by telling a story of four children who wished upon a star:

> The first boy was 4 and he wished for a dollar, but only received a quarter. His 2 year old brother wished for a cookie, but only got half. Their 8 year old twin sisters also received smaller parts of their wishes.

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(This may be a good time to set up a table with your students to see if they can find the pattern)

Age	part of wish
2	½
4	¼
8	?

Can they combine their parts to get a whole wish? Can they combine wishes to get a whole pet? They have to be careful and make sure they're correct (who wants half of a puppy?).

Show how you think they can get a full wish granted.

Allow pairs of (or small groups of) students to puzzle over the problem. Listen for students making sense of the context. Are students using manipulatives and/or making models. Are students using models that are easily broken into fractional parts?

Look for students who are relying heavily on models and manipulatives to solve the problem. Everyone should create a model for the problem as well as use equations, but during the closing part of the lesson, allow some of the students who relied heavily on manipulatives to share first. Allow students who used more abstract thinking (but who also model the problem and solution) to share last.

COMMON MISCONCEPTIONS

Students may add the denominators of the fractions. Students may create models that do not have the same size whole or come from sets of different sizes.

ESSENTIAL QUESTIONS

- When should we use models to solve problems with fractions?
- What connections can we make between the models and equations with fractions?

MATERIALS

- Accessible manipulatives
- Task Sheet
- *The Wishing Club* by Donna Jo Napoli

GROUPING

Pair/SmallGroup

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TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task develops the concept addition of fractions using equivalent fractions with a variety of manipulatives. It is important for students to solve multiple problems over time using concrete models and representing these models on paper. With the representations, it is important for students to write the abstract fraction equations. Students then must explain their thinking, representations, and models.

Comments:

This task was developed from the story *The Wishing Club*, by Donna Jo Napoli. A brief summary of Ms. Napoli's story is found in the teacher notes above, in case this wonderful book is not in your library.

The goal of this task is for students to make sense of and practice solving fraction problems with unlike denominators, where addition is the operation to use. The difficult part for the teacher is to not give students an algorithm (find common denominators) before the task. It is much more beneficial for students to struggle with the idea of how to go about combining two fractions with unlike denominators. This struggle is where the learning happens, but the teacher must provide facilitating questions at the end of the lesson to bring the students' own understanding to light. A student journal question, asking what students learned or what they are still struggling with when adding fractions with unlike denominators would make a good formative assessment and it would give students time to reflect on their own thinking (metacognition) and understanding.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answers are correct?
- How do you know that your model is labeled correctly?
- What other names do you think there are for some of the parts on your model?
- Did you identify any patterns or rules for adding these kinds of fractions? Explain what you have found.

After enough time has been devoted to the task, Each small group partners with other partners to make groups of 4-6 students. Appoint a facilitator to guide the group and make sure everyone shares their thinking. Pay attention to students' talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share with the whole group.

When students have finished, come back to the large group and begin the closing of the lesson. The goal of this closing is to help students make further generalizations about adding fractions and to help them become more fluent. Help students reach this goal, not by telling, but by asking thought provoking questions about the work and asking students to share in an order that moves from least efficient to most efficient, or mostly concrete to more representational and abstract.

Part II

Begin this lesson by reviewing the work from the previous lesson. Review any generalizations made and introduce the next part of the task.

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Make sure everyone understands the context of the task. Let students know that the task they completed previously was a warm-up for the next task. In this task, students will be asked to find other ages of family members that would allow them to make a wish and get a complete pet.

Ask students to share initial thoughts about whether they think the children in their family could wish for and get a whole pet. Ask them to share how they know (some may say they are an only child, so they couldn't get a whole animal, others may point out that they are the youngest, so the pieces everyone else gets are smaller than theirs).

Give pairs of students the task for the day and ask them to find several different ages of children that would allow a wish for a whole pet.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answers are correct?
- How do you know that your model is labeled correctly?
- What did you do to add the fractions together?
- Did you use any patterns or rules for adding these kinds of fractions? Explain what you have found.

After enough time has been devoted to the task, Combine student pairs to make groups of 4-6 students. Appoint a facilitator to guide the group and make sure everyone shares their thinking. Pay attention to students' talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share with the whole group.

When students have finished, come back to the large group and begin the closing of the lesson. The goal of this closing is to help students make generalizations about adding fractions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work and asking students to share in an order that moves from least efficient to most efficient, or mostly concrete to more representational and abstract.

DIFFERENTIATION

Extension

Students should work on contextual problems such as those found in Teaching Student Centered Mathematics, by John Van de Walle, pgs 167-172. Possible student representations are also presented in these pages.

Intervention

Students requiring intervention should also use contextual problems such as those found in Teaching Student Centered Mathematics, by John Van de Walle, pgs 167-172. Students should be talking their way through the problems with teacher support and questioning.

*Intervention Table

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TECHNOLOGY CONNECTION

http://www.counton.org/games/map-fractions/racing/ this is an interactive board game where players race bikes on a game board by adding fractions with like denominators. Students may choose from boards with $\frac{1}{2}$ & $\frac{1}{4}$ or $\frac{1}{2}$, $\frac{1}{4}$, & $\frac{1}{8}$.

http://www.counton.org/games/map-fractions/frosty/ this is an interactive three across board game where players add fractions with like and unlike denominators and place a virtual counter on the sum. The first to get three in a row wins the game. The model provided at the bottom of the game board is a powerful tool for helping children see common denominators and decompose fractions with common denominators.

http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

*http://www.visualfractions.com/add.htm A set of interactive tools that allows students to represent fraction addition with measurement (lines) and area (circles) models.

*https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators Highly interactive tool that allows students/teachers to represent fraction addition with number lines and area models, and also allows the user to input a customized word problem.

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The Wishing Club - Part I

Is there a way for the two year old, the four year old and the twin eight year old sisters to wish for one whole animal? Show your mathematical thinking.

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The Wishing Club - Part II

Can you think of any other combinations of ages in a family that would have allowed them to make a wish and get a whole pet? Show your mathematical thinking.

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Constructing Task: Fraction Addition and Subtraction

Back to Task Table

This task helps students develop the concept of addition with fractions using common and unlike denominators through the use of various manipulatives. It is important for students to solve multiple problems over time using concrete models and representing these models on paper. It is important for students to write the abstract fraction equations however students must be able to explain and justify their thinking through representations and models.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

MGSE5.NF.2 Solve word problems involving addition and subtraction of fractions, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < ½.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students can use any representations they wish (5), but they will always use representations that are familiar to them. Pattern blocks make great fraction manipulatives, but they can be limiting since they imply part to whole more often than part to set (or group). It is necessary for students to have multiple models for fractions in order to facilitate flexibility in representations and computation.

COMMON MISCONCEPTIONS

Students often mix models when adding, subtracting or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when comparing fractions with thirds and fourths. Remind students that the representations need to be from the same whole models with the same shape and size.

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ESSENTIAL QUESTIONS

- How are equivalent fractions helpful when solving problems?
- How can making equivalent fractions and using models help us solve problems?

MATERIALS

- Pencil
- Journal, Learning Log, Student Problem Solving Notebook, etc.
- Accessible manipulatives, rulers may be helpful in this task, but are not necessary.
- Copy of task.

GROUPING

Pair/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments:

The goal of this task is for students to make sense of a problem with unlike denominators, where addition is the operation to use. The difficult part for the teacher is to not give students an algorithm (find common denominators) before the task. It is much more beneficial for students to struggle with the idea of how to go about combining two fractions with unlike denominators. This constructive struggle is where the learning happens, but the teacher must provide facilitating questions at the end of the lesson to bring the students' own understanding to light. A student journal question, asking what students learned, or what they are still struggling with when adding fractions with unlike denominators, would make a good formative assessment and it would give students time to reflect on their own thinking (metacognition) and understanding.

Teacher Notes:

Part I

Cut a strip of paper, see fraction strips at the end of this task and identify/label this as the whole that students will use to complete the task. Cut about 5 more strips the same length. Present the students with the following equations and have them model the equation using blank fraction strips. Have students share their model.

¼ + ¼

1/3 + 1/3

2/3 + 2/3

Students are guided to use the paper strips, fold, cut and join, but some students may already know what the answers are. Encourage students to prove they are correct with the models and see if they can name some of these fractions in other ways. What fractions are equivalent to 1/2? (*show how you know with modeling)*What fractions are equivalent to 4/3? *(show how you know with modeling)* Use more examples if needed to increase students' understanding once students have created the strips, guide them to the next question which leads students to represent their models on a number line. Students may need guidance in creating the number line, locating

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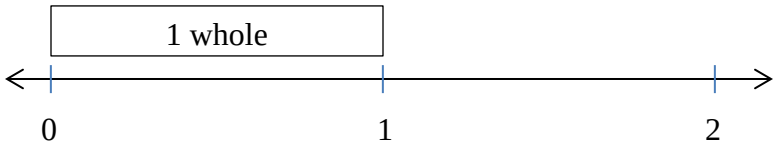
where 1 would be, etc. Have students locate all of their leftover and newly created fractions on the number line. See below:



FORMATIVE ASSESSMENT QUESTIONS

- Using the fraction strips, how can you tell that the fractions you created are equivalent?
- How do you know that your points on the number line are labeled correctly?
- What other names do you think there are for some of your points on the number line?
- Did you identify any patterns or rules for adding these kinds of fractions? Explain what you have found!

After enough time has been devoted to the task, join partners with other partners to create a new group of 4-6 students. Appoint a facilitator to guide the group and make sure everyone shares their thinking. Pay attention to students' talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share with the whole group.
When students have finished, come back to the large group and begin the closing of the lesson. The goal of the closing is to help students make generalizations about adding fractions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work and asking students to share in an order that moves from least efficient to most efficient, or mostly concrete to more representational and abstract.

Part II
Begin this lesson by reviewing the work from the previous lesson. Review any generalizations made and introduce the next part of the task.
Make sure everyone understands the context of the task. Let students know that the task they completed previously was a warm-up for the real task. In this task, student will be asked to do the same thing, but with fractions that are different. Show the fractions and equations:

Sam had 1/2 of a chocolate bar and John gave him 1/3 more of a chocolate bar. How much chocolate bar does he now have?
Sam had 2/3 of a chocolate bar and John gave him 1/2 more of a chocolate bar. How much chocolate bar does he now have?
Sam had 2/3 of a chocolate bar and John gave him 1/2 more of a chocolate bar. How much chocolate bar does he now have?

*****ALLOW TIME FOR A CONSTRUCTIVE STRUGGLE TO TAKE PLACE*****

Ask students what is different about these fractions/equations. Reflect back on the work from the last lesson again with questions like, "I remember when all we had was fourths: ¼ and ¼. That was TWO fourths. I wonder what we'll have to do to combine fractions that are not alike.

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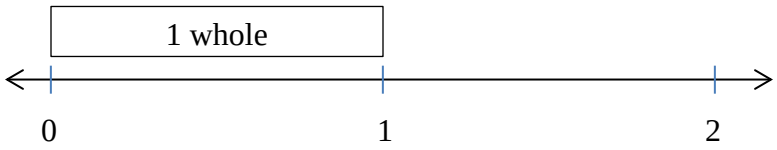
Any ideas? Brainstorm with your students and ask what they could do, since these fractions are not alike.

Students will not likely know any answers right away for this part, but having had the experience in part one, they will likely be successful with this. Students will likely be able to show fractions joined together as unlike denominators but unable to immediately identify the sum. Refer students to the previous lesson and allow them to share, again, what they learned about identifying equivalent fractions. Finding an equivalent fraction for the two parts combined can guide students to made meaningful connections and generalizations.

Students may ask for another strip to cut into smaller pieces. Encourage them to use the pieces they have to make what they need. This builds understandings of how fractions are related (1/2 of 1/3 is 1/6 or a 1/2 piece cut in three makes 3 1/6 pieces) and helps build a fractional number sense. Experiences like this, with teacher guidance and questioning, will also build fluency with fraction computation.

Once students have created the strips for each equation, guide them to the next question which leads students to represent their models on a number line. Some students may still need guidance in creating the number line, locating where 1 would be, etc. See below:



Once students have completed their number line, have them label the point with an equation (1/2 + 1/3) as well as a fraction (10/12 or 5/6).

After enough time has been devoted to the task, group partners with other partners to create a group of 4-6 students. Appoint a facilitator to guide the group and make sure everyone shares their thinking. Pay attention to students' talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share with the whole group.

When students have finished, come back to the large group and begin the closing of the lesson. The goal of the closing is to help students make generalizations about adding fractions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work and asking students to share in an order that moves from least efficient to most efficient, or mostly concrete to more representational.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answers are correct?
- How do you know that your points on the number line are labeled correctly?
- How can we change these problems into ones just like the easy ones we did in the last task?
- What did you learn about combining fractional parts with different denominators?

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- Is there one fraction name for your equation points on the number line?
- Did you identify any patterns or rules for adding these kinds of fractions? Explain what you have found!

DIFFERENTIATION

Extension

Students needing an extension should be given opportunities to investigate addition with fractions with different denominators. This is what they will do in part II, but simple problems with a context should be used and can be found in Teaching Student Centered Mathematics Vol. 2, Grades 3-5, by John A. Van de Walle on pg 162. Samples of possible student representations can also be found on pg 163.

Intervention

Students requiring intervention should use manipulatives and talk their way through the problem with teacher support and questioning.

*Intervention Table

TECHNOLOGY CONNECTION

http://www.bbc.co.uk/schools/ks2bitesize/maths/number/ a lot of games and activities for students to use to practice working with fractions and decimals as well as whole numbers.

http://www.counton.org/games/map-fractions/racing/ this is an interactive board game where players race bikes on a game board by adding fractions with like denominators.

http://www.counton.org/games/map-fractions/frosty/ this is an interactive three across board game where players add fractions with like and unlike denominators and place a virtual counter on the sum. The first to get three in a row wins the game.

http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

*http://www.visualfractions.com/add.htm A set of interactive tools that allows students to represent fraction addition with measurement (lines) and area (circles) models.

*http://www.visualfractions.com/subtract.htm A set of interactive tools that allows students to represent fraction subtraction with measurement (lines) and area (circles) models.

*https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators
*https://www.conceptuamath.com/app/tool/subtracting-fractions-with-uncommon-denominators
Highly interactive tool that allows students/teachers to represent fraction addition/subtraction with number lines and area models, and also allows the user to input a customized word problem.

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Fraction Addition and Subtraction
Part I

1. Cut out a strip of paper, and label it "1 whole." Cut 4 or 5 more of these same size strips. Fold, cut, and join the strips to create new strips that are the following lengths:

$\frac{1}{4} + \frac{1}{4}$ $\frac{1}{3} + \frac{1}{3}$ $\frac{2}{3} + \frac{2}{3}$

2. Use the strips you made in step 1, with diagrams and number lines to help you explain how to add and subtract fractions like these.

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Fraction Addition and Subtraction Part II

1. Cut out a strip of paper, and label it "1 whole." Cut 4 or 5 more of these same size strips. Fold, cut, and join the strips to create new strips that are the following lengths:

$\frac{1}{2} + \frac{1}{3}$ $\frac{2}{3} - \frac{1}{2}$ $\frac{2}{3} + \frac{1}{2}$

2. Use the strips you made in step 1, with diagrams and number lines to help you explain how to add and subtract fractions like these. How is adding (and subtracting) these kinds of fractions different than the fractions in Part I?

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Practice Task – Flip it Over

Back to Task Table

This task was developed to help students develop and use relationships between certain fractions for fraction computation. It is designed to allow students to develop these relationships for fluency and understanding of fractional computation.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can fractions with different denominators be added together?
- What strategies can we use for adding and subtracting fractions with different denominators?
- What models can we use to help us add and subtract fractions with different denominators?
- How do we use equivalent fractions to add and subtract fractions?

COMMON MISCONCEPTIONS

Students may think that they should add the numerators and then the denominators They may lack understanding about the relative value of the fractions themselves.

MATERIALS

- Game board
- Two counters per person
- Available manipulatives

GROUPING

Group/Partner Task

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TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task students will play a game to see who can flip over their cards first. This game will allow students to use their fractional understandings and build their fractional computation strategies. Logical thinking and problem solving skills will begin to develop as students devise strategies for playing the game.

Comments

Multiple fraction models, in addition to those included in the task, should be made available to the students as support for those who need it. In addition, fractional number lines (or open number lines) could benefit many students with this task.

This task could be introduced in a small group or by playing with class as a whole using available technology. In addition, the teachers could role model with a student, or better yet have two students role model how to play for the rest of the class.

Before asking students to work on this task, be sure students are able to:

- Use models to create equivalent fractions (see previous tasks and Teaching Student Centered Mathematics, volume 2, pg 155-156 (Slicing Squares).
- Understand that a whole can be written as a fraction with any number of parts as long as all of those parts are included to make the whole. For example, a whole can be cut into tenths. In order to have a whole, I need all ten tenths (10/10).
- Be able to decompose fraction, for example $^4/_4$ = ½ + ½ or ¼ + ¾
- *A wonderful game for reinforcing all of the foundational skills listed above is Fraction Capture: https://www.georgiastandards.org/Georgia-Standards/Documents/Foundations-of-Algebra/Day-2-Segment-10-Fraction-Capture-from-Everyday-Math.pdf

Task Directions

Students will follow directions below from the Create Three Game activity.

Play the game several times.
Keep track of computation strategies used (use models in explanations during the closing).
Keep track of game playing strategies used (if any develop this first time playing).

FORMATIVE ASSESSMENT QUESTIONS

- What fractions do you find easy to work with? Why?
- Which fraction do you like to spin? Why?
- What strategies do you use when playing this game?

Questions for Teacher Reflection

While planning the task:
What level of support do my struggling students need in order to be successful with this task?
In what way can I deepen the understanding of those students who are competent in this task?
Could this game be played again, or changed in any way?

During and after the students complete the task:

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Which students have developed a strategy based on fraction understandings of numerators and denominators?
Which students are becoming fluent in creating equivalent fractions when adding fractions?
Which students still prefer to use manipulatives and rely heavily on models?

DIFFERENTIATION

Extension

http://www.openmiddle.com/subtracting-fractions/
http://www.openmiddle.com/adding-fractions/
http://www.openmiddle.com/adding-mixed-numbers/
http://www.openmiddle.com/subtracting-mixed-numbers/

Intervention

Have students play Fraction Capture: https://www.georgiastandards.org/Georgia-Standards/Documents/Foundations-of-Algebra/Day-2-Segment-10-Fraction-Capture-from-Everyday-Math.pdf

*Intervention Table

TECHNOLOGY CONNECTION

http://www.counton.org/games/map-fractions/racing/ this is an interactive board game where players race bikes on a game board by adding fractions with like denominators.

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http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

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*https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators
*https://www.conceptuamath.com/app/tool/subtracting-fractions-with-uncommon-denominators
Highly interactive tool that allows students/teachers to represent fraction addition/subtraction with number lines and area models, and also allows the user to input a customized word problem.

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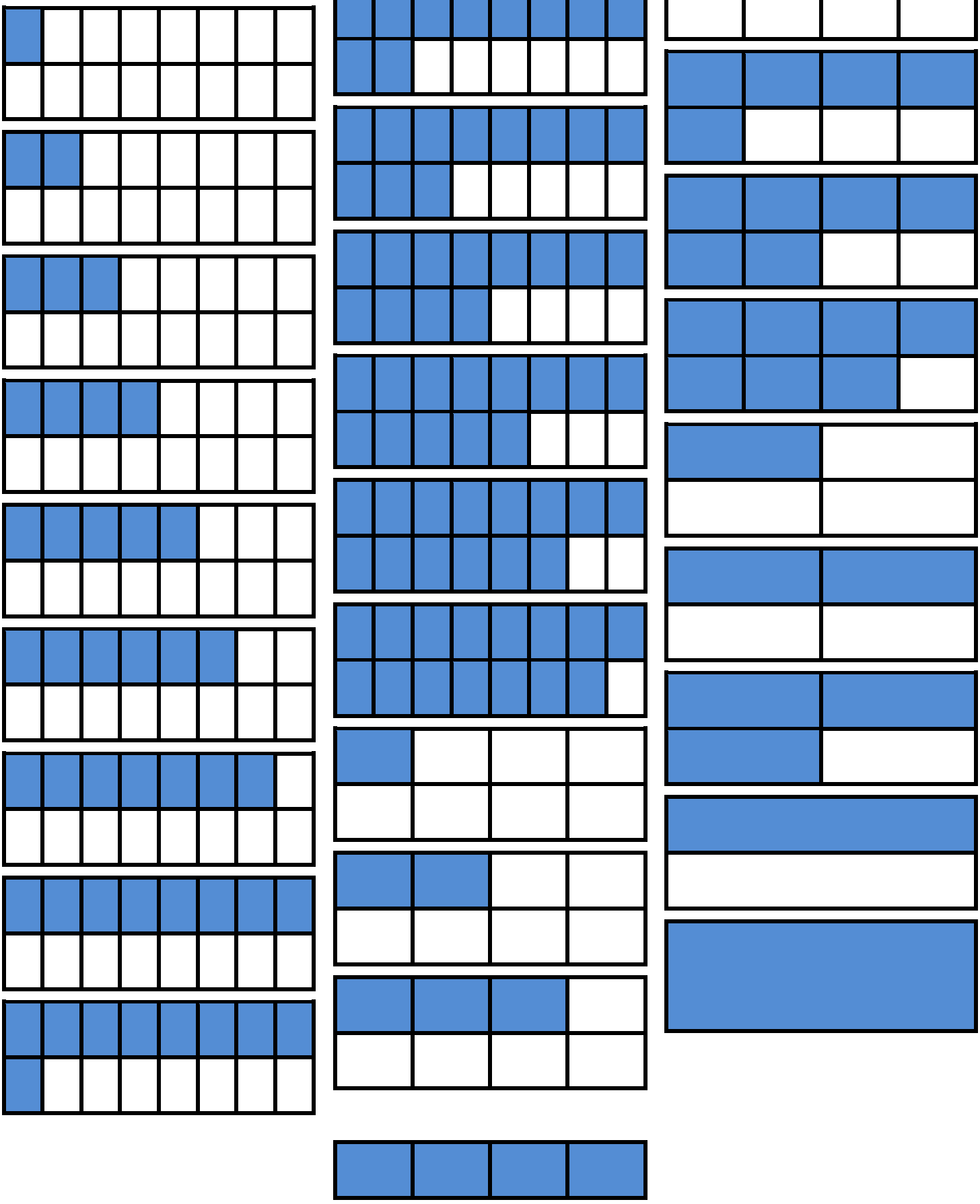

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$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$
$\frac{5}{16}$	$\frac{6}{16}$	$\frac{7}{16}$
$\frac{8}{16}$	$\frac{9}{16}$	$\frac{10}{16}$
$\frac{11}{16}$	$\frac{12}{16}$	$\frac{13}{16}$
1	$\frac{14}{16}$	$\frac{15}{16}$
$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$

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$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$
$\frac{2}{4}$	$\frac{3}{4}$	$\frac{17}{16}$
$\frac{9}{8}$	$\frac{5}{4}$	$\frac{3}{2}$
$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$
$\frac{6}{8}$	$\frac{7}{8}$	

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Fraction Operation Cards

$+\frac{1}{16}$	$+\frac{3}{16}$	$+\frac{5}{16}$
$+\frac{1}{8}$	$+\frac{3}{8}$	$+\frac{7}{16}$
$+\frac{5}{8}$	$+\frac{3}{4}$	$+\frac{9}{16}$
$+\frac{7}{8}$	$+\frac{1}{4}$	$+\frac{1}{2}$

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Flip it Over

Materials: One set of Fraction Picture Cards (attached), One set of Fraction Cards (attached), One set of Fraction Operation Cards, (attached).

Players: two or four players

Directions:

Set up the game:

- Deal all of the fraction number cards evenly to each player.
- Everyone places their fraction number cards face-up in front of them.
- Place the fraction picture cards in a deck, face down, in the center of the playing area.
- Place the set of fraction operation cards loosely in the center of the playing area.
- Decide who plays first.

Playing the game:

- The first player takes the top fraction picture card from the deck in the center, then chooses 3 operation cards.
- Player one chooses one or more of the operation cards to add to the picture card to make a sum equal to one of their fraction number cards.
- If you have a number card that matches the sum, flip it over.
- Return the used operation cards and the picture card to the center.
- Player two takes the left over operation card(s) takes some more to have three operation cards, then chooses a picture card
- Player two chooses one or more of the operation cards to add to the picture card to make a sum equal to one of their fraction number cards.
- If you have a number card that matches the sum, flip it over.
- Play continues in this way for each player.
- The first player to flip over all of his or her cards wins.

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Practice Task: Up and Down the Number Line Back to Task Table

This game was developed from a kindergarten counting game and adapted for use with fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

ESSENTIAL QUESTIONS

- How can fractions with different denominators be added together?
- What strategies can we use for adding and subtracting fractions with different denominators?
- What models can we use to help us add and subtract fractions with different denominators?
- What do equivalent fractions have to do with adding and subtracting fractions?

COMMON MISCONCEPTIONS

Students may not understand why you need to have like denominators before you add or subtract fractions. They may lack the concept of equivalence. Some students may not have had experience with a vertical number line and will need some introduction and practice with it.

MATERIALS

- Game board
- Two counters per person
- Available manipulatives

GROUPING

Partner Task

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TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task students spin a spinner to move their counters up and/or down the number line. The first to move to their side of the number line wins the game. This game will allow students to use their fractional understandings to add and subtract fractions, look for relationships between certain fractions, and as their fractional computation strategies develop, logical thinking and problem solving skills will begin to develop their game playing strategies.

Comments

Multiple fraction models should be made available to the students as support for students who need it. In addition, fractional number lines (other than the ones included with the task) could benefit many students with this task. Students could utilize blank (open) number lines as a strategy as well.

This task could be introduced in a small group or by playing with class as a whole using available technology. In addition, the teacher could role model with a student, or better yet have two students role model how to play for the rest of the class.

Before asking students to work on this task, be sure students are able to:

- Use models to create equivalent fractions (see previous tasks and Teaching Student Centered Mathematics, volume 2, pg 155-156 (Slicing Squares).
- Understand that a whole can be written as a fraction with any number of parts as long as all of those parts are included to make the whole. For example, a whole can be cut into tenths. In order to have a whole, I need all ten tenths (10/10).
- Be able to decompose fraction, for example $^{4}/_{4}$ = ½ + ½ or ¼ + ¾.

Task Directions

Students will follow directions below from the Create Three Game activity.

Play several rounds with each other.
Keep track of computation strategies used (use models in explanations during the closing).
Keep track of game playing strategies you think are helping you.

During and after the students complete the task:

Which students have developed a strategy based on fraction understandings of numerators and denominators?
Which students are becoming fluent in creating equivalent fractions when adding fractions?
Which students still prefer to use manipulatives and rely heavily on models?

FORMATIVE ASSESSMENT QUESTIONS

- What fraction would you like to spin first? Why?
- What strategies do you have for this game?
- If you could change the spinner, what fraction would you like to have on it?
- Do you notice any relationships between any of the fractions?
- Do you think other fractions may have similar relationships? What fractions might have this similar relationship? Explain.

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Questions for Teacher Reflection
While planning the task:
What level of support do my struggling students need in order to be successful with this task?
In what way can I deepen the understanding of those students who are competent in this task?
Could this game be played again, or changed in any way?
After the lesson:
How did my students engage in the 8 mathematical practices today?
How effective was I in creating an environment where meaningful learning could take place?
How effective was my questioning today? Did I question too little or say too much?
Were manipulatives made accessible for students to work through the task?
One positive thing about today's lesson and one thing you will change.

DIFFERENTIATION

Extension

http://www.openmiddle.com/subtracting-fractions/
http://www.openmiddle.com/adding-fractions/
http://www.openmiddle.com/adding-mixed-numbers/
http://www.openmiddle.com/subtracting-mixed-numbers/

Intervention

Have students play Fraction Capture: https://www.georgiastandards.org/Georgia-Standards/Documents/Foundations-of-Algebra/Day-2-Segment-10-Fraction-Capture-from-Everyday-Math.pdf

*Intervention Table

TECHNOLOGY CONNECTION

http://www.counton.org/games/map-fractions/racing/ this is an interactive board game where players race bikes on a game board by adding fractions with like denominators.

http://www.counton.org/games/map-fractions/frosty/ this is an interactive three across board game where players add fractions with like and unlike denominators and place a virtual counter on the sum. The first to get three in a row wins the game.

*http://www.visualfractions.com/add.htm A set of interactive tools that allows students to represent fraction addition with measurement (lines) and area (circles) models.

*http://www.visualfractions.com/subtract.htm A set of interactive tools that allows students to represent fraction subtraction with measurement (lines) and area (circles) models.

*https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators
*https://www.conceptuamath.com/app/tool/subtracting-fractions-with-uncommon-denominators
Highly interactive tool that allows students/teachers to represent fraction addition/subtraction with number lines and area models, and also allows the user to input a customized word problem.

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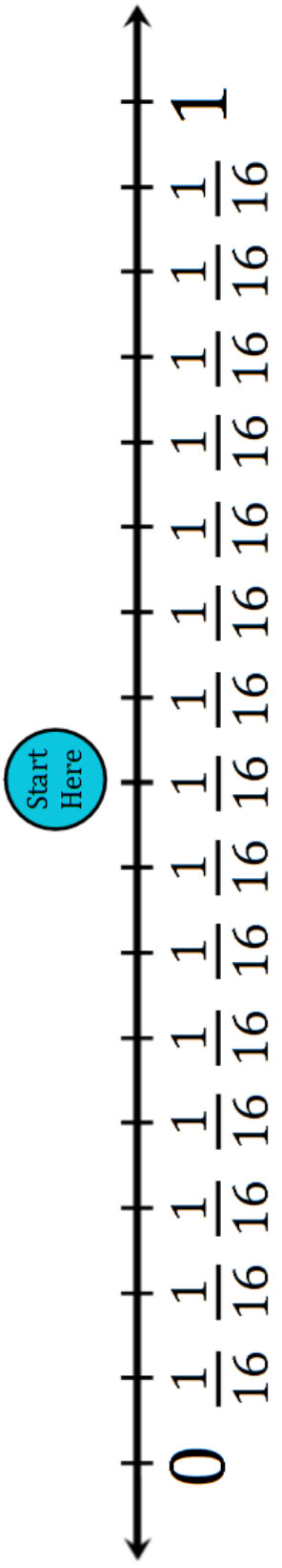
Directions: This game is for two players (or two teams of players). Both players place their counters on the start position. One player is trying to get to get to the one on the number line. The other player is trying to get to the zero on the number line. Player one spins the spinner and moves that distance on the number line from the starting position. Player two then spins and moves the distance shown on the spinner. If the distance on the spinner is too large, the player must move that distance in the opposite direction. The first person to reach their number exactly, wins the game.



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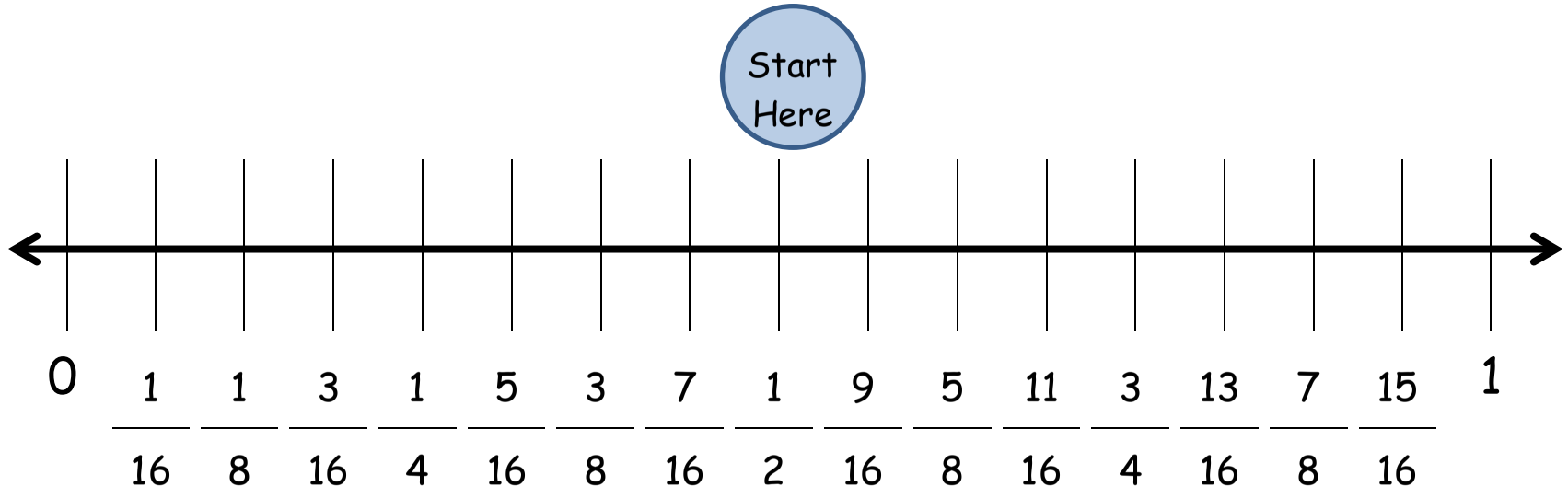
Alternate Game Board

Directions: This game is for two players (or two teams of players). Both players place their counters on the start position. One player is trying to get to get to the one on the number line. The other player is trying to get to the zero on the number line. Player one spins the spinner and moves that distance on the number line from the starting position. Player two then spins and moves the distance shown on the spinner. If the distance on the spinner is too large, the player must move that distance in the opposite direction. The first person to reach their number exactly, wins the game.



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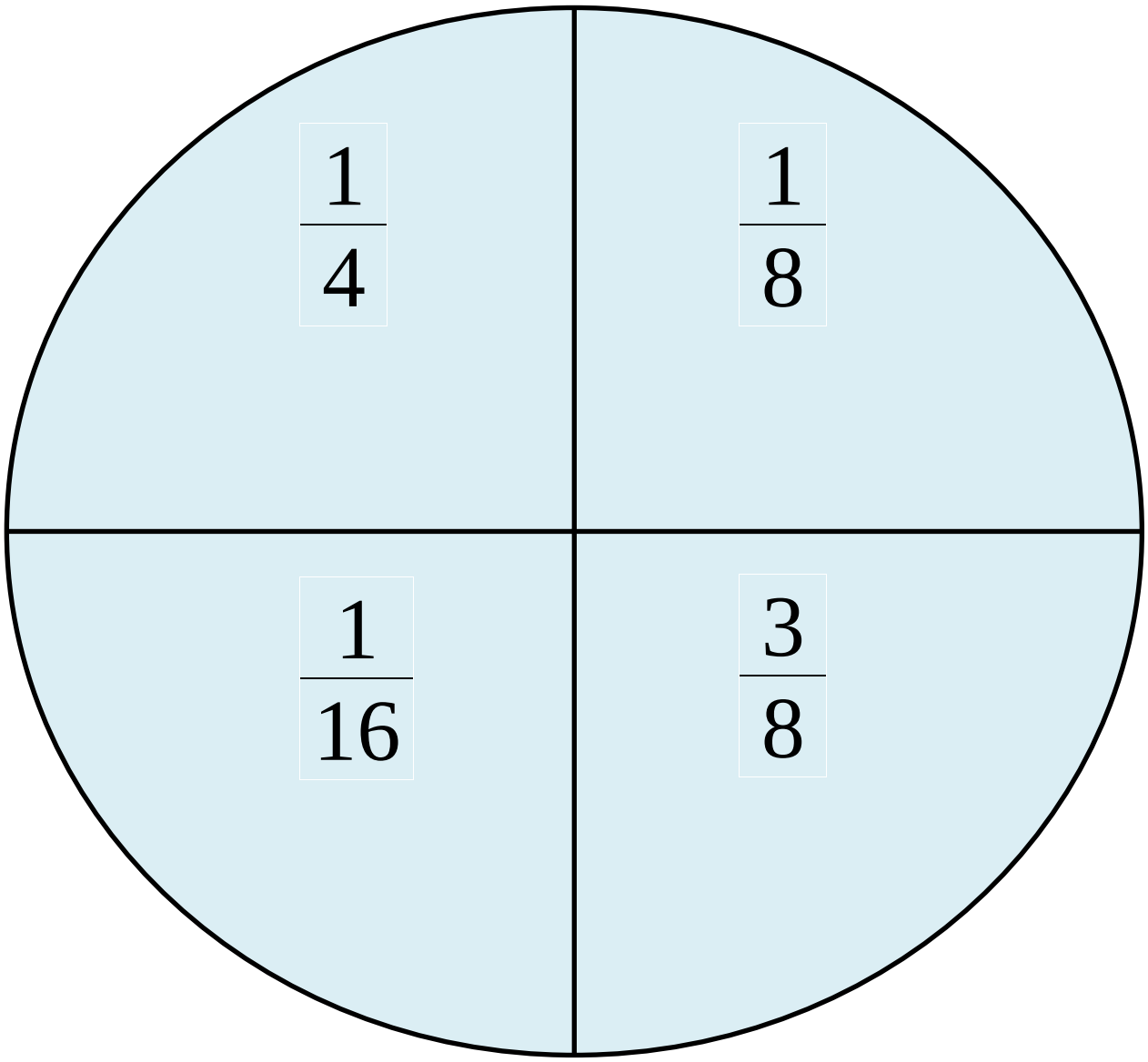

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Practice Task - Create Three

Back to Task Table

In this task students will play a game to be the first to create 3 wholes on the number line and game board.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

ESSENTIAL QUESTIONS

- What strategies can we use for adding and subtracting fractions with different denominators?
- What do equivalent fractions have to do with adding and subtracting fractions?

COMMON MISCONCEPTIONS

Students may not understand the relationships between fractions with different denominators. They may lack the skills to keep up with their scores and develop a strategy for winning the game. Students may need to practice with the number line for keeping score.

MATERIALS

- Game board
- Two counters per person
- Available manipulatives

GROUPING

Group/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

This game will allow students to use their fractional understandings and as their fractional computation strategies develop, logical thinking and problem solving skills will begin to develop their game playing strategies.

Comments

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Multiple fraction models should be made available to the students as support for students who need it. In addition, fractional number lines (other than the ones included with the task) could benefit many students with this task. Students could utilize blank number lines as a strategy for keeping track of their score.

This task could be introduced in a small group or by playing with class as a whole using available technology. In addition, the teacher could model with a student, or better yet have two students role model how to play for the rest of the class.

Before asking students to work on this task, be sure students are able to:

Use models to create equivalent fractions (see previous tasks and Teaching Student Centered Mathematics, volume 2, pg 155-156 (Slicing Squares).
Understand that a whole can be written as a fraction with any number of parts as long as all of those parts are included to make the whole. For example, a whole can be cut into tenths. In order to have a whole, I need all ten tenths (10/10).
decompose fraction, for example $^{4}/_{4}$ = ½ + ½ or ¼ + ¾

Task Directions
Students will follow directions below from the Create Three Game activity.
Play several rounds with each other.
Keep track of computation strategies used (use models in explanations during the closing).
Keep track of game playing strategies used (if any develop this first time playing).

FORMATIVE ASSESSMENT QUESSTIONS

- What strategies do you have for this game?
- If you had to start on 5/8, what would your first move be? Why?

Questions for Teacher Reflection
While planning the task:
What level of support do my struggling students need in order to be successful with this task?
In what way can I deepen the understanding of those students who are competent in this task?
Could this game be played again, or changed in any way?

During and after the students complete the task:
Which students have developed a strategy based on fraction understandings of numerators and denominators?
Which students are becoming fluent in creating equivalent fractions when adding fractions?
Which students still prefer to use manipulatives and rely heavily on models?

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DIFFERENTIATION

Extension

http://www.openmiddle.com/subtracting-fractions/
http://www.openmiddle.com/adding-fractions/
http://www.openmiddle.com/adding-mixed-numbers/
http://www.openmiddle.com/subtracting-mixed-numbers/

Intervention

Have students play Fraction Capture: https://www.georgiastandards.org/Georgia-Standards/Documents/Foundations-of-Algebra/Day-2-Segment-10-Fraction-Capture-from-Everyday-Math.pdf

*Intervention Table

TECHNOLOGY CONNECTION

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http://www.counton.org/games/map-fractions/frosty/ this is an interactive three across board game where players add fractions with like and unlike denominators and place a virtual counter on the sum. The first to get three in a row wins the game.

http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

www.nzmaths.co.nz this site has some virtual problem solving games and others in the materials section under numeracy. There is a lot here, but it's all good, standards-based teaching strategies and tasks.

*http://www.visualfractions.com/add.htm A set of interactive tools that allows students to represent fraction addition with measurement (lines) and area (circles) models.

*http://www.visualfractions.com/subtract.htm A set of interactive tools that allows students to represent fraction subtraction with measurement (lines) and area (circles) models.

*https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators
*https://www.conceptuamath.com/app/tool/subtracting-fractions-with-uncommon-denominators
Highly interactive tool that allows students/teachers to represent fraction addition/subtraction with number lines and area models, and also allows the user to input a customized word problem.

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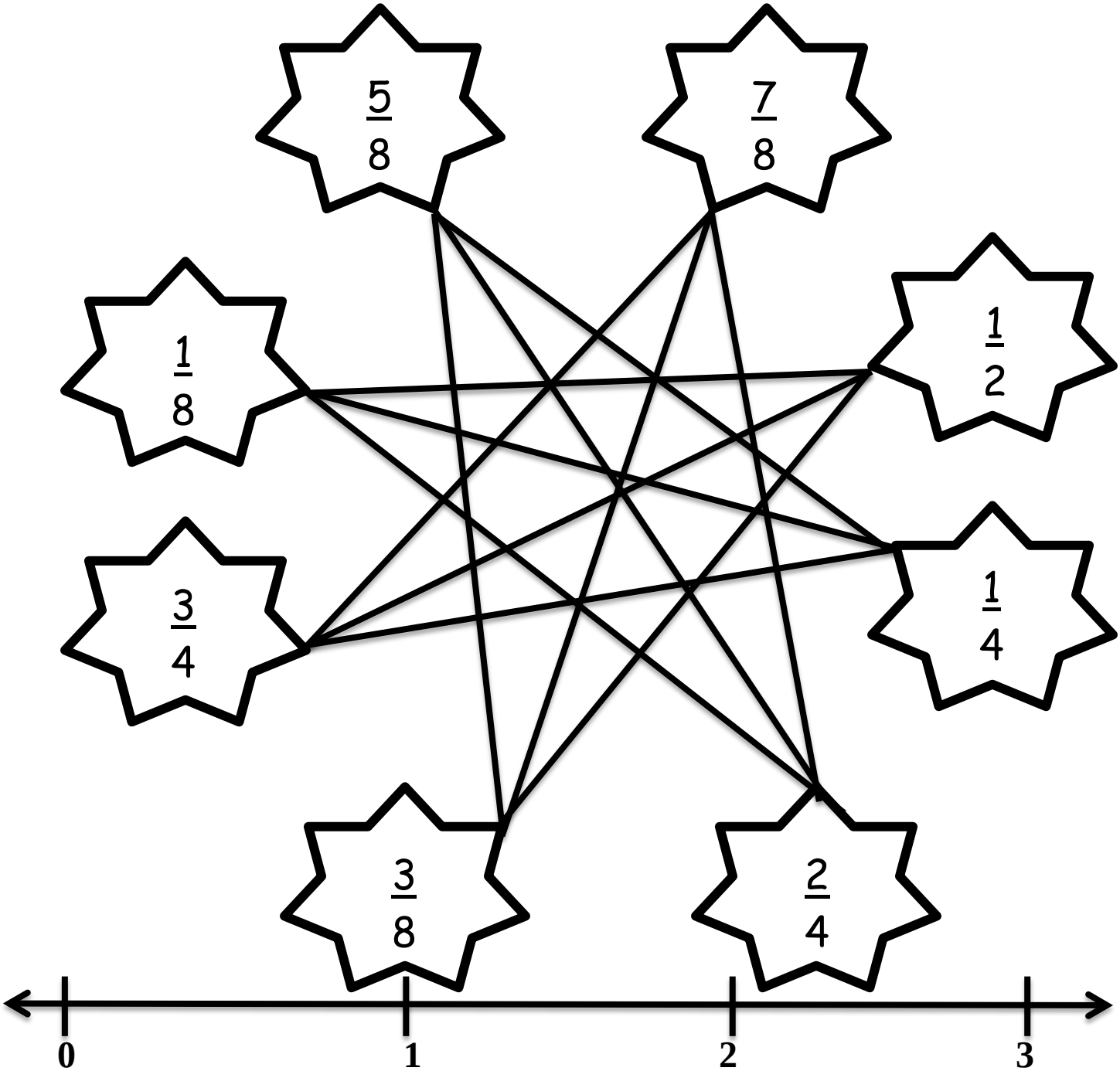
Create Three



Play with a partner. Each player chooses a fraction to place their counter on. Take turns moving your counter to another faction along the lines only. Add the new fraction to your total. The first player to make exactly three is the winner. Go over three and you lose the game. Players use an additional counter to keep a running total along the number line.

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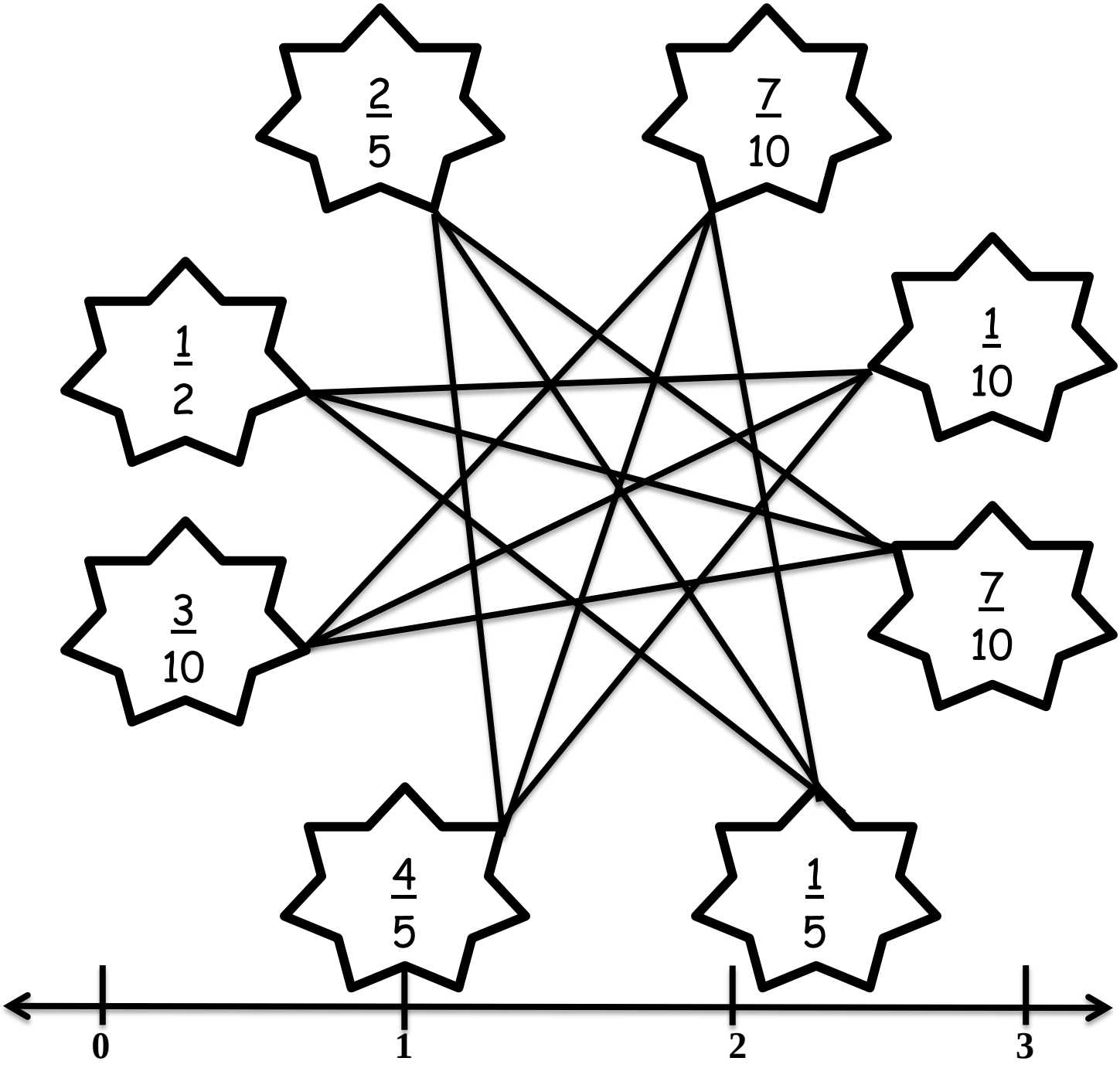
Create Three



Play with a partner. Each player chooses a fraction to place their counter on. Take turns moving your counter to another faction along the lines only. Add the new fraction to your total. The first player to make exactly three is the winner. Go over three and you lose the game. Players use an additional counter to keep a running total along the number line.

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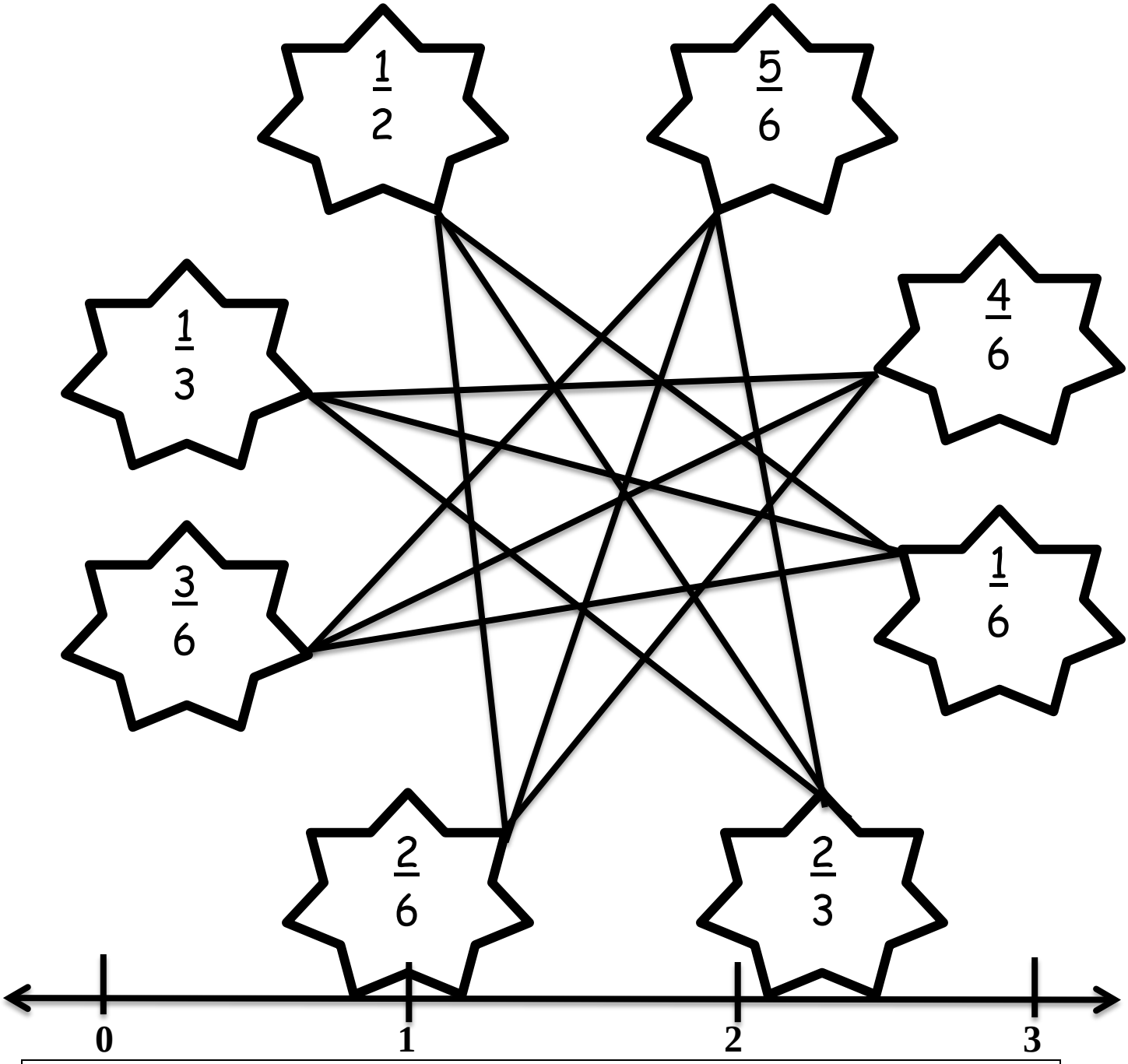
Create Three



Play with a partner. Each player chooses a fraction to place their counter on. Take turns moving your counter to another faction along the lines only. Add the new fraction to your total. The first player to make exactly three is the winner. Go over three and you lose the game. Players use an additional counter to keep a running total along the number line.

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Practice Task - Comparing MP3s

Back to Task Table

This task uses multiplying fractions in a real world application. It also allows students to take the concept of arrays and distributive property one step further by extending their knowledge from whole numbers and decimals to fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

MGSE5.NF.2 Solve word problems involving addition and subtraction of fractions, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < ½.

MGSE5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. Example: $\frac{3}{5}$ can be interpreted as "3 divided by 5 and as 3 shared by 5".

MGSE5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Apply and use understanding of multiplication to multiply a fraction or whole number by a fraction.
   Examples: $\frac{a}{b} \times q$ as $\frac{a}{b} \times \frac{q}{1}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

MGSE5.NF.5 Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. Example 4 x 10 is twice as large as 2 x 10.

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

MGSE5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem

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STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students engaging in this task should be familiar with arrays and part-whole thinking as applied to multiplication. They should have had experiences using the distributive property to split one factor into two parts, multiplying each part by the other factor, and realizing that the product is the sum of the two partial products. Students have used the distributive property before with whole number multiplication and decimal multiplication. In this unit, they can also apply the distributive property to fractions.

Examples of distributive property applied to fraction multiplication:

Example 1

	B	c
a	ab	ac

$5 \times 3\frac{1}{2}$

a = 5
b = 3
c = $\frac{1}{2}$

a x (b+c) = (a x b) + (a x c)
$5 \times (3\frac{1}{2}) = (5 \times 3) + (5 \times \frac{1}{2})$

Example 2

c	d

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a	ac	ad
b	bc	bd

$4\frac{1}{2}$ x $6\frac{1}{3}$

a = 4
b = $\frac{1}{2}$
c = 6
d = $\frac{1}{3}$

(a + b) x (c + d) = (ac + ad) + (bc + bd)
(4 + $\frac{1}{2}$) x (6 + $\frac{1}{3}$) = [(4 x 6) + (4 x $\frac{1}{3}$) + ($\frac{1}{2}$x6) + ($\frac{1}{2}$ x $\frac{1}{3}$)]

COMMON MISCONCEPTION

Look out for students who believe that when you multiply the product is larger than the factors. For example they don't understand that $\frac{1}{2}$ x $\frac{1}{2}$ = $\frac{1}{4}$ and the product $\frac{1}{4}$ is smaller than the factors. Watch out for students who only use some of the partial products when multiplying a mixed number by a mixed number. For example $3\frac{1}{2}$ x $4\frac{1}{2}$ = $12\frac{1}{4}$.

ESSENTIAL QUESTIONS

- How can we model an area with fractional pieces?
- How can decomposing fractions or mixed numbers help us model fraction multiplication?

MATERIALS

- Comparing MP3s Task
- Pencil, ruler
- Task Sheets may want Grid paper
- Accessible manipulatives

GROUPING

Pair/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

This task was developed to give students a real world application of multiplying fractions that would be engaging. It also allows students to take the concept of arrays, which they should have worked with extensively in their study of multiplication, and apply this familiar concept to another area of

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multiplication – fractions. The relationships between factor sizes and products can be discovered and generalized as students study partial products. Once students understand this relationship, they will be better able to attend to precision because they will know if their partial products are reasonable.

Decomposition: Mixed Number = Whole number + Fraction (if fraction is a proper fraction)

x	Whole Number	Fraction
Whole Number	> or = whole #	< whole # > or = Fraction
Fraction	< whole # > or = Fraction	< both Fractions

Students should be allowed to draw representations of their thinking. This allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

Teacher Notes:

Before the lesson, work with students on a number talk involving multiplication. Use the following problems, one at a time, giving think time and time to share after each one. Discuss strategies and give students opportunities to make connections to other areas in mathematics. Students should notice about the size of the product in relation to the factors. The use of "of" may be helpful for students to visualize some of the problems.

2 x 4

$\frac{1}{2}$ x 4 or $\frac{1}{2}$ of 4

$\frac{1}{2}$ x $\frac{1}{2}$ or $\frac{1}{2}$ of $\frac{1}{2}$

$\frac{8}{2}$ x $\frac{8}{2}$

$\frac{1}{2}$ x $\frac{8}{2}$ or $\frac{1}{2}$ of $\frac{8}{2}$

$\frac{1}{2}$ x $4\frac{1}{2}$ or $\frac{1}{2}$ of $4\frac{1}{2}$

$2\frac{1}{2}$ x $4\frac{1}{2}$ or $2\frac{1}{2}$ of $4\frac{1}{2}$

Introduce the problem and be sure everyone is clear with the context. You may wish to use the pictures included at the end of this task to help develop this context. Facilitate a preliminary discussion with the class, before students get to working on the problem. Allow students to share their initial thoughts, then ask them to work in pairs to investigate the following:

One of the most popular MP3 players is Apple's iPod. However, companies such as Samsung and Microsoft are trying to take some business away from Apple with their own products – some of

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which can cost much less. As of right now, Apple's iPod is holding on to its share of the MP3 market, but some of the other MP3 players offer larger screens that might lure consumers from the Apple.

Apple wants our help to make a mock-up. They think if they can make the length of the new iPod Touch 1 ¼ times longer than it is now and the width 1 ½ times longer than it is now, the iPod Touch will still be small enough to fit in a pocket or purse, but will have a screen size that is easier to use and work with and be large enough to compete with the largest touch screen MP3 player out there.

How much larger is each dimension of the prototype iPod than the original?
How much larger is the area of the prototype iPod than the original?
Samsung's model (Galaxy Player 5.0 with wi-fi) has dimensions of 3 1/10 x 5 3/5. Will Apple's new prototype be larger than Samsung's?

Possible struggles students may have can be turned into wonderful inquiries. As students investigate the screen areas, you may notice them:

- Cutting each screen into familiar pieces first, such as wholes, then the fractional pieces - this strategy may lead them to an open array model

- Using multiplication of fractions algorithm - this strategy may promote discussion, so please allow students the freedom to make sense of this in the closing part of the lesson

After enough time has been devoted to the task, ask pairs of students to make posters to prepare for the closing of the lesson. Posters should be clear enough for others in the class to understand their thinking, but should not just be the figuring that was initially done copied over again. The posters should be clear and concise presentations of any important ideas and strategies students wish to present.
Some ideas to encourage discussion about in the presentations of student work:
Estimation, using just the whole number dimensions can help determine whether areas are in the ballpark.
How mixed numbers are decomposed can make a difference.
Students who use any algorithm should also make sense of the algorithm used with some kind of model.

DIFFERENTIATION

Extension

Design an MP3 player with the ideal dimensions and find the area of your MP3 player.

Intervention

Show how to use grid paper and manipulatives. Provide assistance with the distributive property with whole numbers using arrays and partial products before moving to fractions. Be sure to use an array with four parts.

*Intervention Table

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TECHNOLOGY CONNECTION

http://my.hrw.com/math06_07/nsmedia/tools/Decimal_Fractions/Decimal_Fractions.swf Modeling fraction x a fraction or a fraction times a mixed number if you change from add and subtract to multiply.

http://www.bbc.co.uk/schools/ks2bitesize/maths/number/games and activities for students to use for practice working with fractions and decimals, as well as whole numbers.

http://www.counton.org/games/map-fractions/falling/ in this game, students find fractions of whole numbers to collect leaves falling from a tree. The player with the most leaves at the end of the game is the winner.

*http://www.dreambox.com/teachertools **(Scroll down to "Fraction Multiplication 2")** Students multiply two fractions together and use an area model to represent the product.

*http://www.visualfractions.com/multiply.htm A set of interactive tools that allows students to represent fraction multiplication with measurement (lines) and area (circles) models.

*https://www.conceptuamath.com/app/tool/multiplying-fractions-tool
Highly interactive tool that allows students/teachers to represent fraction multiplication with number lines and area models, and also allows the user to input a customized word problem.

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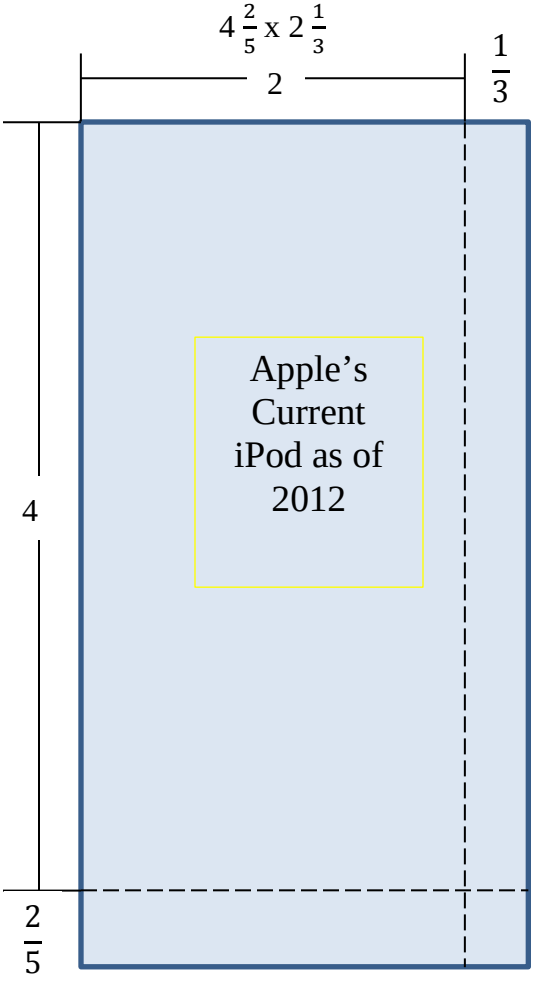
Practice Task - Comparing MP3s

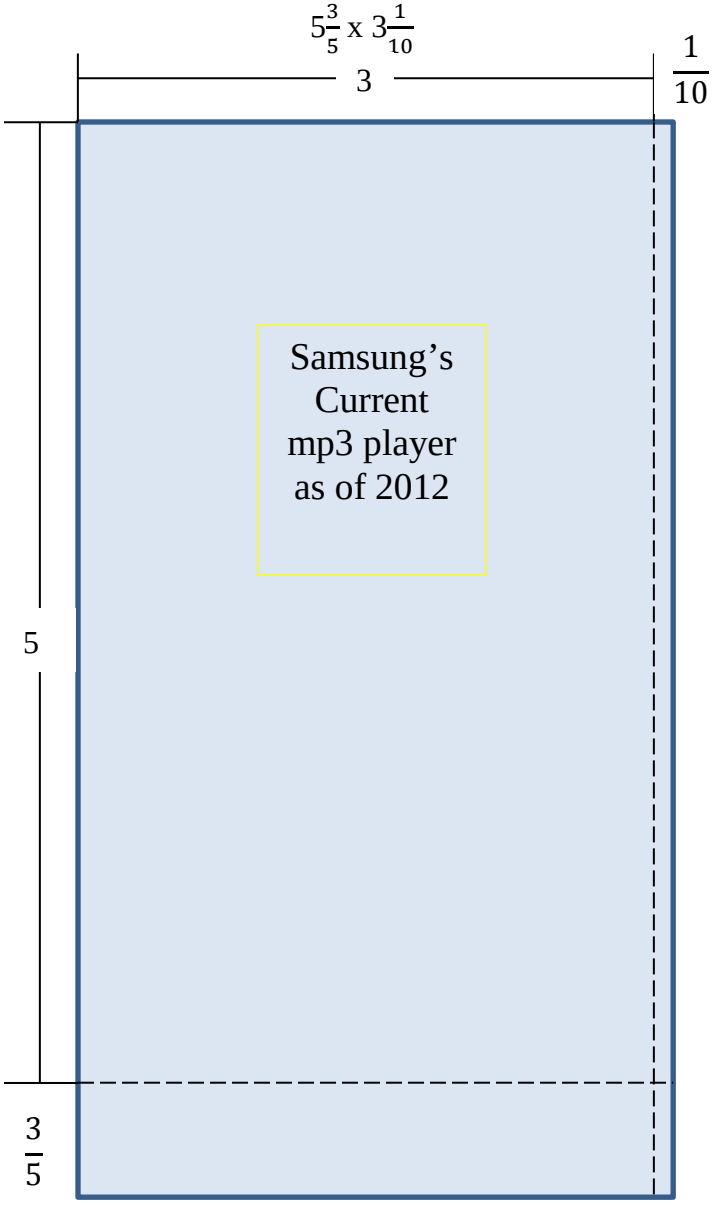
One of the most popular MP3 players is Apple's iPod. However, companies such as Samsung and Microsoft are trying to take some business away from Apple with their own products – some of which can cost much less. As of right now, Apple's iPod is holding on to its share of the MP3 market, but some of the other MP3 players offer larger screens that might lure consumers from the Apple.

Apple wants your help to make a mock-up. They think if they can make the length of the new iPod Touch 1 ¼ times longer than it is now and the width 1 ½ times longer than it is now, the iPod Touch will still be small enough to fit in a pocket or purse, but will have a screen size that is easier to use and work with and be large enough to compete with the largest touch screen MP3 player out there.

Current iPod dimensions:



Current Samsung Galaxy 5.0 dimensions:



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What will the dimensions (length and width) of the new iPod Touch mock-up be, if the length and width are increased according to Apple's specifications?

Use grid paper, a pencil, and a ruler, or a drawing or word processing program to create your mock-up. Use the dimensions you found in step one to create the rectangle dimensions.

Find the area of each MP3 player including the new iPod Touch mock-up and make a decision based on your mathematics as to whether the new iPod Touch mock-up will be able to compete with Samsung's model. Be sure to include the areas of the screens, the differences in their dimensions, which one is larger and whether you think current iPod Touch users would like this change in size in your decision support statement.

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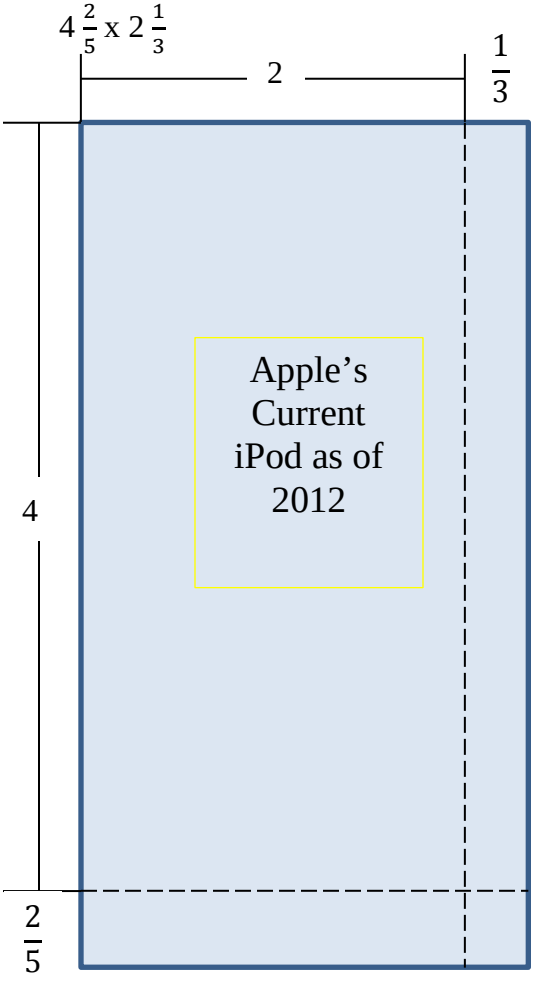
Comparing MP3 Players

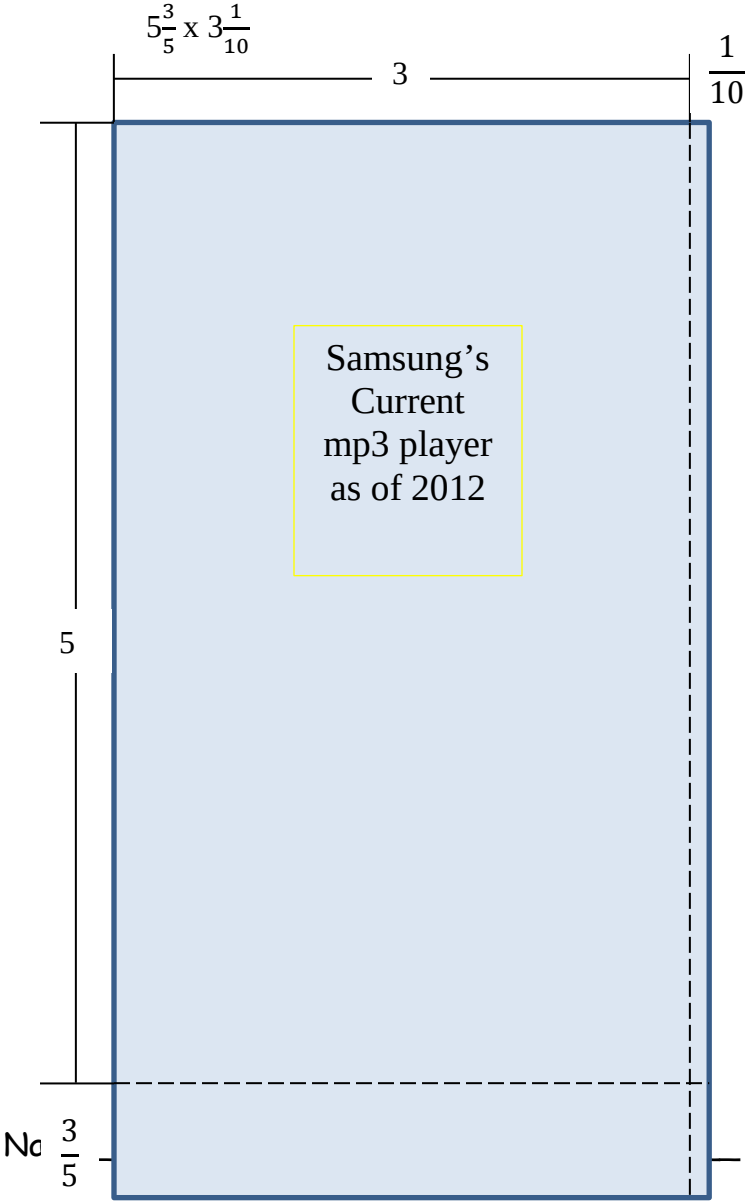
One of the most popular MP3 players is Apple's iPod. However, companies such as Samsung and Microsoft are trying to take some business away from Apple with their own products – some of which can cost much less. As of right now, Apple's iPod is holding on to its share of the MP3 market, but some of the other MP3 players offer larger screens that might lure consumers from the Apple.

Apple wants your help to make a mock-up. They think if they can make the length of the new iPod Touch 1 ¼ times longer than it is now and the width 1 ½ times longer than it is now, the iPod Touch will still be small enough to fit in a pocket or purse, but will have a screen size that is easier to use and work with and be large enough to compete with the largest touch screen MP3 player out there.

Current iPod dimensions:



Current Samsung Galaxy 5.0 dimensions:



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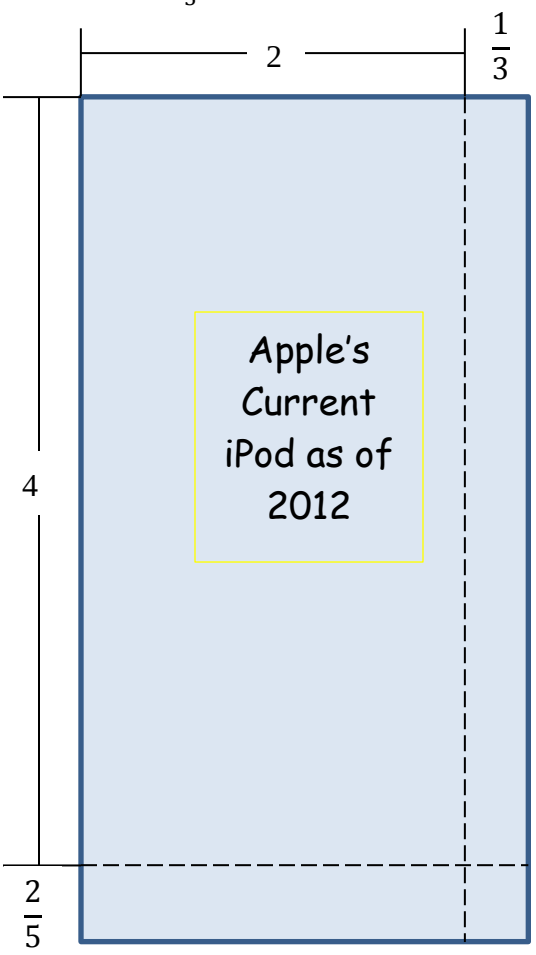
1. What is the area of the iPod?

Current iPod dimensions:
$4\frac{2}{5}$ inches x 2



2. What is the area of the Galaxy?

Current Samsung Galaxy 5.0 dimensions:
$5\frac{3}{5}$ inches x $3\frac{1}{10}$ inches

3. How much larger is the Samsung Galaxy than the iPod?

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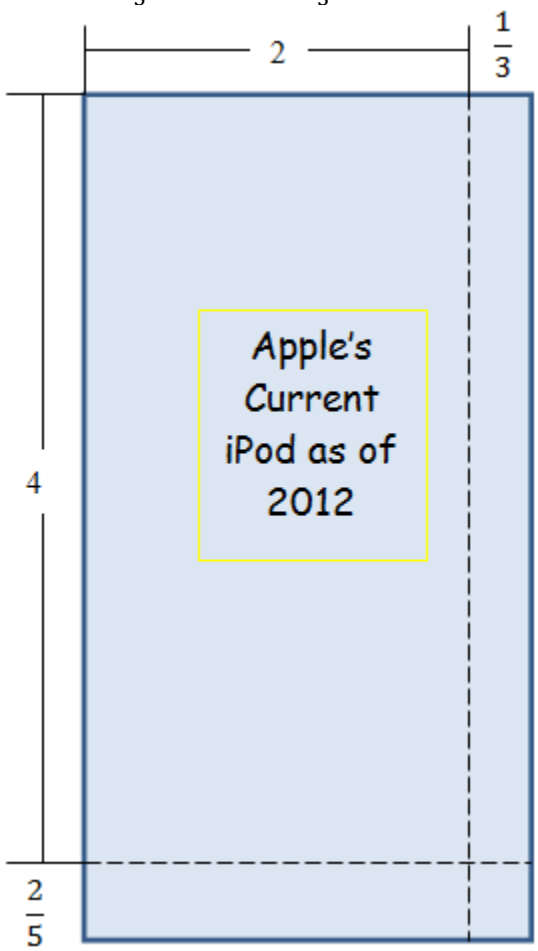
4. Apple wants your help to make a prototype for a larger iPod. They want to make the length of the new iPod Touch 1 $\frac{1}{4}$ times longer than it is now and the width 1 $\frac{1}{2}$ times longer than it is now.

What will the dimensions (length and width) of the new iPod Touch prototype be, if the length and width are increased according to Apple's specifications? Create a model showing the new dimensions.

4 $\frac{2}{5}$ inches x 2 $\frac{1}{3}$ inches



5. How much larger is each dimension of the prototype iPod than the original?

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6. How much larger is the area of the prototype iPod than the original iPod?

7. Which MP3 player will be larger: the new iPod prototype or the Samsung Galaxy? How much larger will is it?

8. Make a suggestion to Apple whether they should proceed with the prototype. Be sure to include information about size, usability, comparability.

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Constructing Task – Measuring for a Pillow

Back to Task Table

This task was developed to give students a real world application of determining what will happen to products when one factor remains the same and the other changes.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Apply and use understanding of multiplication to multiply a fraction or whole number by a fraction.
   Examples: $\frac{a}{b} \times q$ as $\frac{a}{b} \times \frac{q}{1}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

MGSE5.NF.5 Interpret multiplication as scaling (resizing), by:

c. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. Example 4 x 10 is twice as large as 2 x 10.

d. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

MGSE5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
(for descriptors of standard cluster, see beginning of unit)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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BACKGROUND KNOWLEDGE

Students engaging in this task should be familiar with arrays and part-whole thinking as applied to multiplication. See Teaching Student Centered Mathematics, Vol. 2, Slicing Arrays Activity, pg. 66.

Teacher Notes:

Before beginning this task, have a computation discussion with your students using the following computations. It is important for students to have plenty of quiet think time for each individual computation as it is presented. Likewise, after the quiet think time, students should share their strategies before moving to the next problem.

10 x 10
10 x 5
10 x 20
5 x 20
½ x 20
1/5 x 20

After each is complete, if no student offers any thoughts about the products, provoke students to think about the products in each pair:

- Do you notice anything about the products in the pairs?
- Why do you think that's happening?
- What happened to the factors in each pair?
- Do you think this might happen all the time?

In your investigation over the next day or so, you may see and use this mathematical concept.

Part I

Introduce the task. Make sure students understand the concept of the task and what they are expected to do. Allow students to share ideas about the task with the group. Make sure students have construction paper (or fabric) and rulers to measure the correct dimensions. Sample intro idea:

You have been working with your book buddies in Kindergarten for several months now. It might be nice to give your book buddies a nice reading pillow for them to use while reading for the rest of the year. When thinking about pillows to lean against, a 12 ½ x 9 inch pillow for seems like a good size for a Kindergarten student.
Questions for students to focus on as they investigate:
How big (what is the area of) this pillow?
Is this large enough?

Allow students to work in pairs to measure and cut the (fabric) construction paper to the proper dimensions and begin the task.

Listen to student thinking and provide support with thought provoking questions like the ones below.

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Students may use several strategies to solve this problem. Look for students who rely heavily on manipulatives. These students should share first in the closing part of the lesson. You may see students using some of these strategies:

After cutting the paper to the dimensions listed above, students may use 1 inch color tiles to tile the paper, noting that 9 tiles fit one dimension and 12 ½ fit the other dimension. Students may note that 9 x 12 is 108 and 9 halves is 4 1/2, so the total is 112 ½ square inches.

After cutting the paper to the dimensions listed, students may "cut" the paper with lines showing split arrays of 9 x 12 and 9 x ½ or 9 x 10, 9 x 2, and 9 x ½, then find the sums of these partial products.

COMMON MISCONCEPTIONS

Students may only use some of the partial products when multiplying with fractions. Students may believe that when you multiply your product always gets larger than your factors.

ESSENTIAL QUESTIONS

- How can comparing factor size to 1 help us predict what will happen to the product?
- How can modeling an area help us with multiplying fractions?

MATERIALS

- Measuring for a Pillow Task
- Pencil, ruler
- Grid paper
- Fabric and thread or construction paper (optional)
- Accessible manipulatives

GROUPING

Pair/Individual

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

This task was developed to give students a real world application of determining what will happen to products when one factor remains the same and the other changes. It is not necessary to complete the sewing aspect of this task, but if you do, it could elicit student inquiry into other mathematical investigations. This task is meant to involve students in a deeper investigation of the concept of the array with fractions.
Students should be allowed to draw representations of their thinking. Using grid paper may facilitate this. Creating these representations allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

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FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answer is correct?
- How do you know that mark goes there? Show me your thinking.
- How did you find the area of the pillow? Is there another way to think about this?
- Did you develop a shortcut to find your answers?
- What kind of representation will you use to show your thinking?
- Did you identify any patterns or rules? Explain what you have found.

After enough time has been devoted to the task, post the work around the room and have students view and make comments on others' work. Students may ask questions, or make mathematical commentary on post-it notes and stick them to the work. Pay attention to students' talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share.

When students have finished the tour, come back to the large group and begin the closing of the lesson. The goal of this closing is to help students make connections about areas of rectangles with fractional dimensions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work.

Questions for Teacher Reflection

- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- One positive thing about today's lesson and one thing you will change.

Part II

Make sure students have completed and shared work from part I as they will need the area of the pillow for part II. The main purpose of the task is for students to multiply a whole number by a mixed number. Although students will most likely struggle with going from an area to factors, realize it is okay if students are unable to be successful with this as this is not the main purpose of the lesson. If students can apply their knowledge and work backwards from area to factors/dimensions, it will increase their problem solving ability. When beginning part II, make sure students understand the context of the problem and any new vocabulary is understood.

The problem in part II is that students think the pillow is much too small (even for kindergarten students), so they look to increase the pillow size (area).

As students are getting ready to begin this task, give them time to share ideas about how to approach the problem (strategies and general ideas). When they are ready, give them the task and have manipulatives available – including grid paper.

Questions for students to think about as they investigate part II:
Should the area be doubled or quadrupled? Show your mathematical thinking.

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What should the dimensions be for your new pillow?
How do these new dimensions compare to the original pillow dimensions from part I?

Allow pairs of students to begin working on the task.

Listen to student thinking and provide support with thought provoking questions like the ones below. Students may use several strategies to solve this problem. Look for students who rely heavily on manipulatives. These students should share first in the closing part of the lesson. You may see students using some of these strategies:

Students may wish to cut and tape paper to new dimensions and tile with one inch tiles.
After cutting the paper to the dimensions listed, students may "cut" the paper with lines showing split arrays, then find the sums of these partial products.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answer is correct? Show me your thinking.
- How did you find the area of the pillow? Is there another way to think about this?
- Did you develop a shortcut to find your answers?
- What kind of representation will you use to show your thinking?
- Did you identify any patterns or rules? Explain what you have found.

After enough time has been devoted to the task, set the work around the room and have students take some time to view and make comments on others' work. Students may ask questions, or make mathematical commentary on post-it notes and stick them to the work. Pay attention to students' talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share.

When students have finished the tour, come back to the large group and begin the closing of the lesson. The goal of the closing is to help students make connections about areas of rectangles with fractional dimensions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work.

DIFFERENTIATION

Extension

Have students work on finding dimensions for a square picture frame that has an area of $72\frac{1}{4}$ square inches. … $110\frac{1}{4}$ square inches, $132\frac{1}{4}$ square inches, $87\frac{1}{9}$ square inches.

Intervention

Show how to use grid paper and manipulatives. Teachers may want to provide assistance with the distributive property with whole numbers using arrays and partial products before moving to fractions.

*Intervention Table

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TECHNOLOGY CONNECTION

http://www.counton.org/games/map-fractions/falling/ in this game, students find fractions of whole numbers to collect leaves falling from a tree. The player with the most leaves at the end of the game is the winner.

http://www.bbc.co.uk/schools/ks2bitesize/maths/number/ a lot of games and activities for students to use to practice working with fractions and decimals as well as whole numbers.

*http://www.dreambox.com/teachertools **(Scroll down to "Fraction Multiplication 2")** Students multiply two fractions together and use an area model to represent the product.

*http://www.visualfractions.com/multiply.htm A set of interactive tools that allows students to represent fraction multiplication with measurement (lines) and area (circles) models.

*https://www.conceptuamath.com/app/tool/multiplying-fractions-tool
Highly interactive tool that allows students/teachers to represent fraction multiplication with number lines and area models, and also allows the user to input a customized word problem.

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Constructing Task – Measuring for a Pillow – Part I

You have been working with your book buddies in Kindergarten for several months now. It might be nice to give your book buddies a nice reading pillow for them to use while reading for the rest of the year. When thinking about pillows to lean against, a 12 ½ x 9 inch pillow for seems like a good size for a Kindergarten student.

Questions:
How big (what is the area of) this pillow?
Is this large enough?

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Constructing Task – Measuring for a Pillow – Part II

After cutting the fabric for the pillow, it doesn't look like the pillow will be large enough. You can't decide if you should double the area of the pillow, or quadrupled the area. Show the possible dimensions of pillows for each case. Show your mathematical thinking with sketches, words and numbers. Which do you think is a better fit for your book buddy? Explain.

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Constructing Task – Reasoning with Fractions

Back to Task Table

In this task, students will use manipulatives and grid paper to investigate what happens to the product when a whole number is multiplied by 1, by a fraction less than 1, and by a mixed number greater than 1.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Apply and use understanding of multiplication to multiply a fraction or whole number by a fraction.
   Examples: $\frac{a}{b} \times q$ as $\frac{a}{b} \times \frac{q}{1}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

MGSE5.NF.5 Interpret multiplication as scaling (resizing), by:

b. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. Example 4 x 10 is twice as large as 2 x 10.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students engaging in this task should be familiar with arrays and part-whole thinking as applied to multiplication. They should have had experiences using the distributive property to split one factor into two parts, multiplying each part by the other factor, and realizing that the product is the sum of the two partial products. Students have used the distributive property before with whole number multiplication and decimal multiplication. In this unit, they can also apply the distributive to fractions.

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Examples of distributive property applied to fraction multiplication:

Example 1

	b	c
a	ab	ac

5 x 3 ½

a = 5
b = 3
c = ½

a x (b+c) = (a x b) + (a x c)
5 x (3 ½) = (5 x 3) + (5 x ½)

Example 2

	c	d
a	ac	ad
b	bc	bd

$4\frac{1}{2}$ x $6\frac{1}{3}$

a = 4
b = $\frac{1}{2}$
c = 6
d = $\frac{1}{3}$

(a + b) x (c + d) = (ac + ad) + (bc +bd)
$(4 + \frac{1}{2})$ x $(6 + \frac{1}{3})$ = $((4 \times 6) + (4 \times \frac{1}{3})) + ((\frac{1}{2} \times 6) + (\frac{1}{2} \times \frac{1}{3}))$

COMMON MISCONCEPTIONS

Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to see that the results will depend on whether the other factor is less than one, equal to one, or greater than one. Another misconception that students may have is an incomplete application of the distributive property. In the example above, they may multiply 5 x 3, but forget to multiply 5 x ½, so that their resulting product is 15 ½. Explicit modeling of the distributive property and the partial products may help them to realize the importance of multiplying both parts of each factor. An alternative strategy would be to convert each mixed number to an improper fraction before multiplying.

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ESSENTIAL QUESTIONS

- How can comparing factor size to 1 help us predict what will happen to the product?
- How can fraction multiplication be modeled with arrays?

MATERIALS

- Reasoning with Fractions student sheet
- Accessible manipulatives
- Grid paper

GROUPING

Pair/Individual

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments:

This task was developed so that students could investigate what will happen to products when one factor remains the same and the other changes. This task is meant to involve students in a deeper investigation of the concept of fraction multiplication using arrays. Students should use manipulatives such as color tiles to draw represent the arrays. They should record their arrays on grid paper. Creating these representations allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

There are three parts for this task. The teacher should decide whether to do just Part 1 or whether to continue to Parts 2 and 3. The three parts are set up as follows:

- Part 1 – multiply *whole numbers* by 1, by fractions less than 1, and by mixed numbers greater than 1
- Part 2 – multiply *fractions less than 1* by 1, by fractions less than 1, and by mixed numbers greater than 1
- Part 3 – multiply *mixed numbers* by 1, by fractions less than 1, and by mixed numbers greater than 1

Teacher Notes:

Before beginning this task, have a computation discussion with your students using the following computations. It is important for students to have plenty of quiet think time for each individual computation is presented. Likewise, after the quiet think time, students should share their strategies before moving to the next problem.

16 x 1
16 x ½
16 x 1 ½

In each expression, 16 is being multiplied by another factor.
Are any of the products smaller than 16?

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Why do you think that's happening?
Do you think this might happen all the time, even with mixed numbers?

In this task, you will investigate this mathematical concept.

Part 1
Introduce the task and make sure students understand what they are expected to do. Allow students to share ideas about the task with the group. Make sure students have materials necessary for investigating this task.

It is important that students explain why some of the products get smaller than the underlined factors and other products become larger. Evidence must be presented as to why this happens in each case.

Allow students to work in pairs to answer the questions posed.

Listen to student thinking and provide support with thought provoking questions like the ones below.

Students may use several strategies to solve this problem.

Some students may use 1 inch color tiles, initially, but may run into trouble explaining fractions using these manipulatives. It is possible for students to use these manipulatives by assigning a fractional length to each tile. For example, students may decide that the length of each tile represents ¼, rather than 1. This presents its own challenges, but the struggle is where the learning happens.

Other students may use grid paper in the same manner presented above. A variety of grid sizes may be useful for this task.

Parts 2 and 3 – These should be continued in the same manner as Part 1.

After enough time has been devoted to the task, bring pairs of students together to share in groups of 4 to 6 students. As students share, listen for different explanations and look for different representations.

When students have finished the sharing, come back to the large group and begin to close the lesson. The goal of this closing is to help students make connections about areas of rectangles with fractional dimensions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work.

FORMATIVE ASSESSMENT QUESTIONS

- How do you know that your answer is correct?
- What is happening here? Show your thinking.
- What kind of representation will you use to show your thinking?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain what you have found.

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DIFFERENTIATION

Extension

- Students can determine their own factors to investigate.
- Students can write generalizations about what will happen to the product, and give examples and contexts when:
    - both factors are whole numbers
    - one factor is a whole number and the other is a fraction less than one
    - one factor is a whole number and the other is a mixed number greater than one
    - both factors are fractions less than one
    - both factors are mixed numbers greater than one
- Students can write about how the situations above and how they could also apply to decimals.

Intervention

- Use an on-line interactive program that will create the array after the student inputs both factors, such as this website from Annenberg Learner. http://www.learner.org/courses/learningmath/number/session9/part_a/try.html

*Intervention Table

TECHNOLOGY CONNECTIONS

*http://www.dreambox.com/teachertools **(Scroll down to "Fraction Multiplication 2")** Students multiply two fractions together and use an area model to represent the product.

*http://www.visualfractions.com/multiply.htm A set of interactive tools that allows students to represent fraction multiplication with measurement (lines) and area (circles) models.

*https://www.conceptuamath.com/app/tool/multiplying-fractions-tool
Highly interactive tool that allows students/teachers to represent fraction multiplication with number lines and area models, and also allows the user to input a customized word problem.

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Name _____ Date _____

Reasoning with Fractions – Part 1

Directions:

- There are several fraction multiplication problems below. The goal for each is to determine why the product is larger or smaller than the first factor.
- Use manipulatives of your choice to investigate and determine the product.
- Create a written representation, such as an array, to show your results.
- Use words and numbers to explain whether the product is larger or smaller than the underlined number and why.

18×1

$16 \times \frac{2}{2}$

$14 \times \frac{1}{2}$

$24 \times \frac{1}{3}$

$14 \times 1\frac{1}{2}$

$24 \times 2\frac{1}{3}$

Choose two from the last four problems and create a context for the each. Share your story problem with your partner.

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Name _____ Date _____

Reasoning with Fractions – Part 2

Directions:

- There are several fraction multiplication problems below. The goal for each is to determine why the product is larger or smaller than the first factor.
- Use manipulatives of your choice to investigate and determine the product.
- Create a written representation, such as an array, to show your results.
- Use words and numbers to explain whether the product is larger or smaller than the underlined number and why.

$\frac{1}{2} \times 1$

$\frac{1}{3} \times \frac{4}{4}$

$\frac{\mathbf{1}}{\mathbf{2}} \times \frac{\mathbf{1}}{\mathbf{2}}$

$\frac{1}{3} \times \frac{1}{4}$

$\frac{2}{3} \times 2\frac{1}{6}$

$\frac{3}{4} \times 1\frac{1}{2}$

Choose two from the last four problems and create a context for the each. Share your story problem with your partner.

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Name _____ Date _____

Reasoning with Fractions – Part 3

Directions:

- There are several fraction multiplication problems below. The goal for each is to determine why the product is larger or smaller than the first factor.
- Use manipulatives of your choice to investigate and determine the product.
- Create a written representation, such as an array, to show your results.
- Use words and numbers to explain whether the product is larger or smaller than the underlined number and why.

$1\frac{1}{2} \times 1$

$3\frac{1}{4} \times \frac{3}{3}$

$1\frac{3}{4} \times \frac{1}{2}$

$2\frac{1}{3} \times \frac{3}{4}$

$1\frac{2}{3} \times 2\frac{1}{6}$

$5\frac{2}{5} \times 1\frac{1}{3}$

Choose two from the last four problems and create a context for the each. Share your story problem with your partner.

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3 ACT TASK: Sweet Tart Hearts

Back to Task Table

Task adapted from Mike Wiernicki (http://mikewiernicki3act.wordpress.com/)

APPROXIMATE TIME: 1 class period

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

MGSE5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Apply and use understanding of multiplication to multiply a fraction or whole number by a fraction.
   Examples: $\frac{a}{b} \times q$ as $\frac{a}{b} \times \frac{q}{1}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

MGSE5.NF.5 Interpret multiplication as scaling (resizing), by:
b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

MGSE5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. After watching a video about various colored sweet tart hearts placed in a glass, students will "analyze givens, constraints, relationships and goals" as they make sense of the problem presented in Act 1. Before determining a solution, students take time to reason through the ideas in the problem, as well as develop a plan of action for solving the problem. (www.corestandards.org/Math/Practice)

2. Reason abstractly and quantitatively. During Act 2, students will determine what fraction of the glass is filled with each color of sweet tart hearts. Students can request a document with representations of the glass filled with each color of sweet tart hearts that will help them to reason quantitatively about the fraction of the glass that has been used when each color of sweet tart hearts is placed inside. For example, students can partition the glass into thirds and determine that one third of the glass has been filled with pink sweet tart hearts.

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3. Construct viable arguments and critique the reasoning of others. While using the pictorial representations of the glasses in Act 2, students will likely find themselves defending what fraction of the glass they think is filled with sweet tart hearts. This requires clear and precise language and vocabulary so that others can understand and respond to their thinking.

6. Attend to precision. Students will communicate precisely with peers about quantities in the problem using mathematics vocabulary.

7. Look for and express regularity in repeated reasoning. In this problem, students will have the opportunity to repeat similar calculations multiple times. Students can listen and discuss with peers about the strategies they find most efficient and which strategies are not working as well.

ESSENTIAL QUESTIONS

During Act 1, students view a video of a glass being filled with Sweet Tart Heart candies. Students see all the candies in the glass, as well what fraction of the glass is filled with each color of candy. Students must infer the fraction of the glass that is filled using other information in the image or video because the video doesn't give this information. It is imperative that teachers allow students to ask questions of each other and participate in discussion that will lead the students to infer that information during Act 2. The essential questions below can be shared at the beginning of Act 2 to define the emphasis of the problem solving opportunity being presented.

- How can a model help us make sense of a problem?
- What connections can be made between the models and equations with fractions?
- What strategies are most efficient and effective when multiplying fractions?

MATERIALS

Act 1 video "Sweet Tart Hearts" http://mikewiernicki3act.wordpress.com/
Student recording sheet (attached)
Act 2 Sweet Tart Hearts Fractions images (Use the link above to locate images.)

GROUPING

Whole group, partners or small groups

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Students will watch a video of a glass being filled with a bag of Sweet Tart Heart candies. After seeing the glass full of candies, students will see what the glass looks filled with only one color of candy. Each color of candy is shown in the glass. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer one of the questions generated on the chart. Students will be given information to solve the problem based on

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need. When they realize they don't have the information they need, and ask for it, it will be given to them.

BACKGROUND KNOWLEDGE:

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org and the K-5 MGSE Mathematics Wiki.

In fourth grade, students learned about multiplying whole numbers by fractions as an extension of their existing multiplication knowledge. Students' multiplication knowledge is rooted in the idea of repeated addition. Students apply this understanding to their understanding of multiplying a whole number by a fraction. For example, $\frac{2}{3}$ x 4 is equivalent to $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$. In fourth grade, students were also introduced to the interpretation of equations that contain a whole number by a fraction as a fraction of the whole number. In the aforementioned example, the equation $\frac{2}{3}$ x 4 would be interpreted as $\frac{2}{3}$ of 4, or $2\frac{2}{3}$.

In fifth grade, students are continuing to apply this existing knowledge of multiplication of whole numbers and fractions to find the products of two fractions. For example, $\frac{1}{2}$ x $\frac{1}{4}$ is interpreted as half of one fourth or one fourth of half. Students in fifth grade must also make generalizations about the product as a whole numbers and fractions are multiplied by fractions less than one and fractions greater than one.

Making meaning of the equations as a fraction of a whole number or a fraction of a fraction helps students understand why the product is smaller than the whole number factor when the fraction factor is less than one, as well as why the product is greater than the whole number factor when the fraction factor is greater than one. Students should also make generalizations about what happens to the product when multiplying two fraction factors.

COMMON MISCONCEPTIONS:

Students are applying previous knowledge of whole number multiplication to multiplying fractions by whole numbers and multiplying fractions by other fractions. The most common misconception that occurs is overgeneralization of what happens to the product when two factors are multiplied. Students have experienced that when two whole numbers are multiplied, the product is greater than either of the factors. Students often experience great disequilibrium when multiplying a fraction less than one by a whole number (e.g., $\frac{1}{5}$ x 7) and getting a product less than the whole number factor or two fractions less than one (e.g., $\frac{1}{2}$ x $\frac{1}{3}$) and the product is less than both of the factors. It is important

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that students spend time investigating and proving this idea so that they have ownership of the knowledge.

Task Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I video or picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

- Show the Act 1 video "Sweet Tart Hearts" to students. http://mikewiernicki3act.wordpress.com/
- Ask students what they noticed in the video, what they wonder about, and what questions they have about what they saw in the video.
- Give each student a copy of the Student Recording Sheet. Have students record their questions and curiosities in the Act 1 section that asks "What questions come to your mind?" Consider doing a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group. Students may need to watch the video several times.
- Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.
- Share the main question that will be investigated during today's lesson. In the list below it is denoted with an asterisk. (*) Students will record the main question on their recording sheet.
- Ask students to estimate how many total candies are in the mug. Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on a number line.
- Also ask students to estimate how many candies of each color are in the mug. (think-pair-share). Students will write their best estimate for each color candy and place it in the appropriate spot on the number line between the estimated range for the total number of candies in the glass. Space is available on the recording sheet for students to record open number line with all three estimates.

Anticipated questions students may ask and wish to answer:

- How many candies are in the glass?
- *How many candies of each color are in the glass?
- What fraction of the candies are pink?
- Which color has the most candies in the bag?
- How many more pink candies are there than green candies?

**Main question(s) to be investigated*
Act 2 – Student Exploration - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

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- During Act 2, students review the main question from Act 1 and decide on the facts, tools, and other information needed to answer the question. The main question for this task is "How many candies of each color are in the glass?" When students decide what they need to solve the problem, they should ask for those things. The document "Sweet Tart Heart Fractions" contains images of the glass with each colored candy inside so that students can infer the fraction of the glass that is used when each color of sweet tart hearts is placed inside. Copies of the images can be given to the students at their request. The document can be found under "Act 2" using the link to the video. Additionally, the link to the video contains an image of the nutrition facts available on the bag of sweet tart hearts under the section titled "Act 2." It is pivotal to the problem solving process that students decide what is needed without being given the information up front.
- Students can record information that they need to solve the problem, given information, estimates and work on the student recording sheet under Act 2.
- The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
    - What is the problem you are trying to solve?
    - What do you think affects the situation?
    - Can you explain what you've done so far?
    - What strategies are you using?
    - What assumptions are you making?
    - What tools or models may help you?
    - Why is that true?
    - Does that make sense?

Additional Information for Act 2

Sweet Tart Hearts Fractions document and Nutrition Facts label http://mikewiernicki3act.wordpress.com/

Important note: Although students will only investigate the main question for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 3 – Whole Group – Share solutions and strategies.

- Students present their solutions and strategies and compare them.
- Reveal possible solutions by projecting or displaying the Act 2 picture and have students share the solutions they arrived at when finding how many candies of each color were in the glass. Record student solutions on the board, as you would in a Number Talk.
- After collecting the different solutions, have students defend a solution by explaining the fractions used to determine the amount of each colored candy in the glass. Students can

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explain how applied their knowledge of multiplying fractions by whole numbers by saying, "*I believe that the pink candies take up one third of the glass. It looks like the glass can be divided into three equal pieces. To find the number of pink candies I found one third of 286, which was about 95 candies*." Students can also defend their thinking for determining the fraction of the glass each candy used. A student might say, "*Since I think that the pick is one third of the cup, it looks like the green is one third of the pink. One third of one third is one ninth, so the green is probably one ninth of the total candies in the glass.*"

- As the discussion progresses, students can discuss why different solutions were reached as they solved the problem. If students don't pick up on this idea through the course of the discussion, it is appropriate for the teacher to ask students to think about why various solutions were given.
- Have students record their result in the Act 3 section of the student recording sheet. Students can also plot their solution on the open number line on the student recording sheet.
- Lead discussion to compare these, asking questions such as:
    - How reasonable was your estimate?
    - Which strategy was most efficient?
    - Can you think of another method that might have worked?
    - What might you do differently next time?

Act 4, The Sequel - "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

Challenge students to revisit the other student generated questions recorded in Act 1. (Refer students to the list created on their recording sheet or the class chart created during Act 1.) Allow students to choose one of the questions from Act 1 and investigate it independently. Allow students to record thinking about the problem in the same manner as the student recording sheet used during this lesson.

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?
- How was your result different than other results in the class? Give examples that prove or disprove your results.
- What do you know about multiplying fractions that you applied to this problem solving situation?

DIFFERENTIATION

Extension

Students can use a clear, plastic cup and fill it with colored centimeter cubes. Students determine the number of cubes used to fill the cup. Next, they can sort the cubes by color and count how many cubes there are of each color. Then, students place the different colors in the cup one at a time, like

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the video of the Sweet Tart Hearts. Students draw the cup and where it is filled to. Students determine what fraction of the plastic cup each color uses.

Intervention
Students need to work with the document "Sweet Tart Heart Fractions" and partition the glass into equal size pieces so they can think through the fractions being represented by each colored candy in the mug.

*Intervention Table

TECHNOLOGY CONNECTIONS

- http://learnzillion.com/lessons/125-multiply-fractions-by-fractions-finding-a-part-of-a-part This Learnzillion video, "Multiplying Fractions by Fractions: Finding Part of a Part" shows how to draw and partition models to find the product of two fraction factors that are less than one. Students learn about finding a fraction of a fraction as they solve problems in the tutorial.

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Three Act Task Student Recording Sheet Name _____

ACT 1

What questions come to your mind?

Main Question: _____

What is your first estimate and why?

Record an estimate that is too low and an estimate that is too high.

On an empty number line, record all three estimates made above.

ACT 2

What information would you like to know or need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

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If possible, give a better estimation with this information: _____

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

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Where are the cookies?

Back to Task Table

Formative Assessments Lessons (FALs)

What is a Formative Assessment Lesson (FAL)? The Formative Assessment Lesson is designed to be part of an instructional unit typically implemented approximately two-thirds of the way through the instructional unit. The results of the tasks should then be used to **inform** the instruction that will take place for the remainder of the unit. Formative Assessment Lessons are intended to support teachers in formative assessment. They both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

What does a Formative Assessment Lesson look like in action? Videos of Georgia Teachers implementing FALs can be accessed HERE and a sample of a FAL lesson may be seen HERE

Where can I find more information on FALs? More information on types of Formative Assessment Lessons, their use, and their implementation may be found on the Math Assessment Project's guide for teachers.

Where can I find samples of FALs?

Formative Assessment Lessons can also be found at the following sites:

Mathematics Assessment Project
Kenton County Math Design Collaborative
MARS Tasks by grade level

A **sample FAL** with extensive dialog and suggestions for teachers may be found HERE. This resource will help teachers understand the flow and purpose of a FAL.

Where can I find more training on the use of FALs?

All of our Georgia RESAs have had a math specialist trained to provide instruction on the use of formative assessment lessons in the classroom. The request for training should be made through the teacher's local RESA and can be referenced by asking for more information on the Mathematics Design Collaborative (MDC). Also, if done properly, these lessons should take about 120-150 minutes, 2-3 classroom periods.
Sources of Information: Vicki Mixon, Former MDC (Math Design Collaborative) trainer, http://www.reneeyates2math.com/ and from The Mathematics Assessment Project and http://melissatabor.wikispaces.com/Formative+Assessment+Lessons+%28FALs%29 Division and Interpreting Remainders (FAL)

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Practice Task – Dividing with Unit Fractions

Back to Task Table

In this task, students will use reasoning to solve fraction division word problems.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

[1]Students able to multiply fractions can develop strategies to divide fractions by reasoning about the relationship between multiplication and division. But the algorithm for division of a fraction by a fraction is not a requirement at this grade.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students engaging in this task should be familiar with multiple fraction models, including but not limited to, fraction strips, and circle fraction pieces, color tiles, colored beads, and pattern blocks. The problems in this task were adapted from problems found in Teaching Student Centered Mathematics, Volume II, by John A. Van de Walle and LouAnn H. Lovin.

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COMMON MISCONCEPTIONS

Prior experiences with whole number division may lead students to believe that division always results in a smaller number. Using models when dividing with fractions will help student to realize that the results can be larger.

ESSENTIAL QUESTIONS

- What does dividing a unit fraction by a whole number look like?
- What does dividing a whole number by a unit fraction look like?
- How can we model dividing a unit fraction by a whole number with manipulatives and diagrams?
- How can we model dividing a whole number by a unit fraction using manipulatives and diagrams?

MATERIALS

- Dividing with Unit Fractions student sheet
- Accessible manipulatives
- Grid paper

GROUPING

Pair/Individual

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments:

This task was developed to give students an opportunity to make sense of division with fractional divisors and dividends. It is meant to involve students in a deeper investigation of the concept of division with unit fractions. Students should be allowed to use manipulatives such as fraction bars and/or draw representations of their thinking. Using grid paper may facilitate this, but is not necessary. Students may wish to use other representations based on their own understandings. Creating these representations allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

Teacher Notes:

Before beginning this task, have a discussion with your students using the following computations. It is important for students to have plenty of quiet think time for each individual computation is presented. Likewise, after the quiet think time, students should share their strategies before moving to the next problem.

What does it mean when you see the computation 16 ÷ 2?
What do you think when you see the computation 16 ÷ ½?
What do you think of when you see the computation ¼ ÷ 6?

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Part I
Introduce the task and make sure students understand what they are expected to do. Allow students to share ideas about the task with the group. Make sure students have materials necessary for investigating this task.

It is important that students explain why some of the quotients are larger than the divisor and the dividend and why some are smaller. Evidence using representations, words, and numbers must be presented for each case.

Allow students to work in pairs to answer the questions posed.

Listen to student thinking and provide support with thought provoking questions like the ones below.

Students may use several strategies to solve this problem.

Some students may use 1 inch color tiles or snap cubes to represent the whole, or they may find that fraction bars or strips are easier to use. It is possible for students to use color tiles or snap cubes by assigning a fractional length to each tile. For example, students may decide that the length of each tile represents ¼, rather than 1. This presents its own challenges, but the struggle is where the learning happens.

Other students may use grid paper in the same manner presented above. A variety of grid sizes may be useful for this task.

FORMATIVE ASSESSMENT QUESTIONS

- How do you know that your answer is correct?
- What is happening here? Show your thinking.
- What kind of representation will you use to show your thinking?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain what you have found.

After enough time has been devoted to the task, bring pairs of students together to share in groups of 4 to 6 students. As students share, listen for different explanations and look for different representations.

When students have finished the sharing, come back to the large group and begin the closing of the lesson. The goal of this closing is to help students make connections about areas of rectangles with fractional dimensions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work.

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DIFFERENTIATION

Extension

- Students can write their own contextual problems. They can work with a partner to solve and check each other's problems. Their work should be supported with use of manipulatives and/or drawings.
- Students can draw models of the fraction division problems using bar models and using number lines.

Intervention

- Use an on-line interactive program that will create the array after the student inputs both factors, such as this website from the National Library of Virtual Manipulatives. **http://nlvm.usu.edu/en/nav/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html**

*Intervention Table

TECHNOLOGY CONNECTION

http://www.learner.org/courses/learningmath/number/session9/part_a/area_division.html *This resource is for teacher understanding.* This is another lesson about division of fractions. This is where students will be in sixth grade, but this is a great resource to build teacher understanding of fraction division.

http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

*https://www.conceptuamath.com/app/tool/divide-partitive
https://www.conceptuamath.com/app/tool/divide-measurement
Highly interactive tool that allows students/teachers to represent fraction division with number lines and area models, and also allows the user to input a customized word problem.

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Name _____ Date _____

Dividing with Fractions

Directions:

- Use your reasoning skills to solve the following fraction division problems.
- Use manipulatives of your choice to investigate and determine the quotient.
- Create a representation to show your results. Grid paper may be used if you find that helpful.
- Compare the quotient to the dividend and divisor. Explain whether the quotient is larger or smaller and why.

1. The pizza slices served at Connor's Pizza Palace are $\frac{1}{4}$ of a whole pizza. There are three pizzas ready to be served. 14 children come in for lunch. Is there enough pizza for every child? Show your mathematical thinking.

2. I am building a patio. Each section of my patio requires $\frac{1}{3}$ of a cubic yard of concrete. The concrete truck holds 2 cubic yards of concrete. How many sections can I make with the concrete in the truck? Show your mathematical thinking.

3. You have just bought 6 pints of Ben & Jerry's ice cream for a party you are having. If you serve each of your guests $\frac{1}{3}$ of a pint of ice cream, how many guests can you serve? Show your mathematical thinking.

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4. Lura has 4 yards of material. She is making clothes for her American Girl dolls. Each dress requires $\frac{1}{6}$ yards of material. How many dresses will she be able to make from the material she has? Show your mathematical thinking.

5. Ms. Held is wrapping presents. She has $\frac{1}{2}$ yard of ribbon to use for 3 presents. How many yards of ribbon can she use for each present?

6. If 4 people share $\frac{1}{3}$ of a cake, how much of the whole cake will each person get?

7. You and your friend are making an art project. The art teacher has given you $\frac{1}{6}$ of a foot of tape. How many feet of tape will each of you receive if you both get the same amount?

8. A group of 5 students are running a relay race. If the race is $\frac{1}{4}$ of a mile long and each student runs the same distance, how far does each student need to run?

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Culminating Task - Adjusting Recipes

Back to Task Table

The purpose of the task is to introduce real life problem while reinforcing the concepts of fractions taught throughout the unit.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

MGSE5.NF.2 Solve word problems involving addition and subtraction of fractions, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < ½.

MGSE5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. Example: $\frac{3}{5}$ can be interpreted as "3 divided by 5 and as 3 shared by 5".

MGSE5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Apply and use understanding of multiplication to multiply a fraction or whole number by a fraction.
   Examples: $\frac{a}{b} \times q$ as $\frac{a}{b} \times \frac{q}{1}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

MGSE5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

MGSE5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

[1]Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

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b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*
c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Within this unit, students were required to add, subtract and multiply fractions. They should have had experiences dividing whole numbers by unit fractions and dividing unit fractions by whole numbers. They will apply their understanding within this culminating task.

COMMON MISCONCEPTIONS

This is a culminating task that incorporates all standards for the unit. Students may still struggle with misconceptions listed in unit tasks.

ESSENTIAL QUESTIONS

- How do we use all four operations to solve problems with fractions?
- How can we use estimation to determine whether our answers are reasonable?

MATERIALS

- Adjusting a Recipe student sheet
- Accessible manipulatives
- Grid paper

GROUPING

Individual/Partner Task

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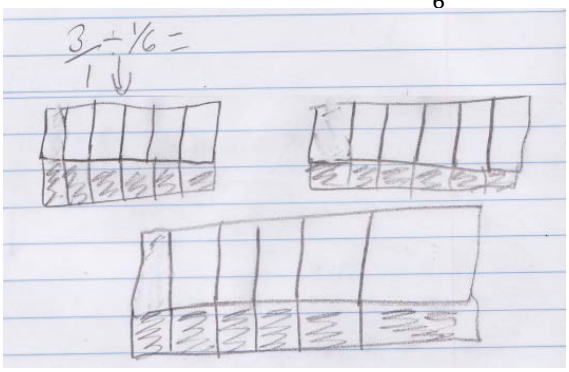
TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

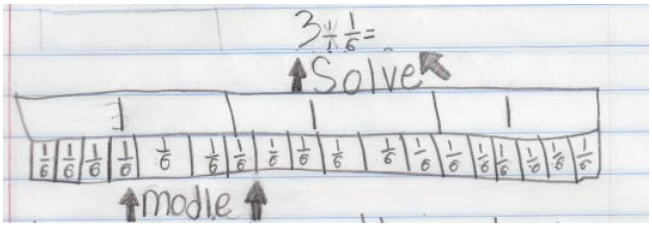
Comments

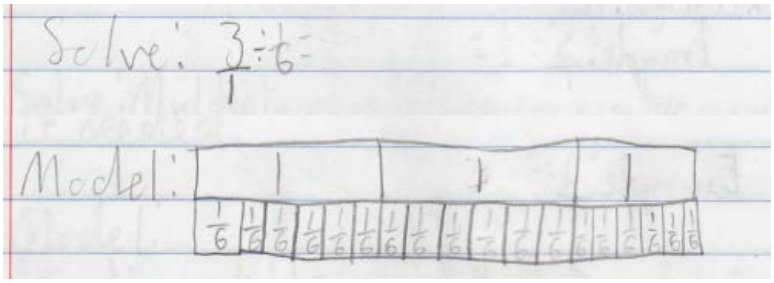
This task was developed as a means to assess students' understanding of operations of fractions in a real world context. It is designed to allow students the freedom to approach the problem using a variety of strategies and allows for many different solutions. Students should be allowed to draw representations of their thinking. This allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

Student Work Samples

The following samples of student work show examples of how fraction division can be modeled. The problem in these examples is $3 \div \frac{1}{6}$.







This example shows a student misconception.

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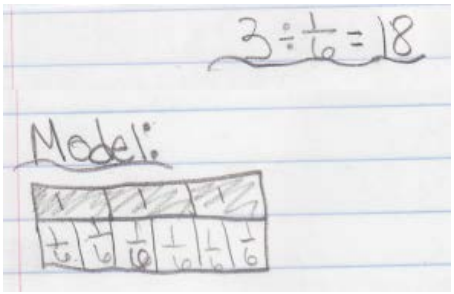

Task Directions
Introduce the problem and be sure everyone is clear with the context. You may wish to use the sample cookie recipe (included at the end of this task) or find another recipe that has several fractions in it to use.
Facilitate a preliminary discussion with the class to make sure students understand all vocabulary as well as the context of the problem, before students get to work. After allowing students to share their initial thoughts, ask them to work in pairs or individually to investigate the following:

How would you rewrite the recipe for twice as many people? Show your mathematical thinking and explain how you know the rewritten recipe is correct.

How would you rewrite the recipe for half as many people? Show your mathematical thinking and explain how you know the rewritten recipe is correct.

Is it possible to adjust the recipe to make 30 cookies? What would you have to do to the measurements of each of the ingredients?

Explain how you would adjust your recipe to feed everyone in our class (don't forget the teacher!) Is it possible to get the exact number of cookies for our class by adjusting the recipe? If not, adjust the recipe to get as close as possible (make sure everyone gets a cookie).

Possible struggles students may have can be turned into wonderful inquiries! As students investigate the screen areas, you may notice them:

Using array models or doubling ideas of multiplication to show how to double a recipe for some ingredients.

Using a multiplication of fractions algorithm. This strategy may promote discussion, so please allow students the freedom to make sense of this in the closing part of the lesson.

Be on the lookout for students who, when working on halving the recipe, divide by ½ rather than by 2. This may be a lack of understanding of the concept of division. Scaffolding the students' learning with thought provoking questions can help students strengthen their conceptual understanding.

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FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answer is correct?
- Does dividing by 2 (or ½) help solve this problem? How do you know?
- Did you develop a strategy to find your answers?
- Did you identify any patterns or rules? Explain!

After enough time has been devoted to the task, ask students to make recipe cards to prepare for the closing of the lesson. Recipe cards should be clear enough for others in the class to understand their thinking, but should not just be the figuring that was initially done copied over again. The recipe cards should be clear and concise presentations of any important ideas and strategies students wish to present.
Some ideas to encourage discussion about in the presentations of student work:
Estimation, using whole number estimates can help determine whether measurements of ingredients are in the ballpark.
How fractions/mixed numbers are decomposed can make a difference.
Students who use any algorithm should also show understanding of the algorithm used with some kind of model.

DIFFERENTIATION:

Extension

- Students can find their own recipes and determine the quantity of ingredients needed to:
  - Double the recipe
  - Make half of the recipe
  - Make 30 cookies
- For an extension of this activity, change to number of persons so that students can analyze the patterns using a different number of guests.

Intervention

- Arrange the tables to seat 48 people, rather than 120. Help students begin the task using an organizational strategy such as is described in the "Background Knowledge" section above.
- Work with a small group or partner.
- Model an easier problem with tables, chair, and students.
- Discuss finding factors and organizing thinking.

*Intervention Table

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Name _____ Date _____

Adjusting Recipes

Some fifth grade classes are making cookies to demonstrate their understanding of fractions. Some classes will make sugar cookies and the other classes will make oatmeal cookies. They need to adjust the recipes to make larger or smaller quantities. They need your help with determining the quantities of ingredients needed.

Sugar Cookie Recipe – makes 12 cookies

$\frac{1}{4}$ c. butter, softened
$\frac{1}{2}$ c. white sugar
1 egg
$\frac{1}{4}$ tsp. vanilla
1 c. flour
$\frac{1}{4}$ tsp. baking soda
$\frac{1}{8}$ tsp. baking powder

Directions:
Preheat oven to 375°F (190°C). In a small bowl, stir together flour, baking soda, and baking powder. Set aside. In a large bowl, cream together the butter and sugar until smooth. Beat in egg and vanilla. Gradually blend in the dry ingredients. Shape dough into balls and place onto ungreased cookie sheets. Bake 8 to 10 minutes in the preheated oven, or until golden. Let stand on cookie sheet two minutes before removing to cool on wire racks.

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Oatmeal Cookie Recipe – makes 24 cookies

$\frac{2}{3}$ c. butter, softened
$\frac{3}{4}$ c. white sugar
1 c. brown sugar
2 eggs
$\frac{1}{3}$ tsp. vanilla
2 c. flour
$\frac{1}{3}$ tsp. baking soda
1 tsp. salt
$\frac{1}{2}$ tsp. cinnamon
3 c. oats

Directions:

Preheat oven to 375°F (190°C). In a small bowl, stir together flour, baking soda, salt and cinnamon. Set aside. In a large bowl, cream together the butter and sugar until smooth. Beat in eggs and vanilla. Gradually blend in the dry ingredients and oats. Shape dough into balls and place onto ungreased cookie sheets. Bake 8 to 10 minutes in the preheated oven, or until golden. Let stand on cookie sheet two minutes before removing to cool on wire racks.

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Use separate paper to answer the following questions. Show your work and explain your answers.

1. If one class needs to change the sugar cookie recipe so that it will make 36 cookies, how much of each ingredient will they need?
2. For a different class, the sugar cookie recipe needs to be changed so that it will make 6 cookies. Divide each ingredient by 2 to determine how much of each ingredient will be needed.
3. The oatmeal cookie recipe needs to be adjusted so that it will make 36 cookies. Will it be possible to do this? If so, how much of each ingredient will be needed?
4. A teacher wants to bring enough of each ingredient to make the sugar cookies and oatmeal cookies as they are written on the recipes. How much of each ingredient will she need?
5. Which recipe uses more white sugar? How much more white sugar does it need?
6. Which recipe uses more baking soda? How much more baking soda does it need?
7. Explain how you would adjust your recipe to feed everyone in your class (don't forget the teacher!) Is it possible to get the exact number of cookies by adjusting the recipe?
8. If not, adjust the recipe to get as close as possible (make sure everyone gets a cookie). How could share the left-over cookie(s) be shared?
9. How many batches of cookies would be needed if every student in the class receives the same number of cookies with no cookies left over?

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