Algebra I
Vocabulary Word Wall Cards

Mathematics vocabulary word wall cards provide a display of mathematics content words and associated visual cues to assist in vocabulary development. The cards should be used as an instructional tool for teachers and then as a reference for all students. **The cards are designed for print use only.**

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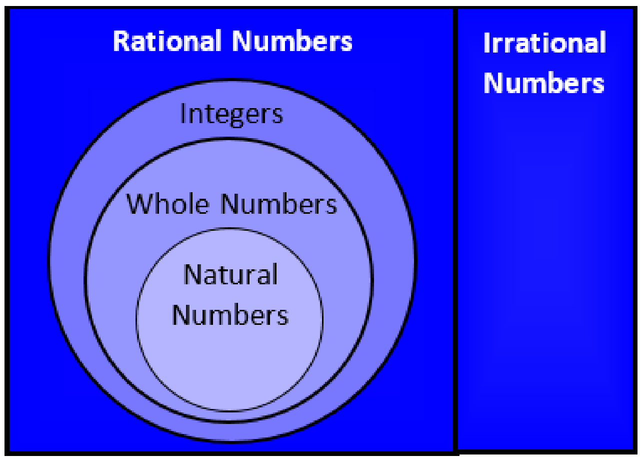
Functions

Statistics

Real Numbers

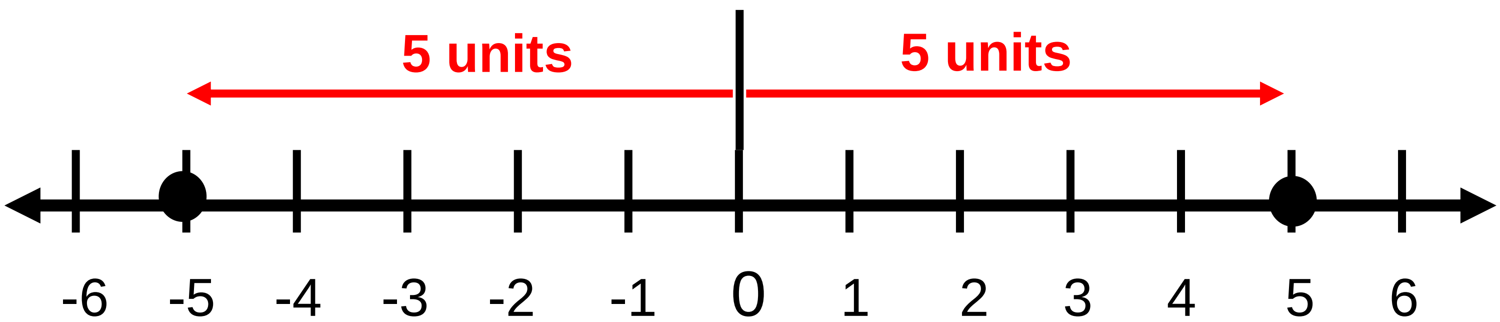
The set of all rational and irrational numbers



Natural Numbers	{1, 2, 3, 4 …}
Whole Numbers	{0, 1, 2, 3, 4 …}
Integers	{… -3, -2, -1, 0, 1, 2, 3 …}
Rational Numbers	the set of all numbers that can be written as the ratio of two integers with a non-zero denominator (e.g., $2\frac{3}{5}$, -5, 0.3, $\sqrt{16}$, $\frac{13}{7}$)
Irrational Numbers	the set of all nonrepeating, nonterminating decimals (e.g, $\sqrt{7}$, π, -.23223222322223…)

Absolute Value

$|5| = 5$ $|-5| = 5$



The distance between a number and zero

Order of Operations

Grouping Symbols	$(\ )$ $\sqrt{\ }$ $\{\ \}$ $\lvert\ \rvert$ $[\ ]$ $-$
Exponents	a^n
Multiplication Division	→ **Left to Right**
Addition Subtraction	→ **Left to Right**

Expression

A representation of a quantity that may contain numbers, variables or operation symbols

$$x$$

$$-\sqrt{26}$$

$$3^4 + 2m$$

$$ax^2 + bx + c$$

$$3(y + 3.9)^2 - \frac{8}{9}$$

Variable

$$2(y + \sqrt{3})$$

$$9 + x = 2.08$$

$$d = 7c - 5$$

$$A = \pi r^2$$

Coefficient

$$(-4) + 2x$$

$$-7y\sqrt{5}$$

$$\frac{2}{3}ab - \frac{1}{2}$$

$$\pi r^2$$

Term

$$\underbrace{3x}+\underbrace{2y}-\underbrace{8}$$

3 terms

$$\underbrace{-5x^2}-\underbrace{x}$$

2 terms

$$\underbrace{\frac{2}{3}ab}$$

1 term

Scientific Notation

$$a \times 10^{n}$$

$1 \leq |a| < 10$ and n is an integer

Examples:

Standard Notation	Scientific Notation
17,500,000	1.75×10^{7}
-84,623	-8.4623×10^{4}
0.0000026	2.6×10^{-6}
-0.080029	-8.0029×10^{-2}
$(4.3 \times 10^{5})(2 \times 10^{-2})$	$(4.3 \times 2)(10^{5} \times 10^{-2}) =$ $8.6 \times 10^{5+(-2)} = 8.6 \times 10^{3}$
$\frac{6.6 \times 10^{6}}{2 \times 10^{3}}$	$\frac{6.6}{2} \times \frac{10^{6}}{10^{3}} = 3.3 \times 10^{6-3} =$ 3.3×10^{3}

Exponential Form

exponent

$$a^n = \underbrace{a \cdot a \cdot a \cdot a}_{n \text{ factors}} \ldots, \; a \neq 0$$

base

n factors

Examples:

$2 \cdot 2 \cdot 2 = 2^3 = 8$
$n \cdot n \cdot n \cdot n = n^4$
$3 \cdot 3 \cdot 3 \cdot x \cdot x = 3^3x^2 = 27x^2$

Negative Exponent

$$a^{-n} = \frac{1}{a^n}, \ a \neq 0$$

Examples:

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
$\frac{x^4}{y^{-2}} = \frac{x^4}{\frac{1}{y^2}} = \frac{x^4}{1} \cdot \frac{y^2}{1} = x^4y^2$
$(2-a)^{-2} = \frac{1}{(2-a)^2}, \ a \neq 2$

Zero Exponent

$$a^0 = 1, \; a \neq 0$$

Examples:

$(-5)^0 = 1$
$(3x + 2)^0 = 1$
$(x^2y^{-5}z^8)^0 = 1$
$4m^0 = 4 \cdot 1 = 4$
$\left(\frac{2}{3}\right)^0 = 1$

Product of Powers Property

$$a^m \cdot a^n = a^{m+n}$$

Examples:

$x^4 \cdot x^2 = x^{4+2} = x^6$
$a^3 \cdot a = a^{3+1} = a^4$
$w^7 \cdot w^{-4} = w^{7+(-4)} = w^3$

Power of a Power Property

$$(a^m)^n = a^{m \cdot n}$$

Examples:

$(y^4)^2 = y^{4 \cdot 2} = y^8$
$(g^2)^{-3} = g^{2 \cdot (-3)} = g^{-6} = \frac{1}{g^6}$

Power of a Product Property

$$(ab)^m = a^m \cdot b^m$$

Examples:

$(-3a^4b)^2 = (-3)^2 \cdot (a^4)^2 \cdot b^2 = 9a^8b^2$
$\dfrac{-1}{(2x)^3} = \dfrac{-1}{2^3 \cdot x^3} = \dfrac{-1}{8x^3}$

Quotient of Powers Property

$$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$$

Examples:

$\frac{x^6}{x^5} = x^{6-5} = x^1 = x$
$\frac{y^{-3}}{y^{-5}} = y^{-3-(-5)} = y^2$
$\frac{a^4}{a^4} = a^{4-4} = a^0 = 1$

Power of Quotient Property

$$\left(\frac{a}{b}\right)^{m} = \frac{a^{m}}{b^{m}}, \; b \neq 0$$

Examples:

$$\left(\frac{y}{3}\right)^{4} = \frac{y^{4}}{3^{4}} = \frac{y}{81}$$

$$\left(\frac{5}{t}\right)^{-3} = \frac{5^{-3}}{t^{-3}} = \frac{\frac{1}{5^{3}}}{\frac{1}{t^{3}}} = \frac{1}{5^{3}} \cdot \frac{t^{3}}{1} = \frac{t^{3}}{5^{3}} = \frac{t^{3}}{125}$$

Polynomial

Example	Name	Terms
7 $6x$	monomial	1 term
$3t - 1$ $12xy^3 + 5x^4y$	binomial	2 terms
$2x^2 + 3x - 7$	trinomial	3 terms

Nonexample	Reason
$5m^n - 8$	variable exponent
$n^{-3} + 9$	negative exponent

Degree of a Polynomial

The largest exponent or the largest sum of exponents of a term within a polynomial

Polynomial	Degree of Each Term	Degree of Polynomial
$-7m^3n^5$	$-7m^3n^5 \rightarrow$ degree 8	8
$2x + 3$	$2x \rightarrow$ degree 1 $3 \rightarrow$ degree 0	1
$6a^3 + 3a^2b^3 - 21$	$6a^3 \rightarrow$ degree 3 $3a^2b^3 \rightarrow$ degree 5 $-21 \rightarrow$ degree 0	5

Leading Coefficient

The coefficient of the first term of a polynomial written in descending order of exponents

Examples:

$7a^3 - 2a^2 + 8a - 1$
$-3n^3 + 7n^2 - 4n + 10$
$16t - 1$

Add Polynomials (Group Like Terms – Horizontal Method)

Example:

$$(2g^2 + 6g - 4) + (g^2 - g)$$

$$= 2g^2 + 6g - 4 + g^2 - g$$

(Group like terms and add)

$$= (2g^2 + g^2) + (6g - g) - 4$$

$$= 3g^2 + 5g - 4$$

Add Polynomials
(Align Like Terms – Vertical Method)

Example:

$$(2g^3 + 6g^2 - 4) + (g^3 - g - 3)$$

(Align like terms and add)

$$\begin{array}{r} 2g^3 + 6g^2 \qquad\quad - 4 \\ +\ \ g^3 \qquad\qquad - g - 3 \\ \hline 3g^3 + 6g^2 - g - 7 \end{array}$$

Subtract Polynomials
(Group Like Terms - Horizontal Method)

Example:

$$(4x^2 + 5) - (-2x^2 + 4x - 7)$$

(Add the inverse.)

$$= (4x^2 + 5) + (2x^2 - 4x + 7)$$

$$= 4x^2 + 5 + 2x^2 - 4x + 7$$

(Group like terms and add.)

$$= (4x^2 + 2x^2) - 4x + (5 + 7)$$

$$= 6x^2 - 4x + 12$$

Subtract Polynomials
(Align Like Terms - Vertical Method)

Example:

$$(4x^2 + 5) - (-2x^2 + 4x - 7)$$

(Align like terms then add the inverse and add the like terms.)

$$\begin{array}{r} 4x^2 \qquad\quad + 5 \\ -\underline{(-2x^2 + 4x - 7)} \end{array} \rightarrow \begin{array}{r} 4x^2 \qquad\quad + 5 \\ \underline{+ 2x^2 - 4x + 7} \\ 6x^2 - 4x + 12 \end{array}$$

Multiply Binomials

Apply the distributive property.

$$(a + b)(c + d) =$$
$$a(c + d) + b(c + d) =$$
$$ac + ad + bc + bd$$

Example: $(x + 3)(x + 2)$

$$= (x + 3)(x + 2)$$
$$= x(x + 2) + 3(x + 2)$$
$$= \mathrm{x}^2 + 2x + 3x + 6$$
$$= x^2 + 5x + 6$$

Multiply Polynomials

Apply the distributive property.

$$(x + 2)(3x^2 + 5x + 1)$$

$$(x + 2)(3x^2 + 5x + 1)$$

$$= x(3x^2 + 5x + 1) + 2(3x^2 + 5x + 1)$$

$$= x \cdot 3x^2 + x \cdot 5x + x \cdot 1 + 2 \cdot 3x^2 + 2 \cdot 5x + 2 \cdot 1$$

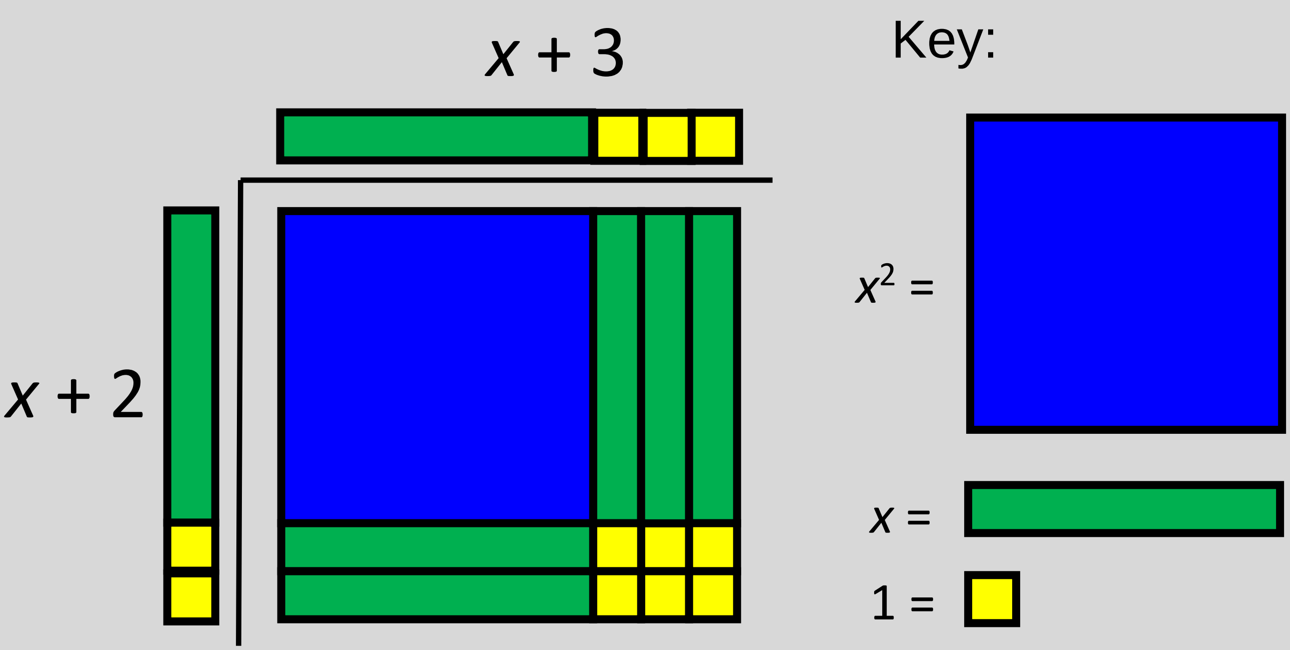
$$= 3x^3 + 5x^2 + x + 6x^2 + 10x + 2$$

$$= 3x^3 + 11x^2 + 11x + 2$$

Multiply Binomials
(Model)

Apply the distributive property.

Example: $(x + 3)(x + 2)$



$x^2 + 2x + 3x + 6 = x^2 + 5x + 6$

Multiply Binomials

(Graphic Organizer)

Apply the distributive property.

Example: $(x + 8)(2x - 3)$

$= (x + 8)(2x + -3)$

	$2x$	$+ \ -3$
x	$2x^2$	$-3x$
$+ \ 8$	$16x$	-24

$2x^2 + 16x + -3x + -24 = 2x^2 + 13x - 24$

Multiply Binomials
(Squaring a Binomial)

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Examples:

$$(3m + n)^2 = 9m^2 + 2(3m)(n) + n^2$$
$$= 9m^2 + 6mn + n^2$$

$$(y - 5)^2 = y^2 - 2(5)(y) + 25$$
$$= y^2 - 10y + 25$$

Multiply Binomials (Sum and Difference)

$$(a + b)(a - b) = a^2 - b^2$$

Examples:

$(2b + 5)(2b - 5) = 4b^2 - 25$
$(7 - w)(7 + w) = 49 - w^2$

Factors of a Monomial

The number(s) and/or variable(s) that are multiplied together to form a monomial

Examples:	**Factors**	**Expanded Form**
$5b^2$	$5 \cdot b^2$	$5 \cdot b \cdot b$
$6x^2y$	$6 \cdot x^2 \cdot y$	$2 \cdot 3 \cdot x \cdot x \cdot y$
$\frac{-5p^2q^3}{2}$	$\frac{-5}{2} \cdot p^2 \cdot q^3$	$\frac{1}{2} \cdot (-5) \cdot p \cdot p \cdot q \cdot q \cdot q$

Factoring
(Greatest Common Factor)

Find the greatest common factor (GCF) of all terms of the polynomial and then apply the distributive property.

Example: $20a^4 + 8a$

$$2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot a \cdot a + 2 \cdot 2 \cdot 2 \cdot a$$

common factors

$$\text{GCF} = \underbrace{2 \cdot 2 \cdot a} = 4a$$

$$20a^4 + 8a = 4a(5a^3 + 2)$$

Factoring

(By Grouping)

For trinomials of the form

$ax^2 + bx + c$

Example: $3x^2 + 8x + 4$

ac = 3 • 4 = 12

Find factors of ac that add to equal b

12 = 2 • 6 ⟶ 2 + 6 = 8

$3x^2 + 2x + 6x + 4$ — Rewrite $8x$ as $2x + 6x$

$(3x^2 + 2x) + (6x + 4)$ — Group factors

$x(3x + 2) + 2(3x + 2)$ — Factor out a common binomial

$(3x + 2)(x + 2)$

Factoring

(Perfect Square Trinomials)

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Examples:

$$x^2 + 6x + 9 = x^2 + 2 \cdot 3 \cdot x + 3^2$$
$$= (x + 3)^2$$

$$4x^2 - 20x + 25 = (2x)^2 - 2 \cdot 2x \cdot 5 + 5^2$$
$$= (2x - 5)^2$$

Factoring
(Difference of Squares)

$$a^2 - b^2 = (a + b)(a - b)$$

Examples:

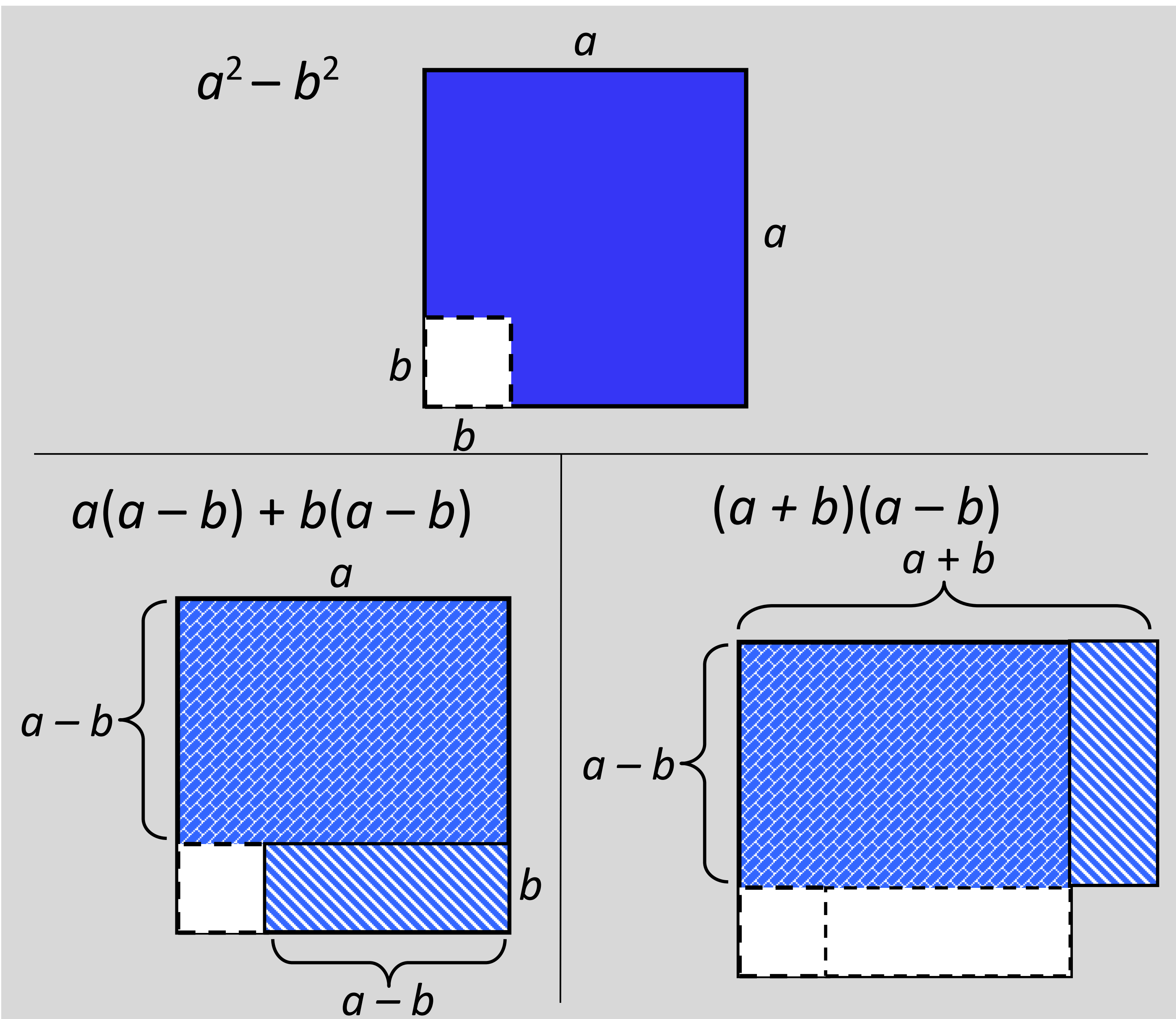
$$x^2 - 49 = x^2 - 7^2 = (x + 7)(x - 7)$$

$$4 - n^2 = 2^2 - n^2 = (2 - n)\ (2 + n)$$

$$9x^2 - 25y^2 = (3x)^2 - (5y)^2$$
$$= (3x + 5y)(3x - 5y)$$

Difference of Squares
(Model)

$$a^2 - b^2 = (a + b)(a - b)$$



Divide Polynomials
(Monomial Divisor)

Divide each term of the dividend by the monomial divisor

Example:

$$(12x^3 - 36x^2 + 16x) \div 4x$$

$$= \frac{12x^3 - 36x^2 + 16x}{4x}$$

$$= \frac{12x^3}{4x} - \frac{36x^2}{4x} + \frac{16x}{4x}$$

$$= 3x^2 - 9x + 4$$

Divide Polynomials
(Binomial Divisor)

Factor and simplify

Example:

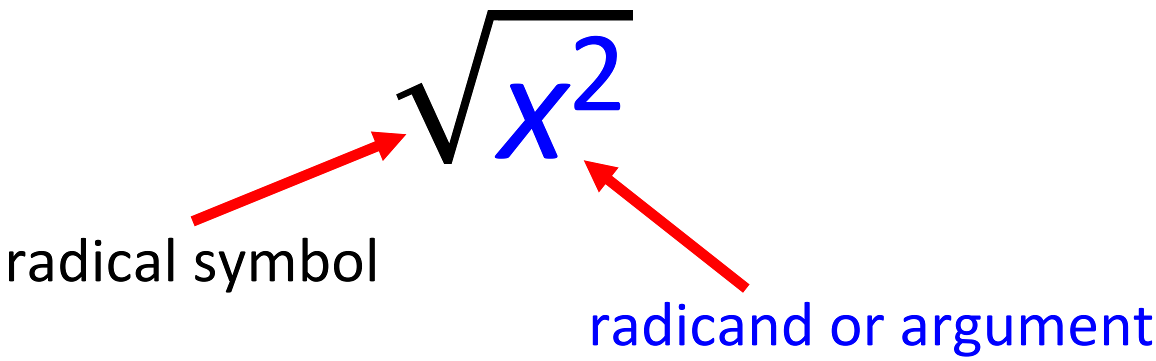
$$(7w^2 + 3w - 4) \div (w + 1)$$

$$= \frac{7w^2 + 3w - 4}{w + 1}$$

$$= \frac{(7w - 4)(w + 1)}{w + 1}$$

$$= 7w - 4$$

Square Root



Simplify square root expressions.

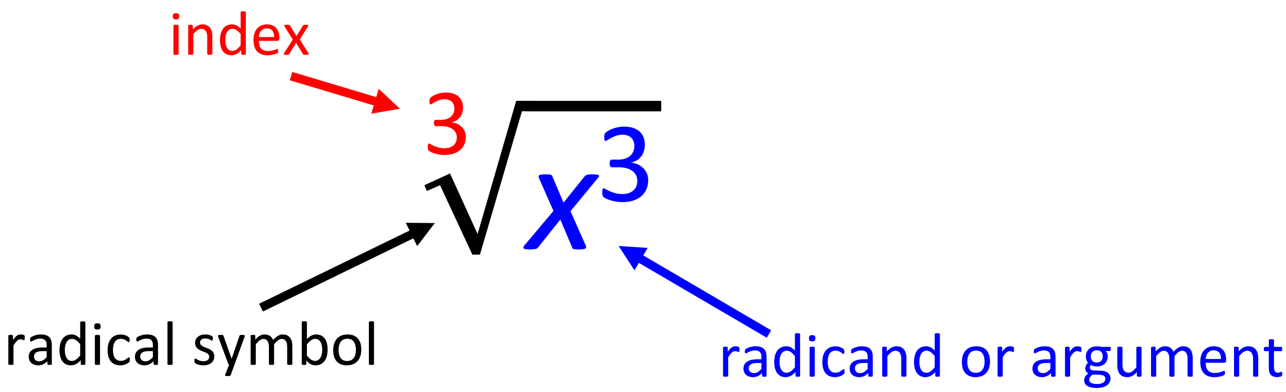
Examples:

$$\sqrt{9x^2} = \sqrt{3^2 \cdot x^2} = \sqrt{(3x)^2} = 3x$$

$$-\sqrt{(x-3)^2} = -(x-3) = -x+3$$

Squaring a number and taking a square root are inverse operations.

Cube Root



Simplify cube root expressions.

Examples:

$\sqrt[3]{64} = \sqrt[3]{4^3} = 4$
$\sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$
$\sqrt[3]{x^3} = x$

Cubing a number and taking a cube root are inverse operations.

Simplify Numerical Expressions Containing Square or Cube Roots

Simplify radicals and combine like terms where possible.

Examples:

$$\frac{1}{2} - \sqrt{32} - \frac{11}{2} + \sqrt{8}$$
$$= -\frac{10}{2} - 4\sqrt{2} + 2\sqrt{2}$$
$$= -5 - 2\sqrt{2}$$

$$\sqrt{18} - 2\sqrt[3]{27} = 3\sqrt{2} - 2(3)$$
$$= 3\sqrt{2} - 6$$

Add and Subtract Monomial Radical Expressions

Add or subtract the numerical factors of the like radicals.

Examples:

$6\sqrt[3]{5} - 4\sqrt[3]{5} - \sqrt[3]{5}$ $= (6 - 4 - 1)\sqrt[3]{5} = \sqrt[3]{5}$
$2x\sqrt{3} + 5x\sqrt{3}$ $= (2 + 5)x\sqrt{3} = 7x\sqrt{3}$
$2\sqrt{3} + 7\sqrt{2} - 2\sqrt{3}$ $= (2 - 2)\sqrt{3} + 7\sqrt{2} = 7\sqrt{2}$

Product Property of Radicals

The nth root of a product equals the product of the nth roots.

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$a \geq 0$ and $b \geq 0$

Examples:

$\sqrt{4x} = \sqrt{4} \cdot \sqrt{x} = 2\sqrt{x}$
$\sqrt{5a^3} = \sqrt{5} \cdot \sqrt{a^3} = a\sqrt{5a}$
$\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$

Quotient Property of Radicals

The nth root of a quotient equals the quotient of the nth roots of the numerator and denominator.

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$a \geq 0$ and $b > 0$

Example:

$$\sqrt{\frac{5}{y^2}} = \frac{\sqrt{5}}{\sqrt{y^2}} = \frac{\sqrt{5}}{y},\ y \neq 0$$

Zero Product Property

If $ab = 0$,
then $a = 0$ or $b = 0$.

Example:

$$(x + 3)(x - 4) = 0$$

$$(x + 3) = 0 \text{ or } (x - 4) = 0$$

$$x = -3 \text{ or } x = 4$$

The solutions or roots of the polynomial equation are -3 and 4.

Solutions or Roots

$$x^2 + 2x = 3$$

Solve using the zero product property.

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = -3 \text{ or } x = 1$$

The solutions or roots of the polynomial equation are -3 and 1.

Zeros

The zeros of a function $f(x)$ are the values of x where the function is equal to zero.

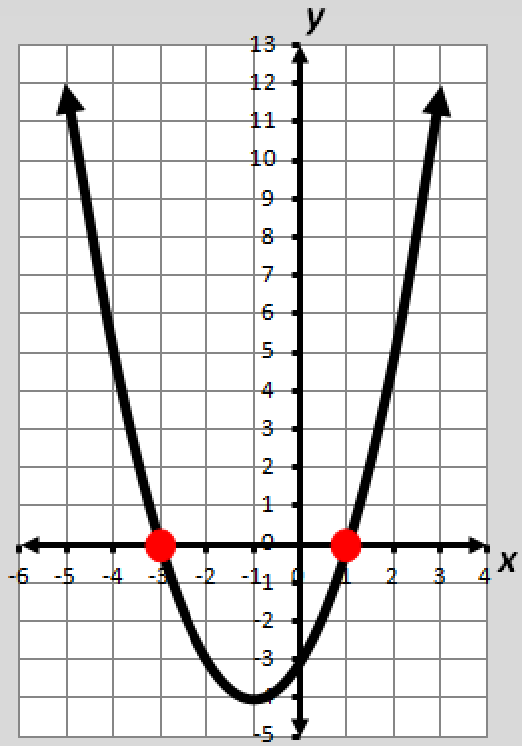
$f(x) = x^2 + 2x - 3$

Find $f(x) = 0$.

$0 = x^2 + 2x - 3$

$0 = (x + 3)(x - 1)$

$x = -3$ or $x = 1$



The zeros of the function $f(x) = x^2 + 2x - 3$ are -3 and 1 and are located at the x-intercepts (-3,0) and (1,0).

The zeros of a function are also the solutions or roots of the related equation.

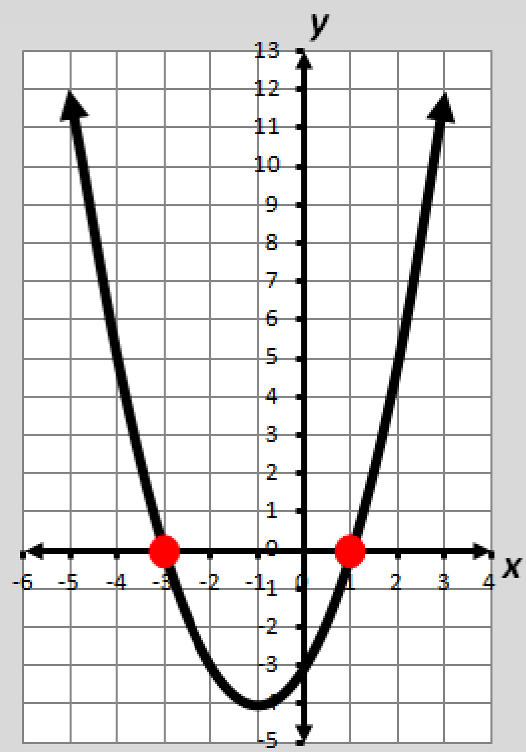
x-Intercepts

The *x*-intercepts of a graph are located where the graph crosses the x-axis and where $f(x) = 0$.

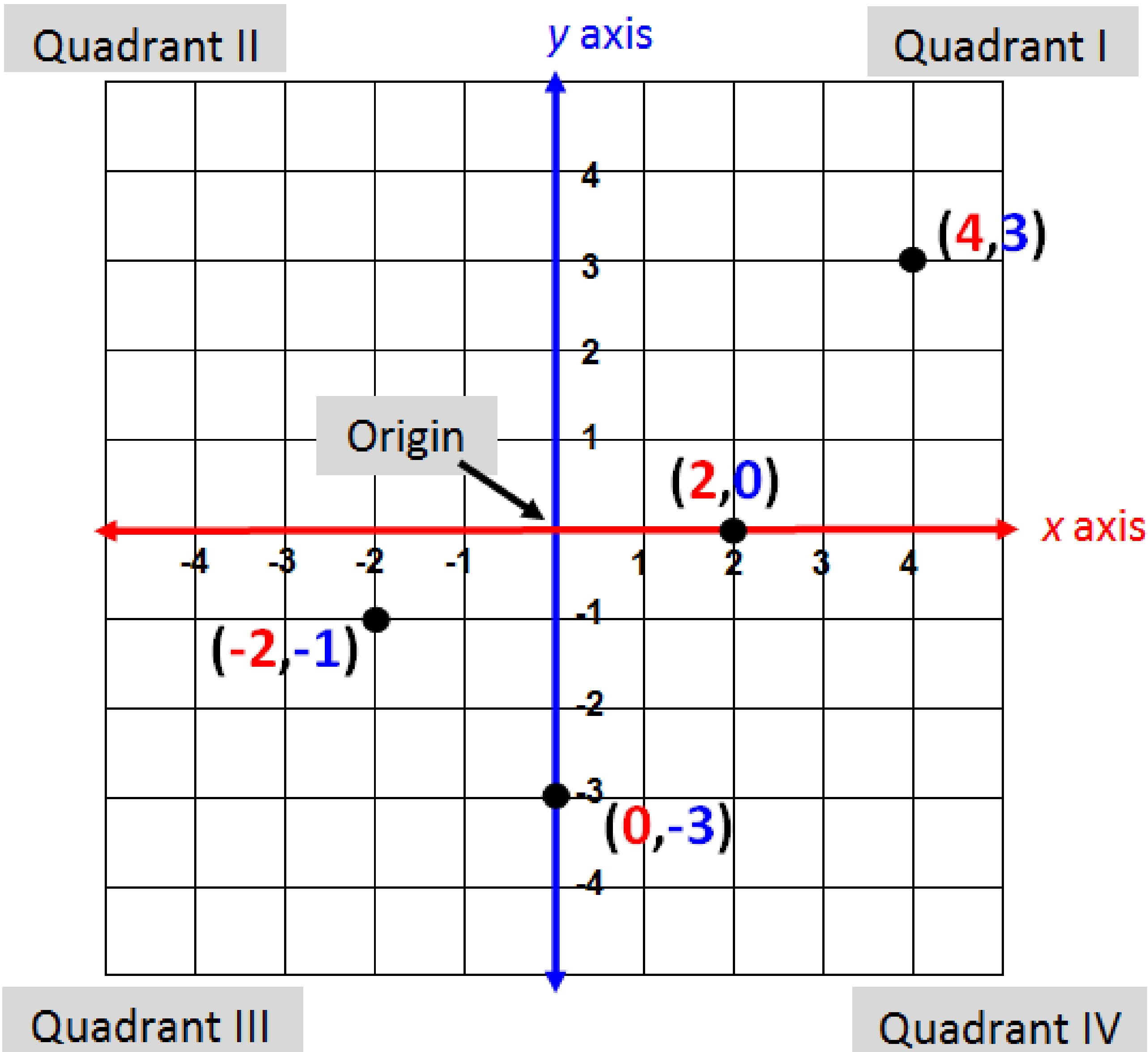
$$f(x) = x^2 + 2x - 3$$

$$0 = (x + 3)(x - 1)$$
$$0 = x + 3 \text{ or } 0 = x - 1$$
$$x = -3 \text{ or } x = 1$$

The zeros are -3 and 1.
The *x*-intercepts are:
-3 or (-3,0)
and
1 or (1,0)



Coordinate Plane



ordered pair (x,y)

Literal Equation

A formula or equation that consists primarily of variables

Examples:

$$Ax + By = C$$

$$A = \frac{1}{2}bh$$

$$V = lwh$$

$$F = \frac{9}{5}C + 32$$

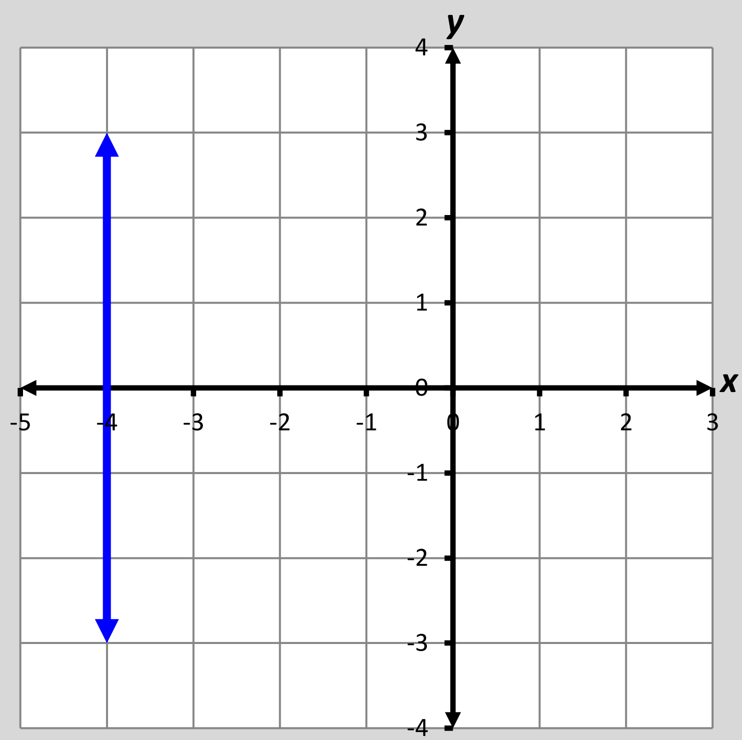
$$A = \pi r^2$$

Vertical Line

$$x = a$$

(where a can be any real number)

Example: $x = -4$



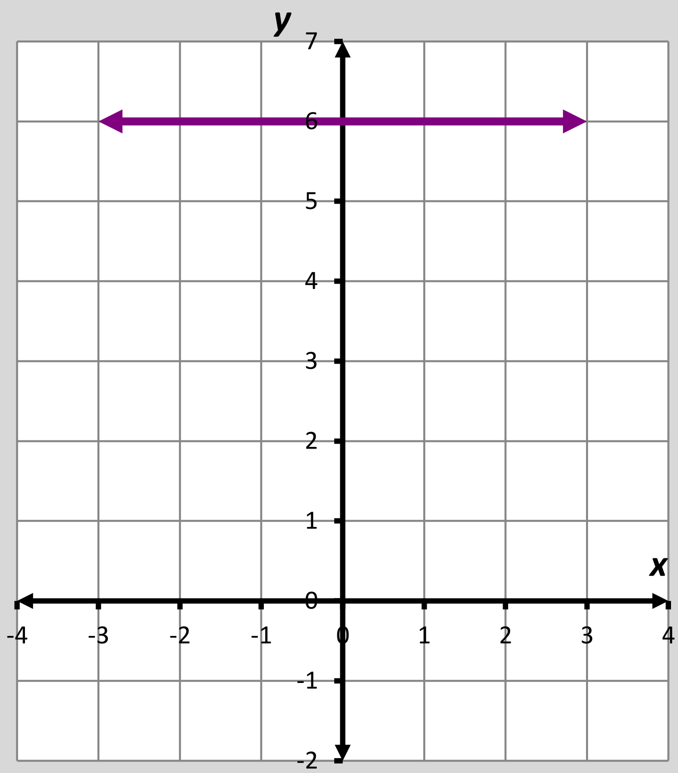
Vertical lines have undefined slope.

Horizontal Line

$$y = c$$

(where c can be any real number)

Example: $y = 6$



Horizontal lines have a slope of 0.

Quadratic Equation
(Solve by Factoring)

$$ax^2 + bx + c = 0$$

$$a \neq 0$$

Example solved by factoring:

$x^2 - 6x + 8 = 0$	Quadratic equation
$(x - 2)(x - 4) = 0$	Factor
$(x - 2) = 0$ or $(x - 4) = 0$	Set factors equal to 0
$x = 2$ or $x = 4$	Solve for x

Solutions to the equation are 2 and 4.
Solutions are {2, 4}

Quadratic Equation

(Solve by Graphing)

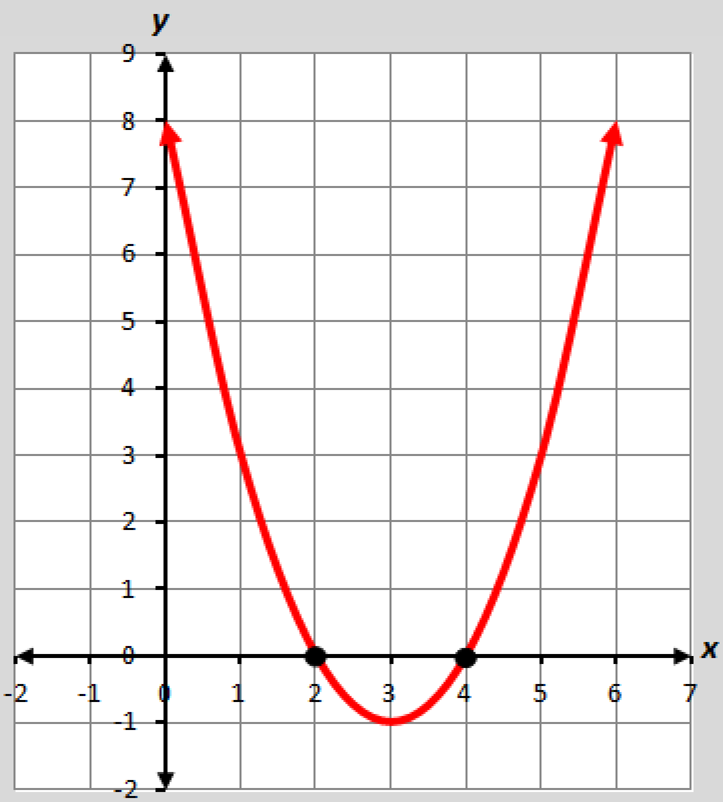
$$ax^2 + bx + c = 0$$

$$a \neq 0$$

Example solved by graphing:

$$x^2 - 6x + 8 = 0$$

Graph the related function $f(x) = x^2 - 6x + 8$.

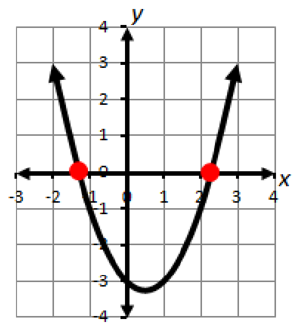
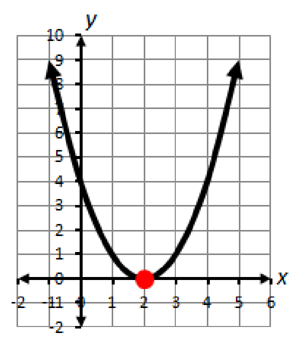
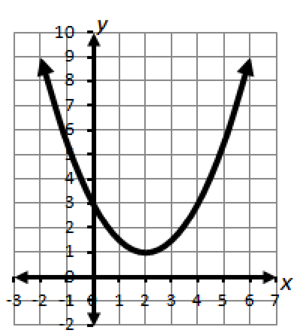


Solutions to the equation are the x-coordinates {2, 4} of the points where the function crosses the x-axis.

Quadratic Equation

(Number/Type of Real Solutions)

$ax^2 + bx + c = 0$, $a \neq 0$

Examples	Graph of the related function	Number and Type of Solutions/Roots
$x^2 - x = 3$		**2 distinct Real roots** (crosses *x*-axis twice)
$x^2 + 16 = 8x$		**1 distinct Real root with a multiplicity of two (double root)** (touches *x*-axis but does not cross)
$\frac{1}{2}x^2 - 2x + 3 = 0$		**0 Real roots**

Inequality

An algebraic sentence comparing two quantities

Symbol	Meaning
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to
$\neq$	not equal to

Examples: $-10.5 > -9.9 - 1.2$

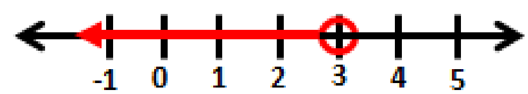
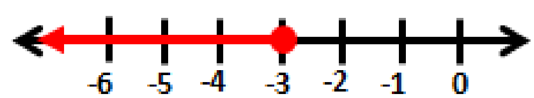
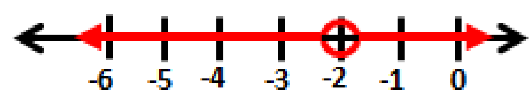
$8 < 3t + 2$

$x - 5y \geq -12$

$x \leq -11$

$r \neq 3$

Graph of an Inequality

Symbol	Example	Graph
$<$; $>$	$x < 3$	
$\leq$; $\geq$	$-3 \geq y$	
$\neq$	$t \neq -2$	

Transitive Property of Inequality

If	Then
$a < b$ and $b < c$	$a < c$
$a > b$ and $b > c$	$a > c$

Examples:

If $4x < 2y$ and $2y < 16$,
then $4x < 16$.

If $x > y - 1$ and $y - 1 > 3$,
then $x > 3$.

Addition/Subtraction Property of Inequality

If	Then
$a > b$	$a + c > b + c$
$a \geq b$	$a + c \geq b + c$
$a < b$	$a + c < b + c$
$a \leq b$	$a + c \leq b + c$

Example:

$$d - 1.9 \geq -8.7$$

$$d - 1.9 + 1.9 \geq -8.7 + 1.9$$

$$d \geq -6.8$$

Multiplication Property of Inequality

If	Case	Then
$a < b$	$c > 0$, positive	$ac < bc$
$a > b$	$c > 0$, positive	$ac > bc$
$a < b$	$c < 0$, negative	$ac > bc$
$a > b$	$c < 0$, negative	$ac < bc$

Example: If $c = -2$

$$5 > -3$$

$$5(-2) < -3(-2)$$

$$-10 < 6$$

Division Property of Inequality

If	Case	Then
$a < b$	$c > 0$, positive	$\frac{a}{c} < \frac{b}{c}$
$a > b$	$c > 0$, positive	$\frac{a}{c} > \frac{b}{c}$
$a < b$	$c < 0$, negative	$\frac{a}{c} > \frac{b}{c}$
$a > b$	$c < 0$, negative	$\frac{a}{c} < \frac{b}{c}$

Example: If $c = -4$

$$-90 \geq -4t$$

$$\frac{-90}{-4} \leq \frac{-4t}{-4}$$

$$22.5 \leq t$$

Linear Equation

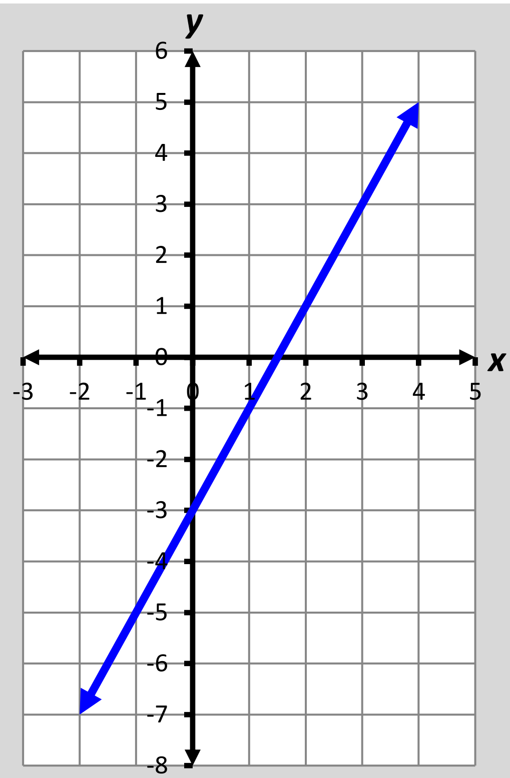
(Standard Form)

$$Ax + By = C$$

(A, B and C are integers; A and B cannot both equal zero)

Example:

$-2x + y = -3$



The graph of the linear equation is a straight line and represents all solutions (x, y) of the equation.

Linear Equation
(Slope-Intercept Form)

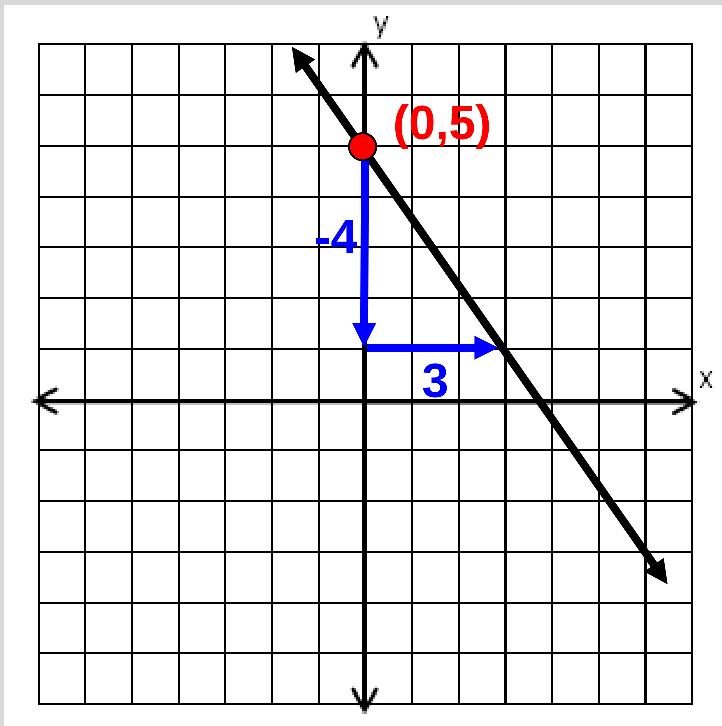
$$y = mx + b$$

(slope is m and y-intercept is b)

Example: $y = \frac{-4}{3}x + 5$

$m = \frac{-4}{3}$

$b = 5$



Linear Equation
(Point-Slope Form)

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1,y_1) is the point

Example:
Write an equation for the line that passes through the point $(^-4,1)$ and has a slope of 2.

$$y - 1 = 2(x - (^-4))$$

$$y - 1 = 2(x + 4)$$

$$y = 2x + 9$$

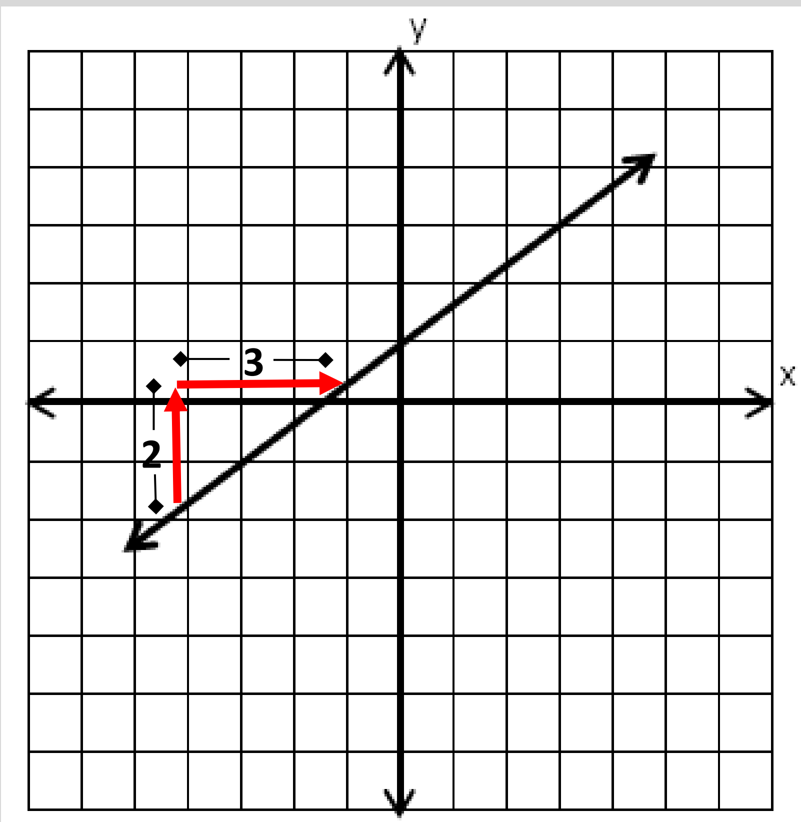
Equivalent Forms of a Linear Equation

Forms of a Linear Equation

Example	$3y = 6 - 4x$
Slope-Intercept $y = mx + b$	$y = -\frac{4}{3}x + 2$
Point-Slope $y - y_1 = m(x - x_1)$	$y - (-2) = -\frac{4}{3}(x - 3)$
Standard $Ax + By = C$	$4x + 3y = 6$

Slope

A number that represents the rate of change in *y* for a unit change in *x*

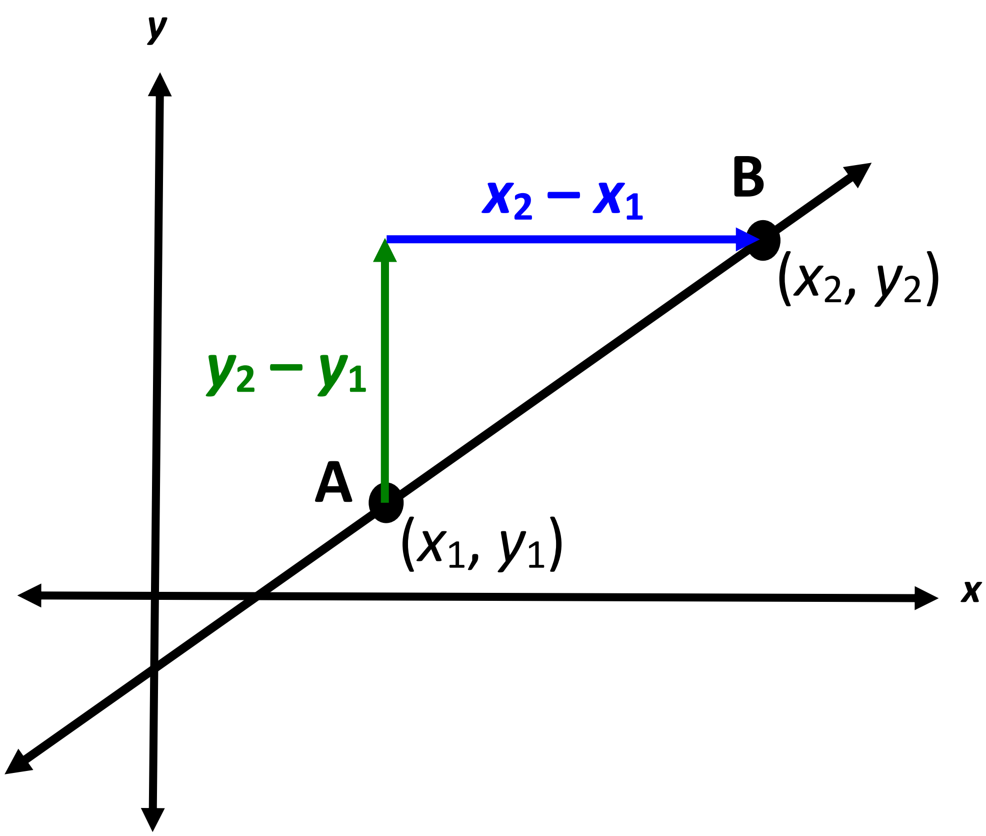


Slope = $\frac{2}{3}$

The slope indicates the steepness of a line.

Slope Formula

The ratio of vertical change to horizontal change

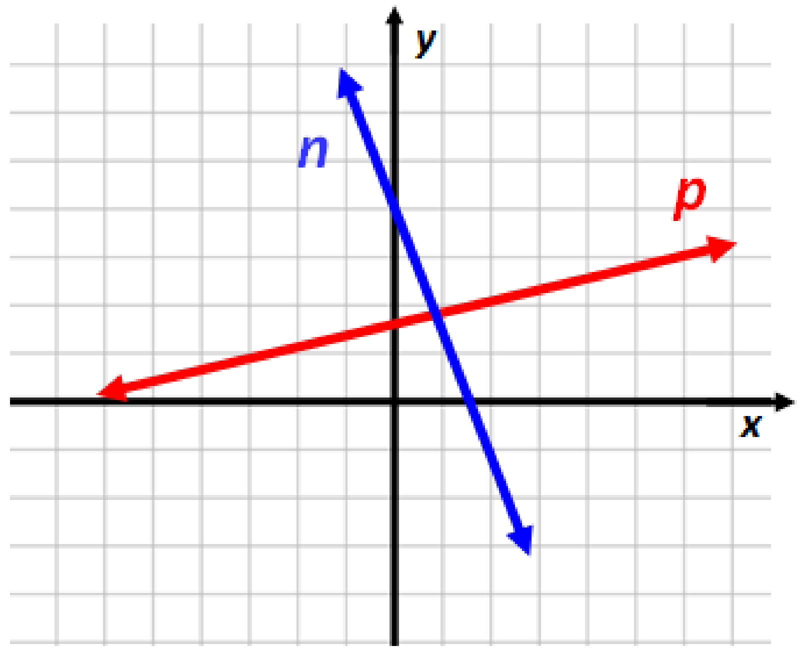


$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slopes of Lines

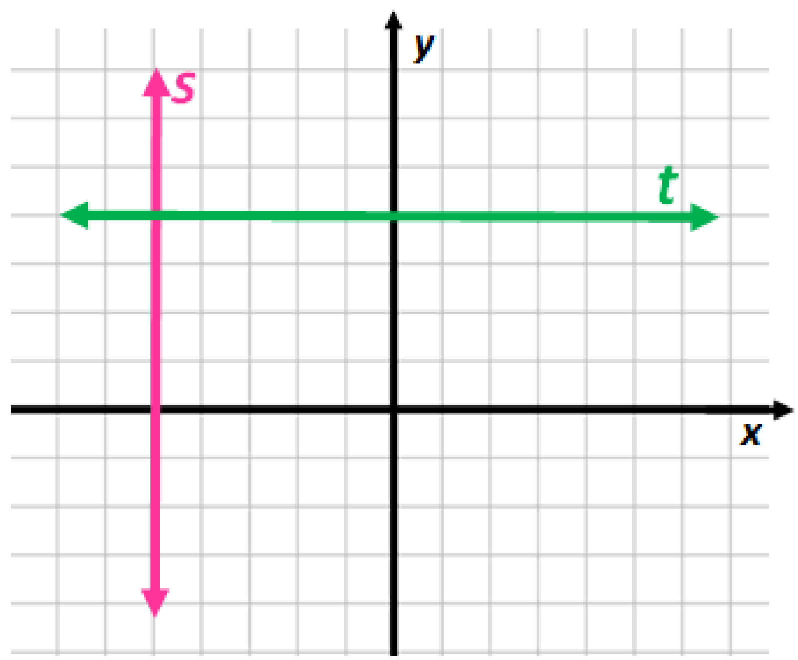
Line ***p***
has a positive slope.

Line ***n***
has a negative
slope.



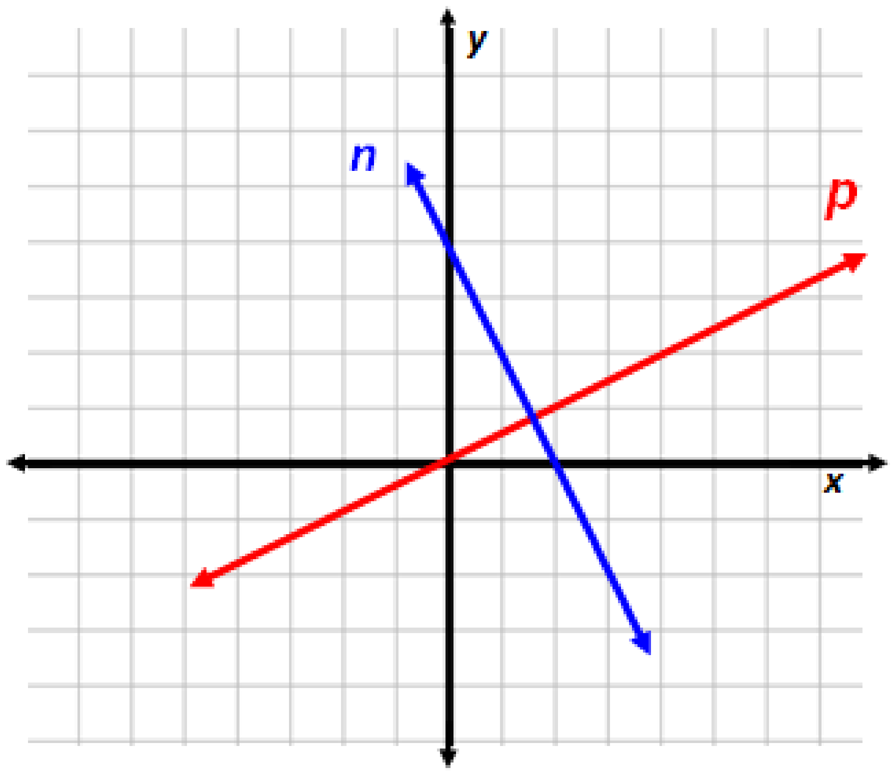
Vertical line ***s*** has an undefined slope.

Horizontal line ***t***
has a zero slope.



Perpendicular Lines

Lines that intersect to form a right angle



Perpendicular lines (not parallel to either of the axes) have slopes whose product is -1.

Example:

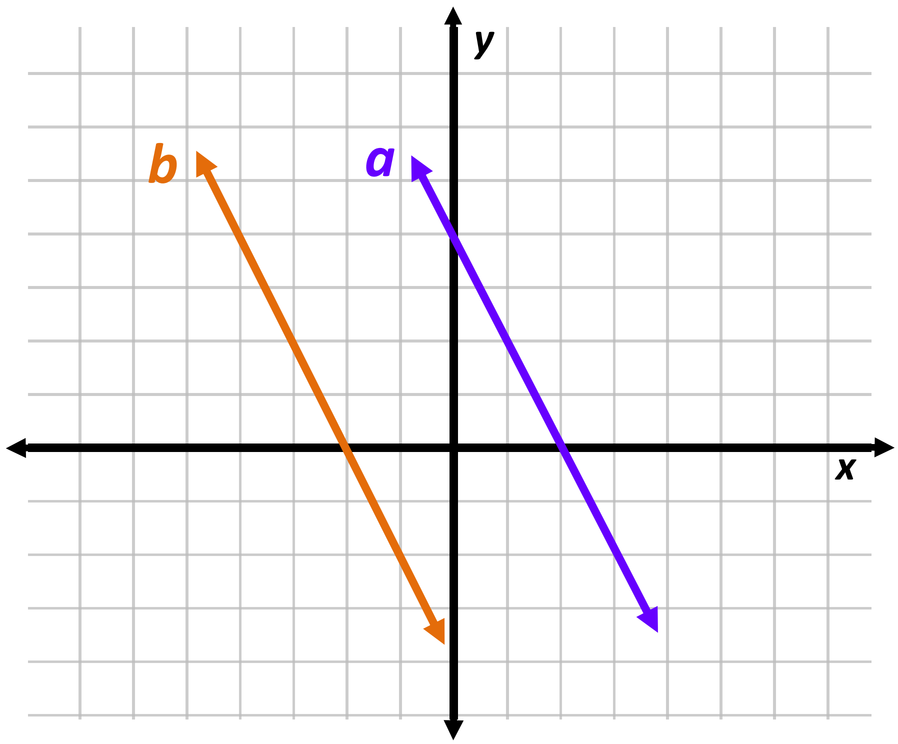
The slope of line *n* = -2. The slope of line *p* = $\frac{1}{2}$.

$-2 \cdot \frac{1}{2} = -1$, therefore, *n* is perpendicular to *p*.

Parallel Lines

Lines in the same plane that do not intersect are parallel.

Parallel lines have the same slopes.



Example:

The slope of line $a = -2$.

The slope of line $b = -2$.

$-2 = -2$, therefore, a is parallel to b.

Mathematical Notation

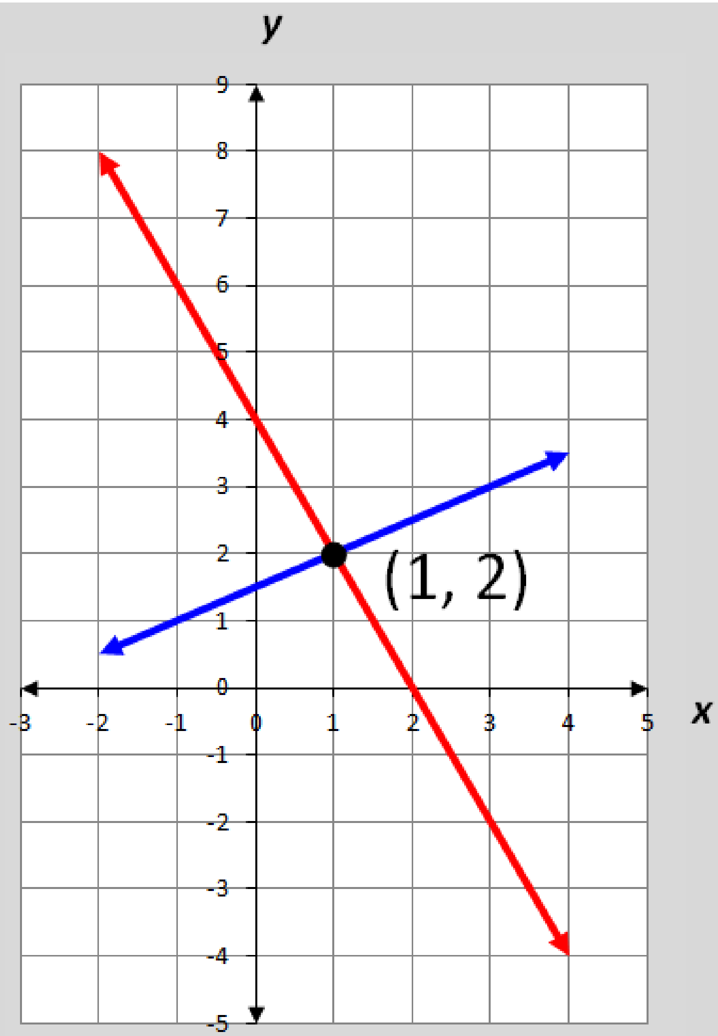
Equation/Inequality	Set Notation
$x = -5$	$\{-5\}$
$x = 5 \text{ } or \text{ } x = -3.4$	$\{5, -3.4\}$
$y > \frac{8}{3}$	$\left\{y : y > \frac{8}{3}\right\}$
$x \leq 2.34$	$\{x \mid x \leq 2.34\}$
Empty (null) set $\emptyset$	$\{\ \}$
All Real Numbers $\mathbb{R}$	$\{x : x \in \mathbb{R}\}$ $\{\text{All Real Numbers}\}$

System of Linear Equations

(Graphing)

$$\begin{cases} -x + 2y = 3 \\ 2x + y = 4 \end{cases}$$

The **solution**, (1, 2), is the only ordered pair that satisfies both equations (the point of intersection).



System of Linear Equations

(Substitution)

$$\begin{cases} x + 4y = 17 \\ y = x - 2 \end{cases}$$

Substitute $x - 2$ for y in the first equation.

$$x + 4(x - 2) = 17$$

$$x = 5$$

Now substitute 5 for x in the second equation.

$$y = 5 - 2$$

$$y = 3$$

The **solution** to the linear system is (5, 3), the ordered pair that satisfies both equations.

System of Linear Equations

(Elimination)

$$\begin{cases} -5x - 6y = 8 \\ 5x + 2y = 4 \end{cases}$$

Add or subtract the equations to eliminate one variable.

$$\begin{array}{r} -5x - 6y = 8 \\ +\ 5x + 2y = 4 \\ \hline -4y = 12 \\ y = -3 \end{array}$$

Now substitute -3 for *y* in either original equation to find the value of *x*, the eliminated variable.

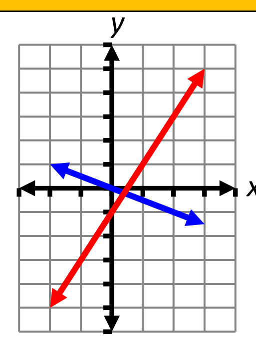
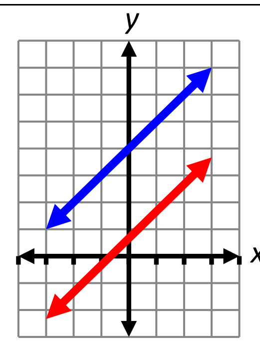
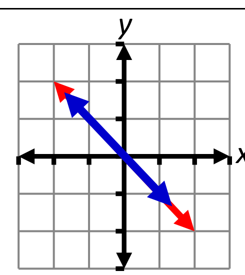
$$-5x - 6(-3) = 8$$

$$x = 2$$

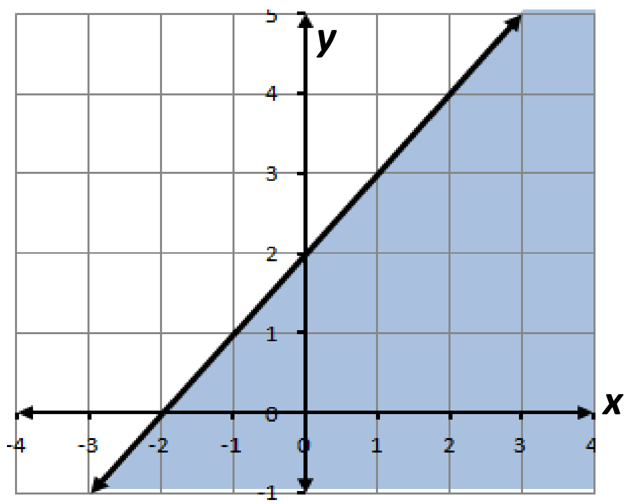
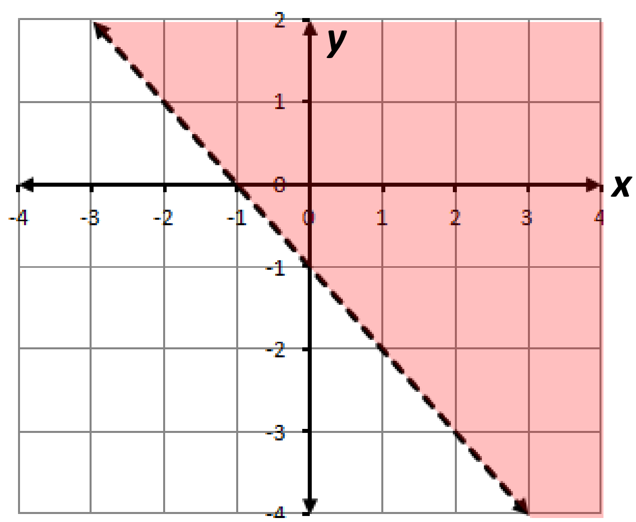
The solution to the linear system is (2,-3), the ordered pair that satisfies both equations.

System of Linear Equations

(Number of Solutions)

Number of Solutions	**Slopes and *y*-intercepts**	**Graph**
One solution	Different slopes	
No solution	Same slope and different - intercepts	
Infinitely many solutions	Same slope and same *y*-intercepts	

Graphing Linear Inequalities

Example	Graph
$y \leq x + 2$	
$y > -x - 1$	

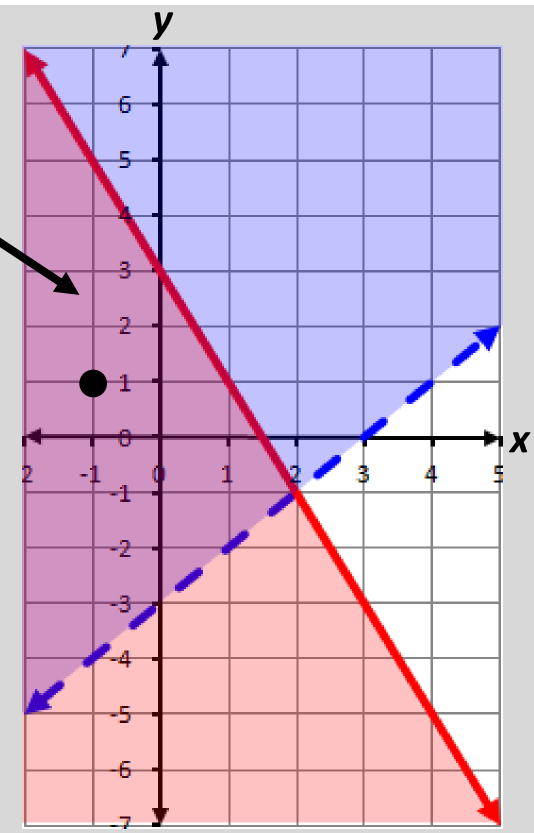
The graph of the solution of a linear inequality is a half-plane bounded by the graph of its related linear equation. Points on the boundary are included unless the inequality contains only < or >.

System of Linear Inequalities

Solve by graphing:

$$\begin{cases} y > x - 3 \\ y \leq -2x + 3 \end{cases}$$

The solution region contains all ordered pairs that are solutions to both inequalities in the system.



(-1,1) is <u>one</u> of the solutions to the system located in the solution region.

Dependent and Independent Variable

***x*,** independent variable
(input values or domain set)

***y*,** dependent variable
(output values or range set)

Example:

$$y = 2x + 7$$

Dependent and Independent Variable
(Application)

Determine the distance a car will travel going 55 mph.

$$d = 55h$$

independent

h	d
0	0
1	55
2	110
3	165

dependent

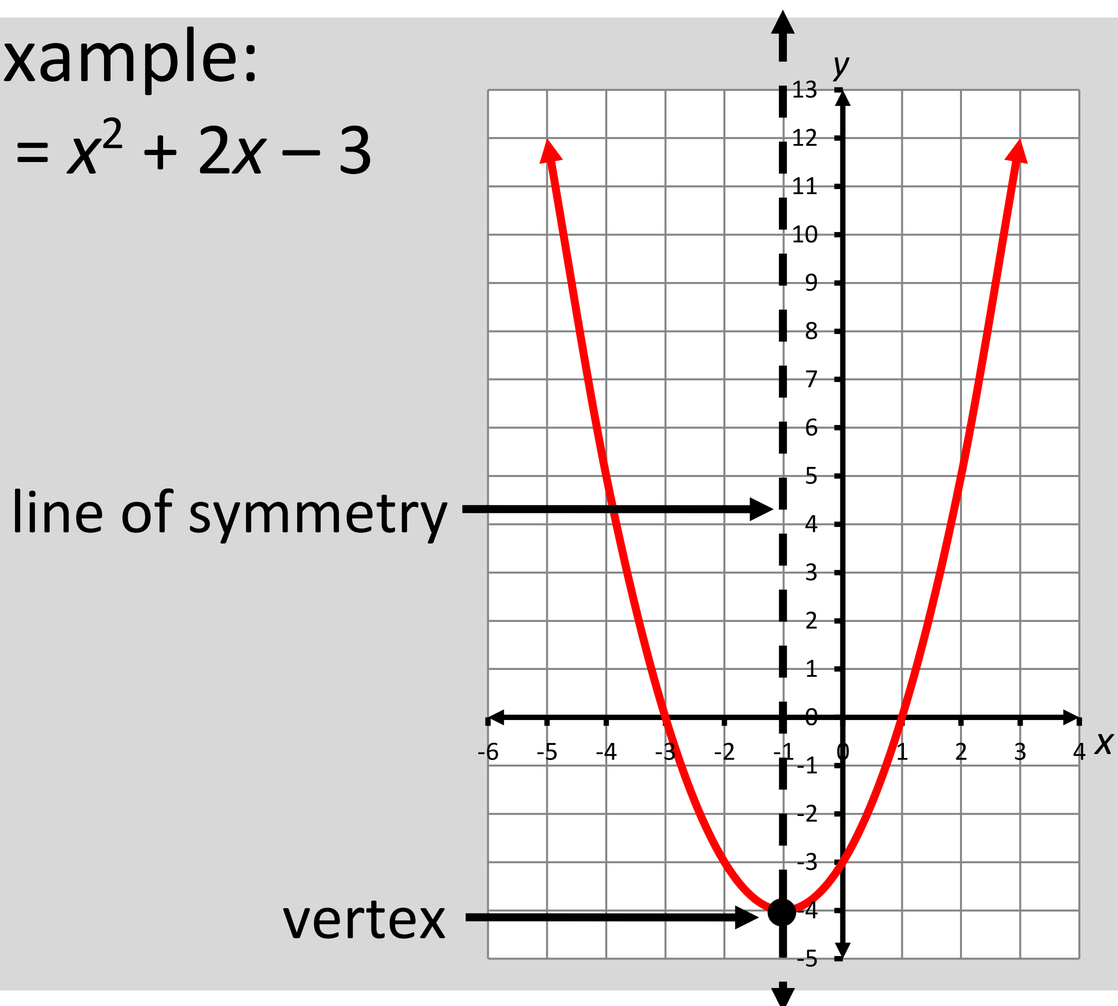
Graph of a Quadratic Equation

$$y = ax^2 + bx + c$$

$$a \neq 0$$

Example:

$y = x^2 + 2x - 3$



The graph of the quadratic equation is a curve (parabola) with one line of symmetry and one vertex.

Vertex of a Quadratic Function

For a given quadratic $y = ax^2 + bx + c$, the vertex (h, k) is found by computing $h = \frac{-b}{2a}$ and then evaluating y at h to find k.

Example: $y = x^2 + 2x - 8$

$$h = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$$

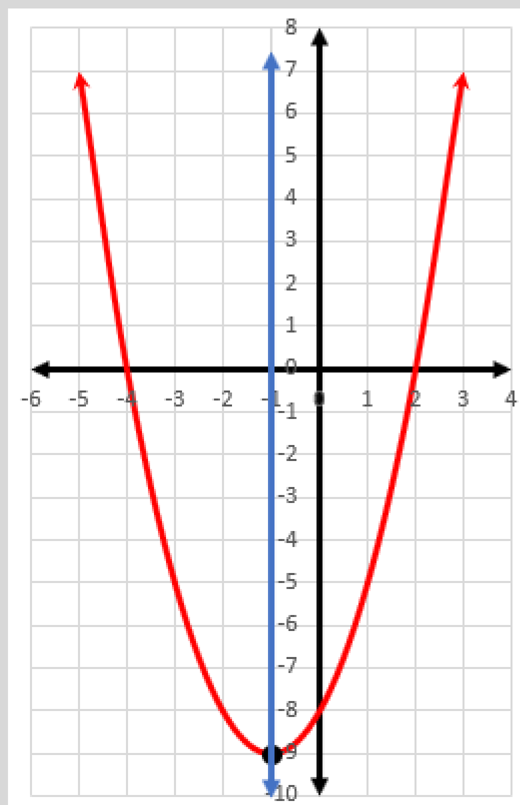
$k = (-1)^2 + 2(-1) - 8$

$k = -9$

The vertex is (-1,-9).

Line of symmetry is $x = h$.

$x = -1$



Quadratic Formula

Used to find the solutions to any quadratic equation of the form,

$$f(x) = ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: $g(x) = 2x^2 - 4x - 3$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{2 + \sqrt{10}}{2}, \frac{2 - \sqrt{10}}{2}$$

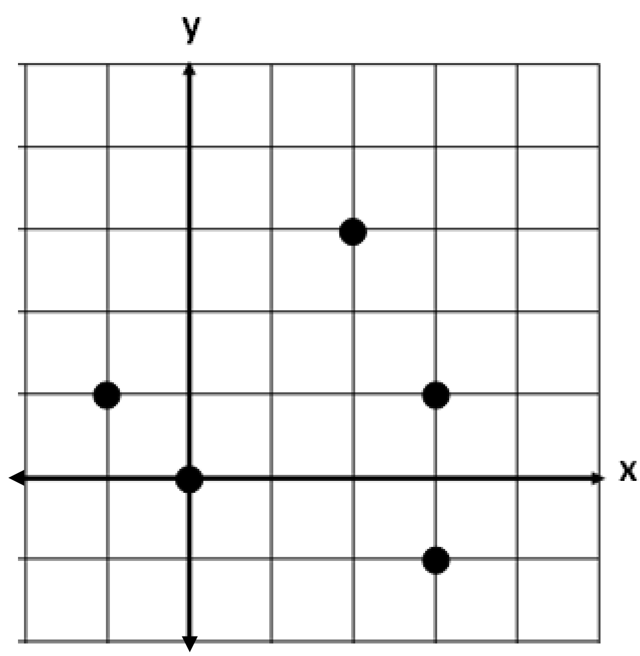
Relation

A set of ordered pairs

Examples:

x	*y*
-3	4
0	0
1	-6
2	2
5	-1

Example 1



Example 2

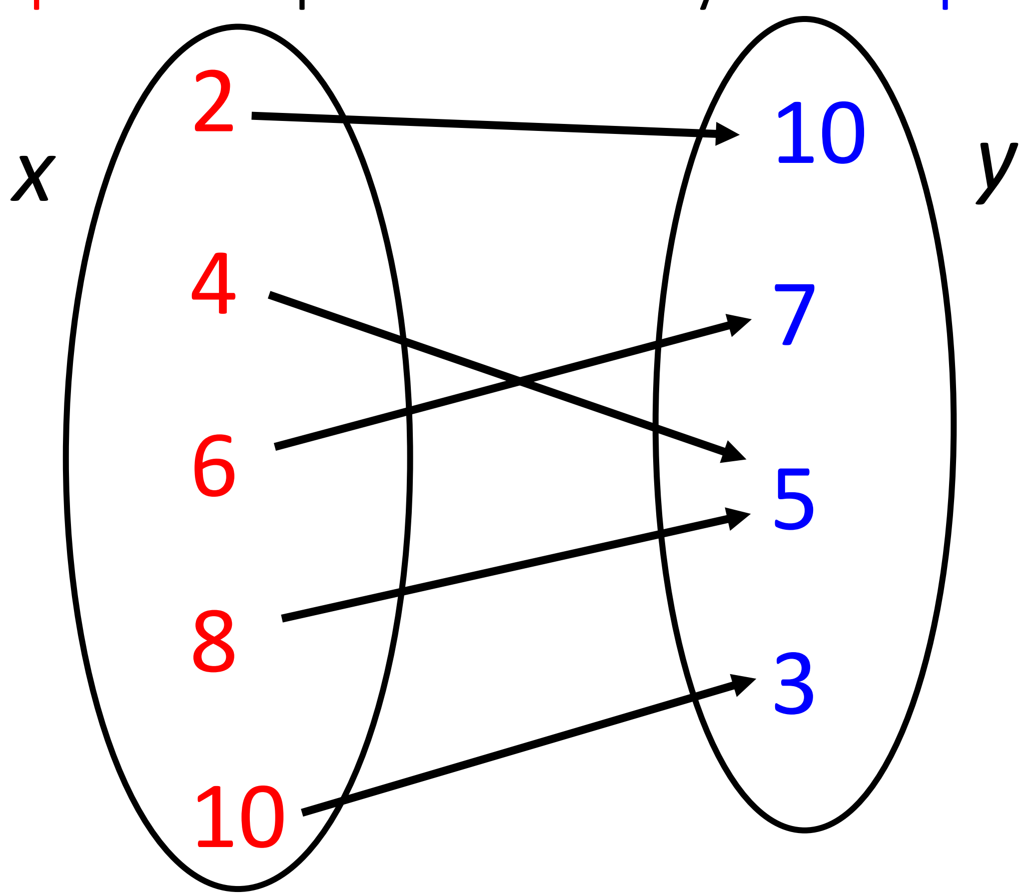
{(0,4), (0,3), (0,2), (0,1)}

Example 3

Function

(Definition)

A relationship between two quantities in which every input corresponds to exactly one output



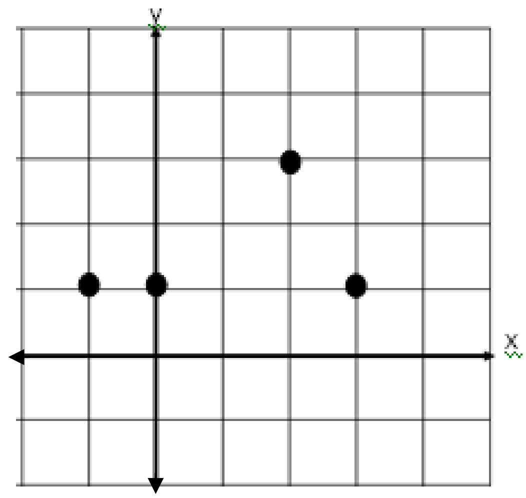
A relation is a function if and only if each element in the domain is paired with a unique element of the range.

Functions

(Examples)

x	*y*
3	2
2	4
0	2
-1	2

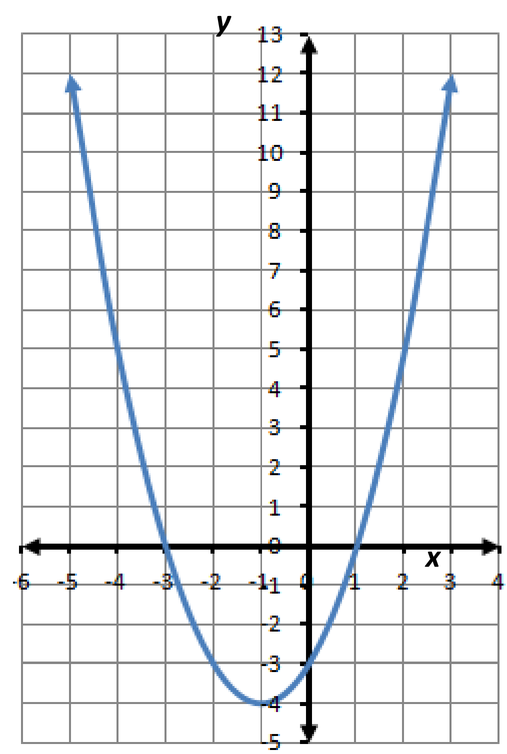
Example 1



Example 2

{(-3,4), (0,3), (1,2), (4,6)}

Example 3



Example 4

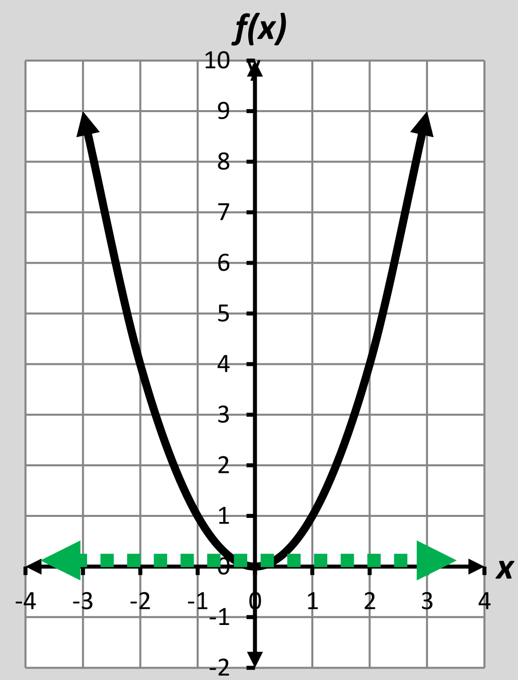
Domain

A set of input values of a relation

Examples:

input	output
x	***g*(*x*)**
-2	**0**
-1	**1**
0	**2**
1	**3**

The **domain** of *g*(*x*) is {**-2, -1, 0, 1**}.



The **domain** of *f*(*x*) is **all real numbers**.

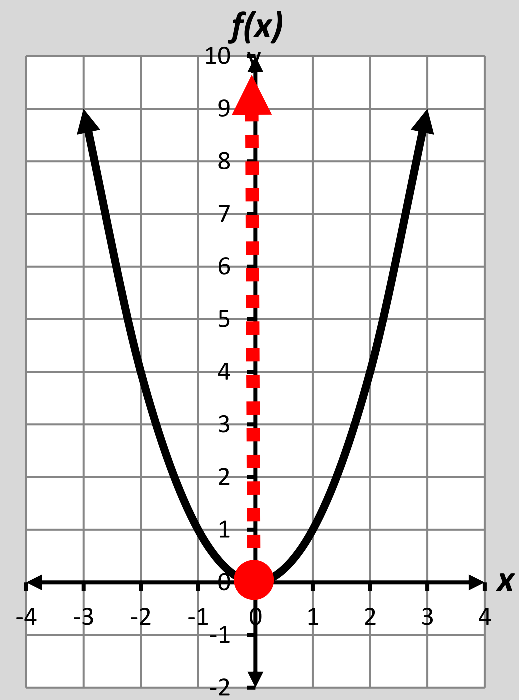
Range

A set of output values of a relation

Examples:

input	output
x	***g*(*x*)**
-2	**0**
-1	**1**
0	**2**
1	**3**

The **range** of g(x) is {**0, 1, 2, 3**}.



The **range** of *f*(*x*) is **all real numbers greater than or equal to zero**.

Function Notation

$f(x)$

$f(x)$ is read

"the value of f at x" or "f of x"

Example:

$f(x) = -3x + 5$, find $f(2)$.

$f(2) = -3(2) + 5$

$f(2) = -6 + 5$

$f(2) = -1$

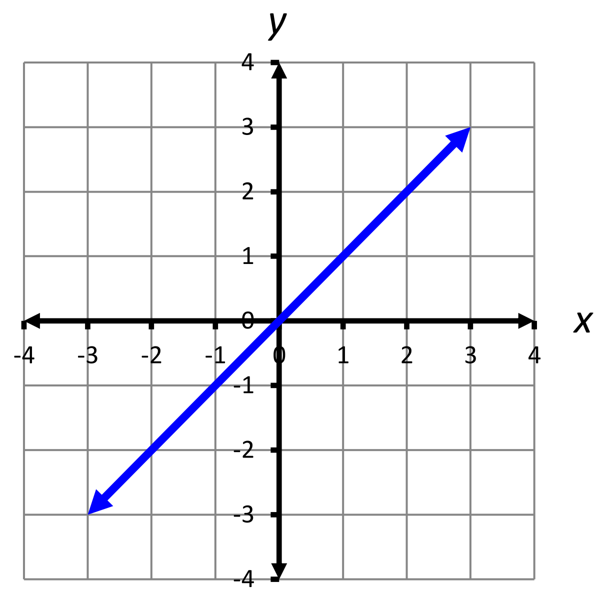
Letters other than f can be used to name functions, e.g., $g(x)$ and $h(x)$

Parent Functions

(Linear, Quadratic)

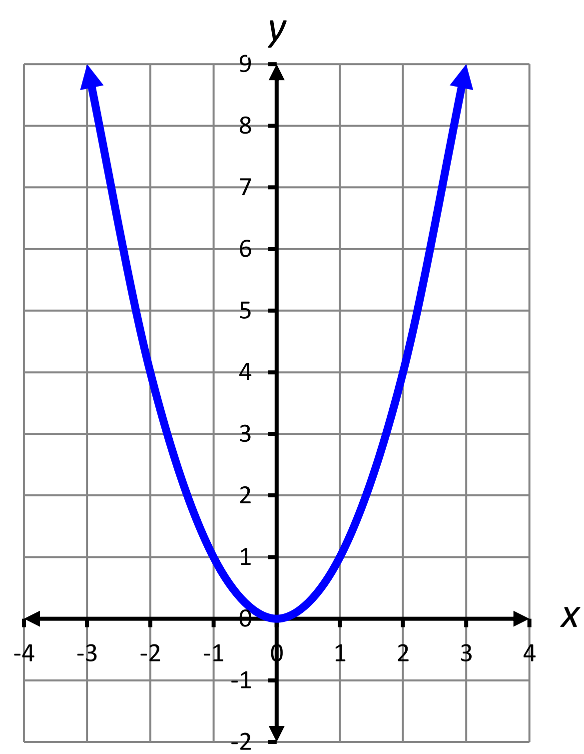
Linear

$f(x) = x$



Quadratic

$f(x) = x^2$



Transformations of Parent Functions (Translation)

Parent functions can be transformed to create other members in a family of graphs.

Translations	$g(x) = f(x) + k$ is the graph of $f(x)$ **translated vertically** –	k units **up** when $k > 0$.
		k units **down** when $k < 0$.
	$g(x) = f(x - h)$ is the graph of $f(x)$ **translated horizontally** –	h units **right** when $h > 0$.
		h units **left** when $h < 0$.

Transformations of Parent Functions (Reflection)

Parent functions can be transformed to create other members in a family of graphs.

Reflections	***g*(*x*) = -*f*(*x*)** is the graph of *f*(*x*) –	**reflected** over the ***x*-axis**.
	***g*(*x*) = *f*(-*x*)** is the graph of *f*(*x*) –	**reflected** over the ***y*-axis**.

Transformations of Parent Functions

(Vertical Dilations)

Parent functions can be transformed to create other members in a family of graphs.

Dilations	$g(x) = a \cdot f(x)$ is the graph of $f(x)$ –	**vertical dilation** (stretch) if $a > 1$. Stretches away from the x-axis
		vertical dilation (compression) if $0 < a < 1$. Compresses toward the x-axis

Linear Function

(Transformational Graphing)

Translation

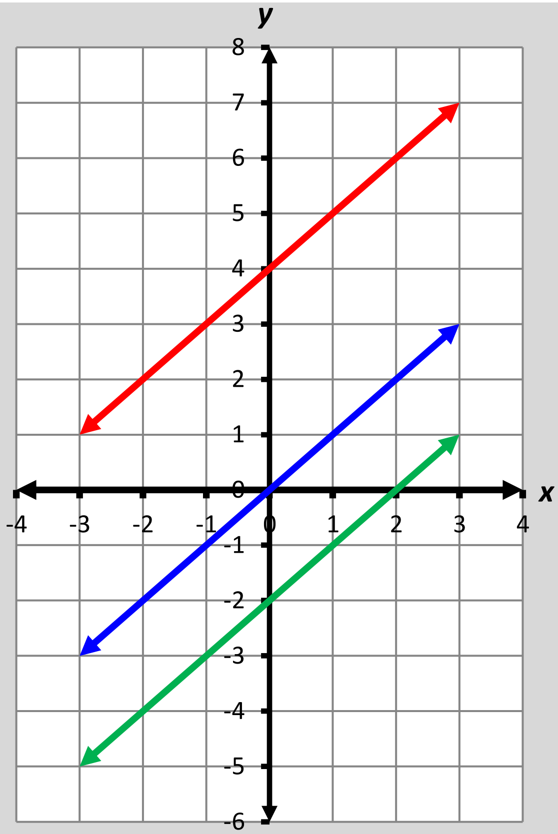
$g(x) = x + b$

Examples:

$f(x) = x$

$t(x) = x + 4$

$h(x) = x - 2$



Vertical translation of the parent function,

$f(x) = x$

Linear Function

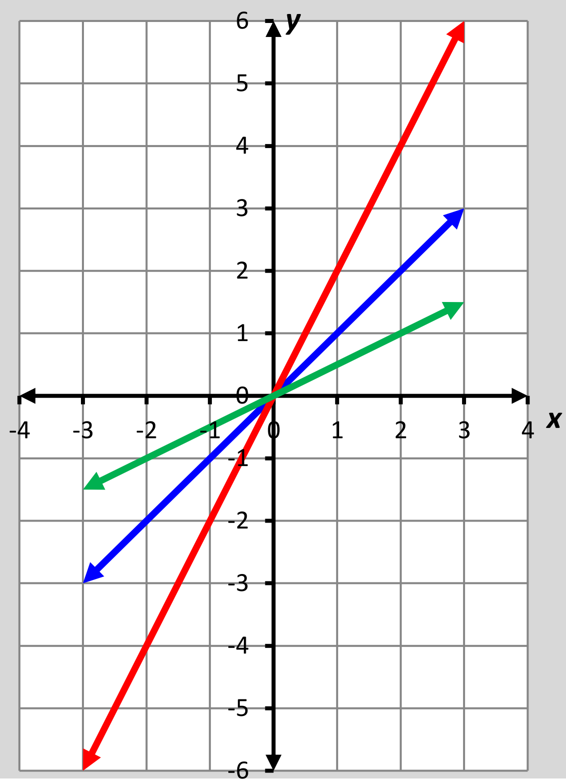
(Transformational Graphing)
Vertical Dilation ($m > 0$)
$g(x) = mx$

Examples:

$f(x) = x$

$t(x) = 2x$

$h(x) = \frac{1}{2}x$



Vertical dilation (stretch or compression) of the parent function, $f(x) = x$

Linear Function

(Transformational Graphing)

Vertical Dilation/Reflection ($m < 0$)

$g(x) = mx$

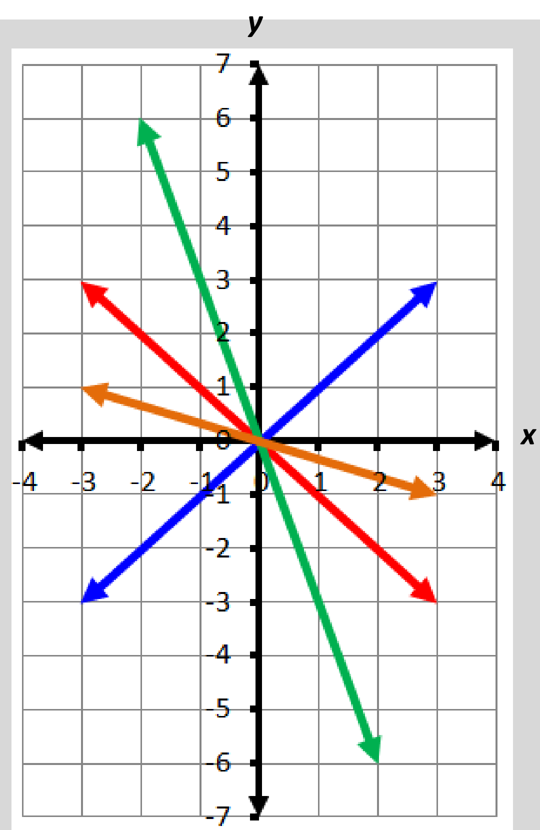
Examples:

$f(x) = x$

$t(x) = -x$

$h(x) = -3x$

$d(x) = -\frac{1}{3}x$



Vertical dilation (stretch or compression) with a reflection of $f(x) = x$

Quadratic Function

(Transformational Graphing)

Vertical Translation

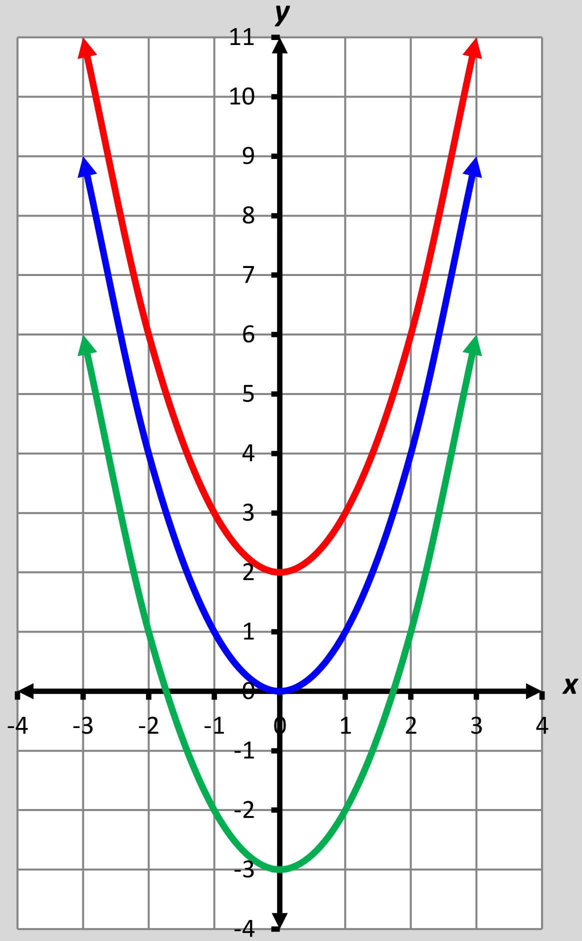
$h(x) = x^2 + c$

Examples:

$f(x) = x^2$

$g(x) = x^2 + 2$

$t(x) = x^2 - 3$



Vertical translation of $f(x) = x^2$

Quadratic Function

(Transformational Graphing)

Vertical Dilation (a>0)

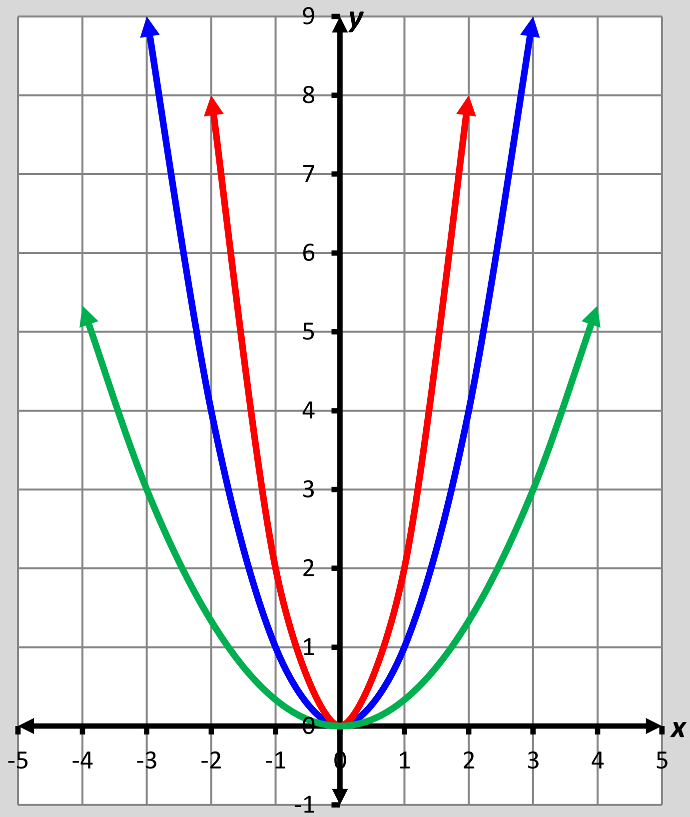
$h(x) = ax^2$

Examples:

$f(x) = x^2$

$g(x) = 2x^2$

$t(x) = \frac{1}{3}x^2$



Vertical dilation (stretch or compression) of $f(x) = x^2$

Quadratic Function

(Transformational Graphing)

Vertical Dilation/Reflection (a<0)

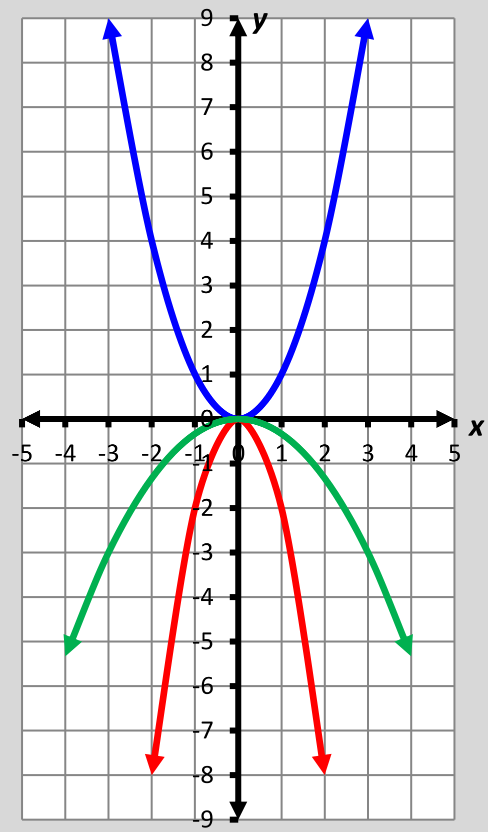
$h(x) = ax^2$

Examples:

$f(x) = x^2$

$g(x) = -2x^2$

$t(x) = -\frac{1}{3}x^2$



Vertical dilation (stretch or compression) with a reflection of $f(x) = x^2$

Quadratic Function

(Transformational Graphing)

Horizontal Translation

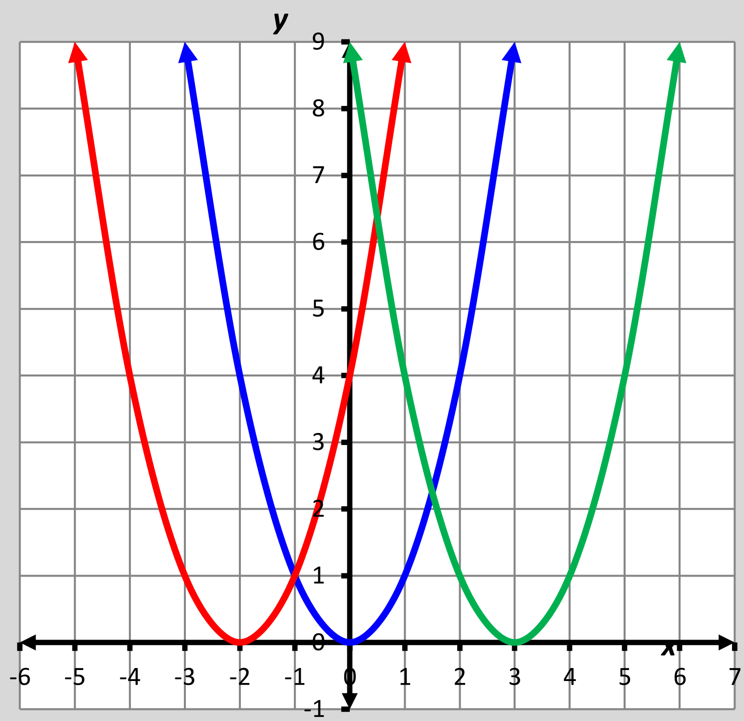
$h(x) = (x \pm c)^2$

Examples:

$f(x) = x^2$

$g(x) = (x + 2)^2$

$t(x) = (x - 3)^2$



Horizontal translation of $f(x) = x^2$

Multiple Representations of Functions

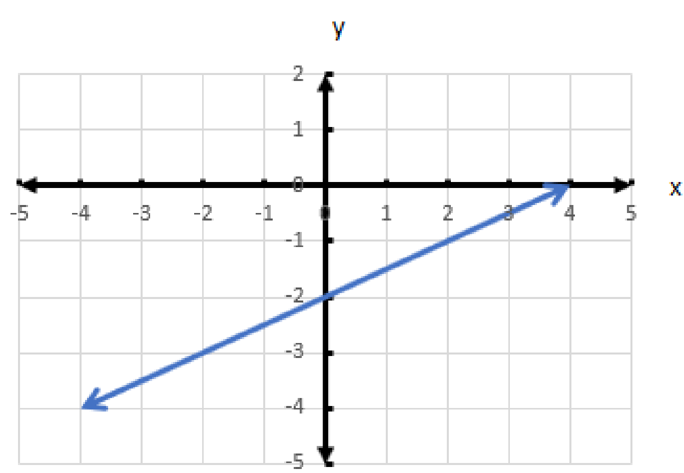
Equation

$$y = \frac{1}{2}x - 2$$

Table

x	y
-2	-3
0	-2
2	-1
4	0

Graph



Words

y equals one-half *x* minus 2

Direct Variation

$$y = kx \text{ or } k = \frac{y}{x}$$

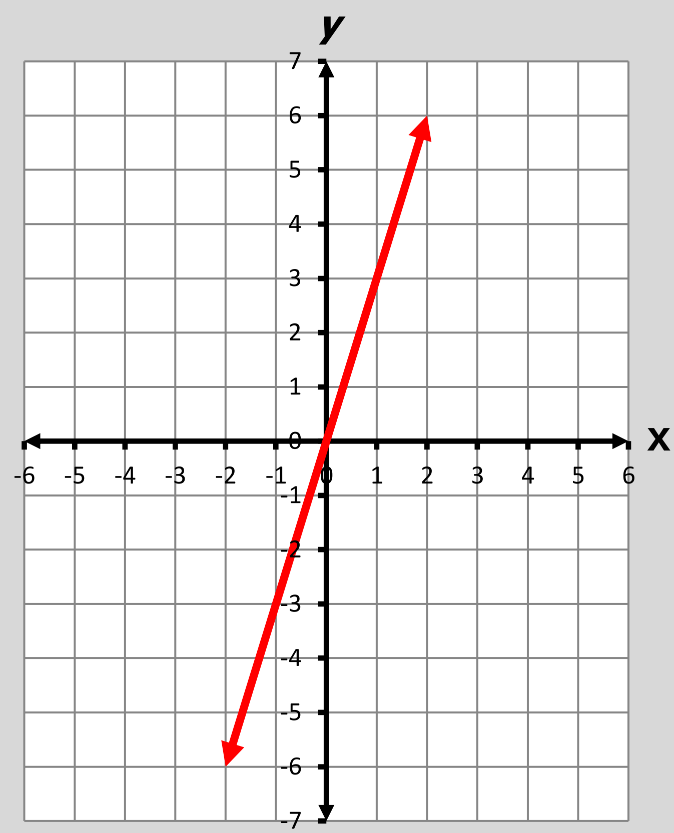
constant of variation, $k \neq 0$

Example:

$y = 3x$ or $3 = \frac{y}{x}$

x	y
-2	-6
-1	-3
0	0
1	3
2	6

$$3 = \frac{-6}{-2} = \frac{-3}{-1} = \frac{3}{1} = \frac{6}{2}$$



The graph of all points describing a direct variation is a line passing through the origin.

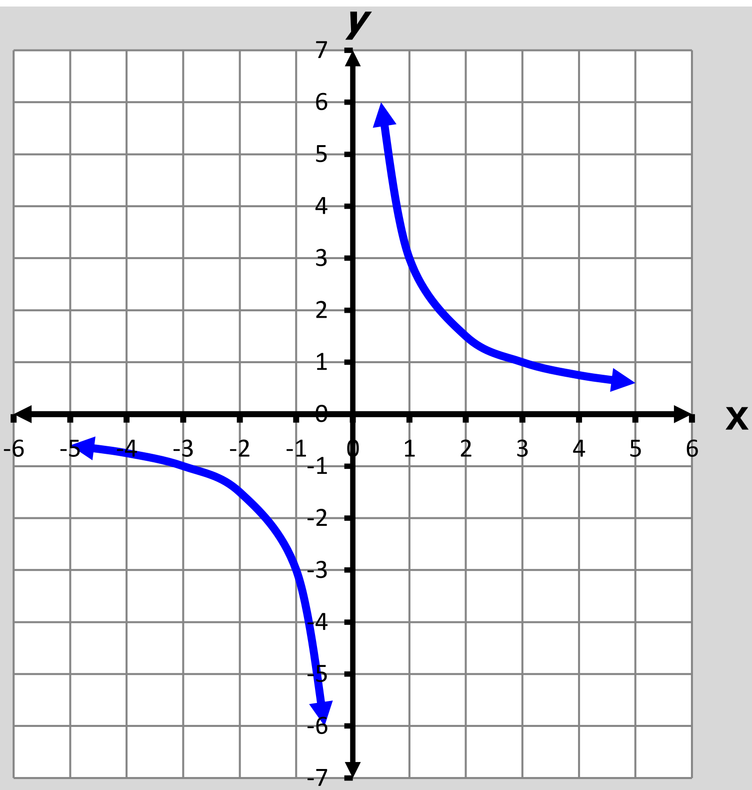
Inverse Variation

$$y = \frac{k}{x} \quad \text{or} \quad k = xy$$

constant of variation, $k \neq 0$

Example:

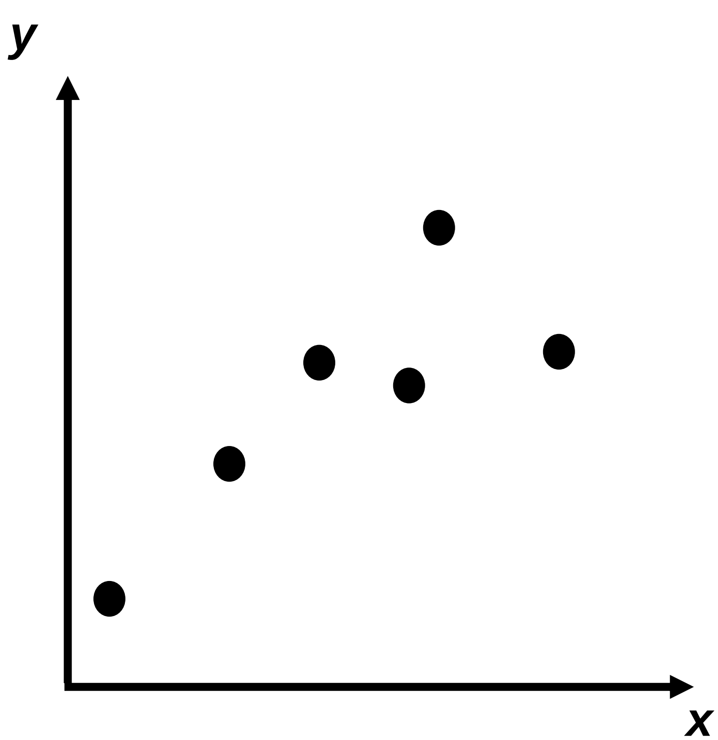
$$y = \frac{3}{x} \quad \text{or} \quad xy = 3$$



The graph of all points describing an inverse variation relationship are two curves that are reflections of each other.

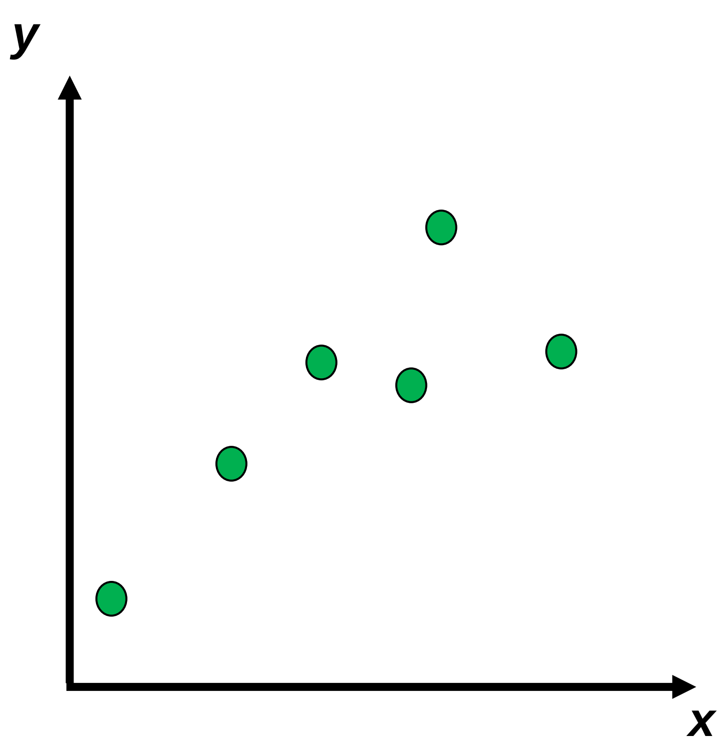
Scatterplot

Graphical representation of the relationship between two numerical sets of data



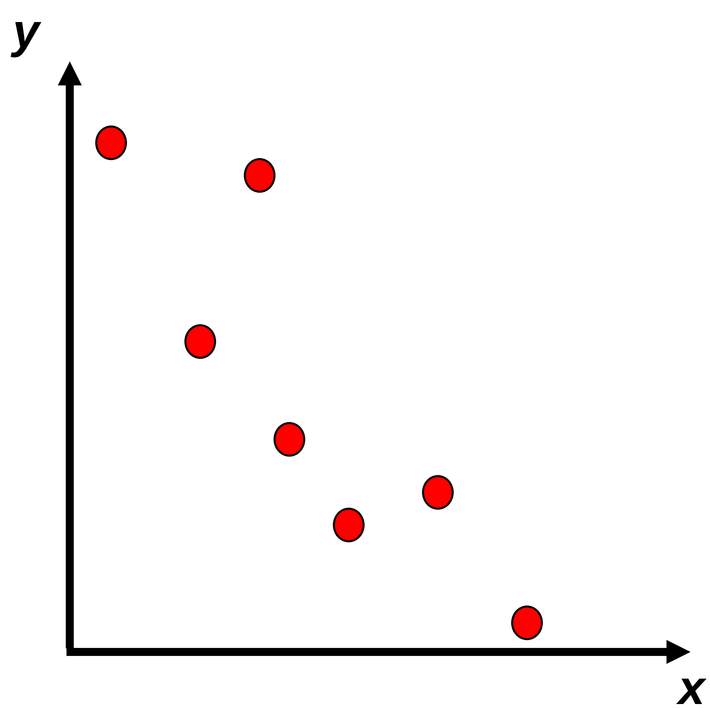
Positive Linear Relationship (Correlation)

In general, a relationship where the dependent (*y*) values increase as independent values (*x*) increase



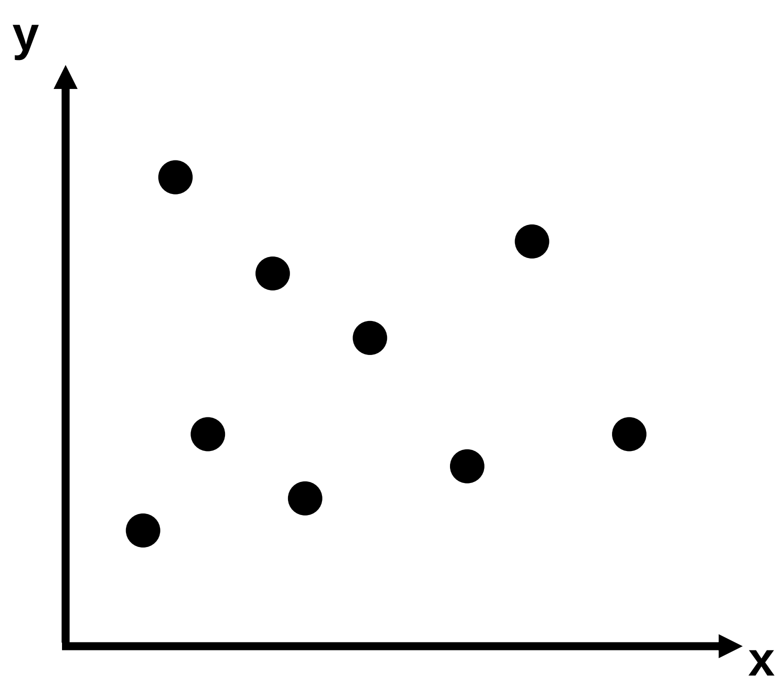
Negative Linear Relationship (Correlation)

In general, a relationship where the dependent (y) values decrease as independent (x) values increase.



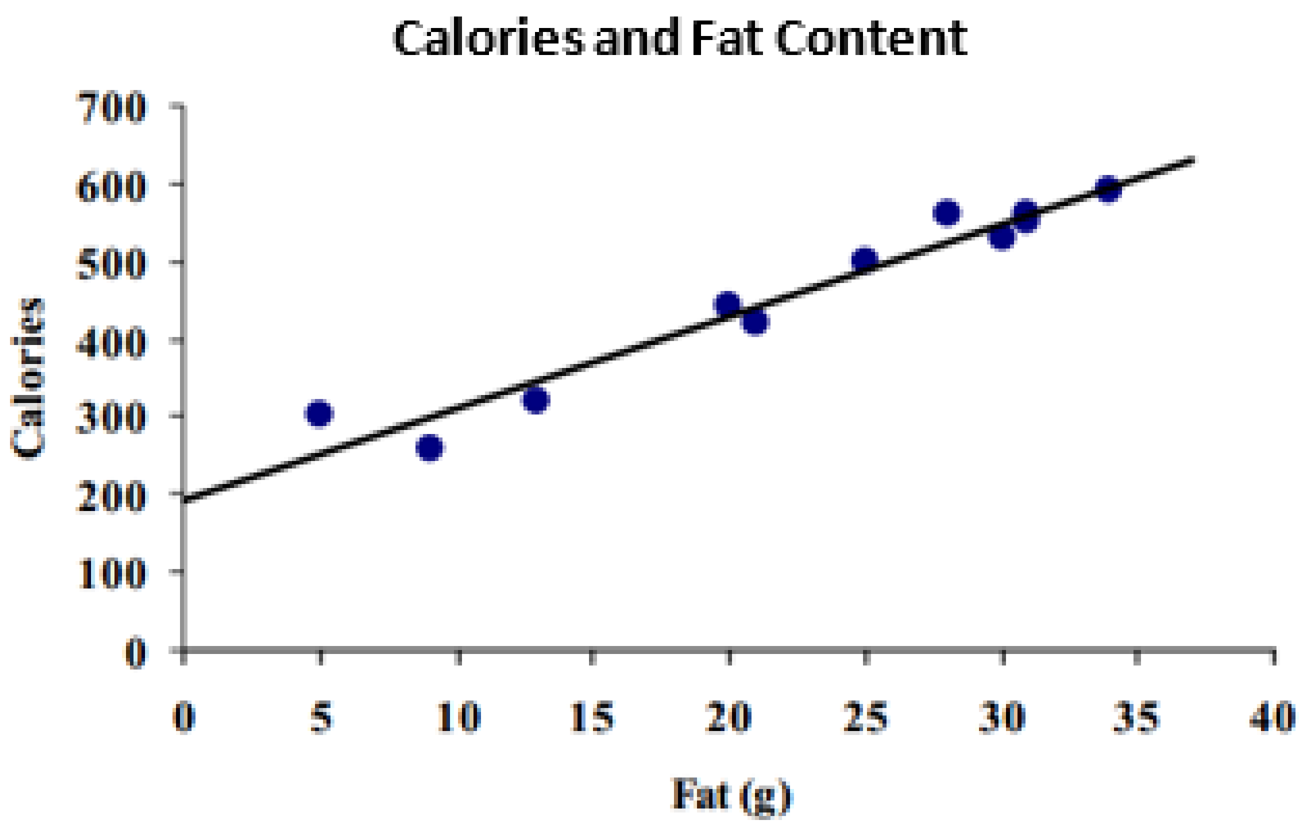
No Linear Relationship (Correlation)

No relationship between the dependent (*y*) values and independent (*x*) values.



Curve of Best Fit

(Linear)

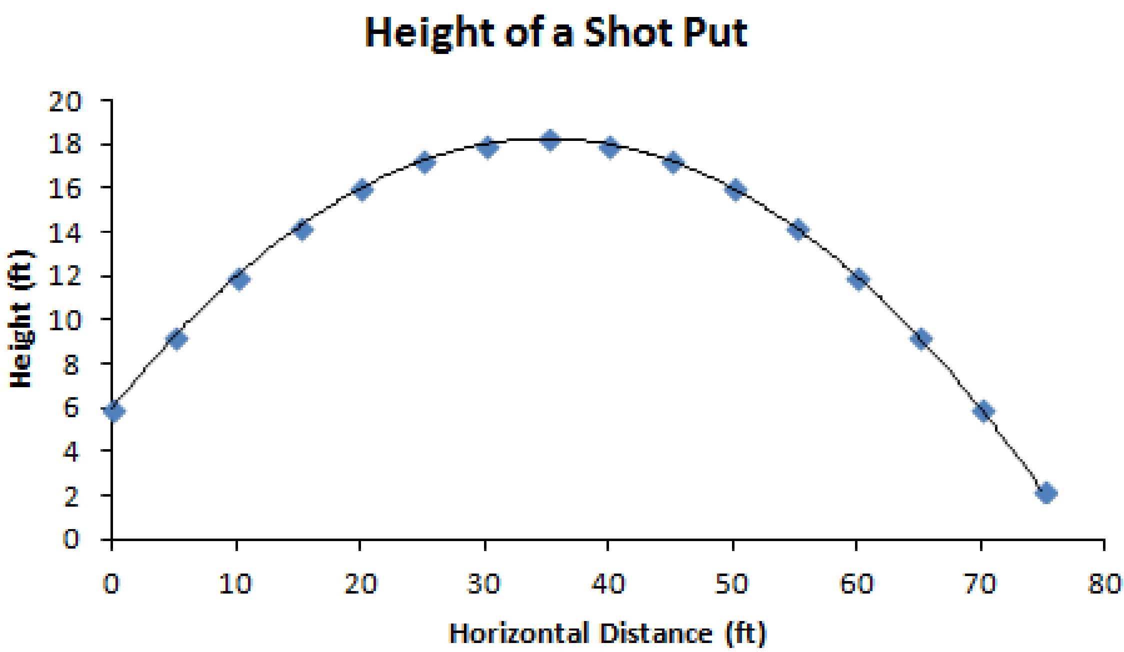


Equation of Curve of Best Fit

$y = 11.731x + 193.85$

Curve of Best Fit

(Quadratic)



Equation of Curve of Best Fit

$$y = -0.01x^2 + 0.7x + 6$$

Outlier Data

(Graphic)