



CCGPS Frameworks Student Edition

Mathematics

Fourth Grade Unit Three Fraction Equivalents



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"Making Education Work for All Georgians"

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Unit 2
FRACTION EQUIVALENTS

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OVERVIEW

In this unit students will:

- understand representations of simple equivalent fractions
- compare fractions with different numerators and different denominators

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight **STANDARDS FOR MATHEMATICAL PRACTICE**: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively, constructing viable arguments and critiquing the reasoning of others, modeling mathematics, using appropriate tools strategically, attending to precision, looking for and making use of structure, and looking for and expressing regularity in repeated reasoning, should be addressed continually as well. The first unit should establish these routines, allowing students to gradually enhance their understanding of the concept of number and to develop computational proficiency.

To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under “Evidence of Learning” be reviewed early in the planning process. A variety of resources should be utilized to supplement. The tasks in these units illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

STANDARDS FOR MATHEMATICAL PRACTICE

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. This list is not exhaustive and will hopefully prompt further reflection and discussion.

1. **Make sense of problems and persevere in solving them.** Students make sense of problems involving equivalent fractions and comparing fractions.
2. **Reason abstractly and quantitatively.** Students demonstrate abstract reasoning about relative size of fractions.
3. **Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding the equivalency of fractions.
4. **Model with mathematics.** Students use constructed fraction strips to demonstrate understanding of equivalent fractions.
5. **Use appropriate tools strategically.** Students select and use tools such as fraction strips and number lines to identify equivalent fractions.
6. **Attend to precision.** Students attend to the language of real-world situations to determine if one fraction is greater than another.
7. **Look for and make use of structure.** Students relate the structure of fractions to the same whole to compare fractions.
8. **Look for and express regularity in repeated reasoning.** Students relate the structure of fractions to the same whole to identify multiple equivalent fractions.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson.*****

ENDURING UNDERSTANDINGS

- Mixed numbers and improper fractions can be used interchangeably.
- Fractions can be represented visually and in written form.
- Fractions with differing parts can be the same size.
- Fractions of the same whole can be compared.

ESSENTIAL QUESTIONS Choose a few questions based on the needs of your students.

- What is a fraction and how can it be represented?
- How can equivalent fractions be identified?
- How can we find equivalent fractions?
- In what ways can we model equivalent fractions?
- How can identifying factors and multiples of denominators help to identify equivalent fractions?
- How can we find equivalent fractions?
- In what ways can we model equivalent fractions?

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- How can we find equivalent fractions and simplify fractions?
- How can benchmark fractions be used to compare fractions?
- What are benchmark fractions?
- How are benchmark fractions helpful when comparing fractions?
- How can we use fair sharing to determine equivalent fractions?
- How do we know fractional parts are equivalent?
- What happens to the value of a fraction when the numerator and denominator are multiplied or divided by the same number?
- How are equivalent fractions related?
- How can you compare and order fractions?
- How are benchmark fractions helpful when comparing fractions?
- How do I compare fractions with unlike denominators?
- How can you compare fraction quantities without adding fractions?
- How do you know fractions are equivalent?
- What can you do to decide whether your answer is reasonable?
- How can identifying factors and multiples of denominators help to identify equivalent fractions?
- How do we represent a fraction that is greater than one?
- How do we locate fractions on a number line?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Identify and give multiple representations for the fractional parts of a whole (area model) or of a set, using halves, thirds, fourths, sixths, eighths, tenths and twelfths.
- Recognize and represent that the denominator determines the number of equally sized pieces that make up a whole.
- Recognize and represent that the numerator determines how many pieces of the whole are being referred to in the fraction.
- Compare fractions with denominators of 2, 3, 4, 6, 10, or 12 using concrete and pictorial models.

COMMON MISCONCEPTIONS

NF.1 & NF.2 - Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing $\frac{1}{2}$ to sixths. They would multiply the denominator by 3 to get $\frac{1}{6}$, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the whole fraction.

Students need to use a fraction in the form of one such as $\frac{3}{3}$ so that the numerator and denominator do not contain the original numerator or denominator.

SELECTED TERMS AND SYMBOLS

Note – At the elementary level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks. Common Core Standards glossary of mathematical terms: <http://www.corestandards.org/Math/Content/mathematics-glossary/glossary>.

The terms below are for teacher reference only and are not to be memorized by the students.

- common fraction
- denominator
- equivalent sets
- improper fraction
- increment
- mixed number
- numerator
- proper fraction
- term
- unit fraction
- whole number

STRATEGIES FOR TEACHING AND LEARNING

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols, and words.
- Interdisciplinary and cross curricular strategies should be used to reinforce and extend the learning activities.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.
- Books such as *Fraction Action* (2007) written and illustrated by Loreen Leedy and *Working with Fractions* (2007) by David A. Adler and illustrated by Edward Miller, are useful resources to have available for students to read during the instruction of these concepts.
- Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
 - What level of support do my struggling students need in order to be successful with this unit?
 - In what way can I deepen the understanding of those students who are competent in this unit?
 - What real life connections can I make that will help my students utilize the skills practiced in this unit?

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Represent and read proper fractions, improper fractions, and mixed numbers in multiple ways.
- Represent equivalent common fractions
- Use mixed numbers and improper fractions interchangeably.
- Compare fractions and express their relationships using the symbols, $>$, $<$, or $=$.

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all fourth grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as performance tasks, they also may be used for teaching and learning.

Scaffolding Task	Tasks that build up to the learning task.
Learning Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
CTE Classroom Tasks	Designed to demonstrate how the Common Core and Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.

Task Name	Task Type/Grouping Strategy	Content Addressed	Standard(s)
Fraction Kits	Scaffolding Task <i>Individual Task</i>	Recognize equivalent fractions	MCC4.NF.1 MCC4.NF.2
Red Rectangles	Constructing Task <i>Individual/Partner Task</i>	Exploring equivalent fractions through an area model	MCC4. NF.1 MCC4.NF.2
Pattern Block Puzzles	Constructing Task <i>Individual/Partner Task</i>	Demonstrate an understanding of equivalent fractions	MCC4.NF.1 MCC4.NF.2
Benchmark Fractions	Constructing Task <i>Small Group/Partner Task</i>	Comparing fractions	MCC4.NF.1 MCC4.NF.2
More or Less	Practice Task <i>Small Group/Partner Task</i>	Comparing fractions	MCC4.NF.2
Closest to 0, 1/2, or 1	Practice Task <i>Small Group/Partner Task</i>	Comparing fractions	MCC4.NF.1 MCC4.NF.2
Their Fair Share	Performance Task <i>Individual Task</i>	Recognize equivalent fractions, comparing fractions	MCC4.NF1
Equivalent Fractions	Practice Task <i>Individual/Partner Task</i>	Using multiplicative/finding equivalent fractions	MCC4.NF.1 MCC4.NF.2
Making Fractions	Practice Task <i>Partner Task</i>	Compare and order fractions	MCC4.NF.2
Write About Fractions	Performance Task <i>Small Group/Partner Task</i>	Write fraction number sentences and explain equivalence	MCC4.NF.1 MCC4.NF.2
Pattern Block Puzzles Revisited	Culminating Task <i>Individual Task</i>	Write fraction number sentences, identify and represent equivalence	MCC4.NF.1 MCC4.NF.2

FORMATIVE ASSESSMENT LESSONS (FALS)

Formative Assessment Lessons are designed for teachers to use in order to target specific strengths and weaknesses in their students’ mathematical thinking in different areas. A Formative Assessment Lesson (FAL) includes a short task that is designed to target mathematical areas specific to a range of tasks from the unit. Teachers should give the task in advance of the delineated tasks and the teacher should use the information from the assessment task to differentiate the material to fit the needs of the students. The initial task should not be graded. It is to be used to guide instruction.

Teachers may use the following Formative Assessment Lessons (FALS) Chart to help them determine the areas of strengths and weaknesses of their students in particular areas within the unit.

Formative Assessments	FALS (Supporting Lesson Included)	Content Addressed	Pacing (Use before and after these tasks)
Picking Fractions		Equivalent Fractions	Fraction Kits Write about Fractions More or Less Closest to 0, $\frac{1}{2}$, 1 Equivalent Fractions

Scaffolding Task: Fraction Kits

Inspired by Math Solutions Publications Teaching Arithmetic: Lessons for Introducing Fractions by Marilyn Burns

TASK CONTENT: In this task, students construct their own set of fraction strips.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

STANDARDS FOR MATHEMATICAL PRACTICES TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Constructing the idea that fractions are relationships, and that the size or amount of the whole matters, is a critical step in understanding fractions. Fair sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. Students have worked with the concept of fair share or partitioning from 2nd grade, with standards which refer to same-sized shares or equal shares. Students should have knowledge of vocabulary terms such as: *numerator and denominator*.

Some common misconceptions, found in *Math Misconceptions*, that children have include:

- Dividing nontraditional shapes into thirds, such as triangles, is the same as dividing a rectangle into thirds. If they are only used to dividing traditional shapes – circles, squares, and rectangles – they begin to think that all shapes are divided similarly.

- Children often do not recognize groups of objects as a whole unit. Instead they will incorrectly identify the objects. For example, there may be 2 cars and 4 trucks in a set of 6 vehicles. The student may mistakenly confuse the set of cars as $\frac{2}{4}$ of the unit instead of $\frac{2}{6}$ or $\frac{1}{3}$ (Bamberger, Oberdorf, & Schultz-Ferrell, 2010).

Therefore, it is important that students are exposed to multiple units of measure, various shapes, and denominators other than halves, thirds, and fourths. Additionally, the denominator used as an expression of the whole is a key concept to express for mastery.

ESSENTIAL QUESTIONS

- What is a fraction and how can it be represented?
- How can equivalent fractions be identified?

MATERIALS

- Construction paper strips of equal length (8 different colors prepared prior to beginning task with students)

GROUPING

Individual

NUMBER TALKS

NUMBER TALKS

Number Talks should now be firmly established as part of the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem “strings” that could be included in a number talk to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Comments: Be sure to cut the strips of construction paper accurately before asking students to make kits. Students should create their own kits since there are basic concepts about fractions to be learned from the process of creating the kit. It is important to discuss the concept of fair sharing. Ask them how they would share the strips of colored paper among various groups of people (two, three, four, six, eight, and sixteen). It is important that you allow time for students to determine the appropriate way to equally share the strips of paper among the various groups. For students who are unable to determine the appropriate measures, ensure they are exposed to strategies discovered by their peers through classroom discussions.

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- To cut fourths, first have students cut halves, then fold and cut each half into fourths.
- To cut eighths, first have students cut fourths (as above), then cut each fourth in half to make eighths.
- To cut sixteenths, have students cut eighths (as above), then cut each eighth in half to make sixteenths.
- To cut thirds, use the concept of measurement. Tell students that the strip is 18 inches long. How many inches for each piece if we want thirds (shares with three people)? Students should have time for small group discussion and sharing.
- To cut sixths, follow the same idea as with thirds, but for sharing with six people. Again, students should have time for small group discussion and sharing.

This task is one that enables the construction of a manipulative that can be used with subsequent tasks. Therefore, it is important that all students are provided with an opportunity to construct their own fraction kit.

Task Directions:

Cut 12-by-18-inch construction paper lengthwise into 3-by-18-inch strips. For the fractions 1 whole, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$, you will need 5 different colors. To include $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{12}$, you will need an additional 3 colors (for a total of 8 colors.)

- Ask students to take a strip of a particular color (that you choose/ or the class agrees upon), fold it in half, and cut it into two pieces. Have them label each piece $\frac{1}{2}$ (and possibly also with the name ‘one half’) and discuss why this label is appropriate (because the pieces are the same size, each is one of the two pieces, which we represent as $\frac{1}{2}$).
- Choose a color for a second strip and have the students fold and cut it into four equal pieces. Instruct students to label each piece $\frac{1}{4}$ (and possibly with the name ‘one fourth’). Have students explain why the label is appropriate.
- Have students fold, cut, and label a third strip in eighths and a fourth strip in sixteenths. (For the sixteenths, students may need to fold a strip in half, cut it, and then fold each half into eighths.)
- Students leave one of the strips whole and label it 1 or $\frac{1}{1}$ (ask students first what they should label it).
- For creating thirds, students would probably be inaccurate in folding. One strategy is to measure the strip, divide the length into three equal segments, and then measure and cut and label. For sixths, they can cut thirds and then fold the thirds in half. (The issue of why this works would be a good topic for class discussion.) Use a similar process for twelfths.

● **FORMATIVE ASSESSMENT QUESTIONS**

- What was the initial cut you made to your colored strips? Why?
- Are your labels appropriate? Why or why not? How do you know?
- Could you use the strips for halves or fourths as a template for making thirds or sixths? Why or why not?
- Which fraction piece is bigger, $\frac{1}{2}$ or $\frac{1}{4}$? Next, $\frac{1}{3}$ or $\frac{1}{6}$?

DIFFERENTIATION

Extension

- Ask students to explain why making other fractions such as $\frac{1}{7}$ would be difficult. Can you name other fractions that would be difficult? How would you go about making them?

Intervention

- Begin by having students work only with halves, thirds, and fourths only.
- Discuss how these pieces compare to the whole.

TECHNOLOGY

- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L540> This lesson has students make and use a set of fraction strips. It can be used for remediation purposes.
- http://www.eastsideliteracy.org/tutorsupport/documents/HO_Fractions.pdf This document provides a template for a fraction kit. It can be used with this task or for remediation purposes.

Constructing Task: Red Rectangles

(Adapted from ETA Cuisenaire's Super Source activity entitled Building Rectangles)

TASK CONTENT: For this activity students are asked to build several rectangles that are colored red over the same proportional area.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

BACKGROUND KNOWLEDGE

Encourage students to organize their thinking to be sure they have found as many ways as possible to represent equivalent fractions. Possible equivalent fractions are shown below.

Before asking students to work on this task, be sure students are able to:

- identify the number of equal pieces needed cover $\frac{2}{3}$ of a rectangle or array
- show equivalent fractions with colored tiles
- record on the student sheet equivalent fractions either by coloring or gluing die cut color tiles
- write an equation which shows the equivalent fractions

ESSENTIAL QUESTIONS

- How can we find equivalent fractions?
- In what ways can we model equivalent fractions?
- How can identifying factors and multiples of denominators help to identify equivalent fractions?

MATERIALS

- Color Tiles
- “Red Rectangles” student recording sheet
- Die cut color tiles or pencils and crayons for recording findings

GROUPING

individual or partner

NUMBER TALK

Continue utilizing the different strategies in number talks and revisiting based on the needs of your students.

Students can once again utilize the friendly number strategies to begin discussing multiples and specifically combining multiples to arrive at a larger product. For example, students could solve the following string of numbers:

$$1 \times 12 = 12$$

$$2 \times 12 = 24$$

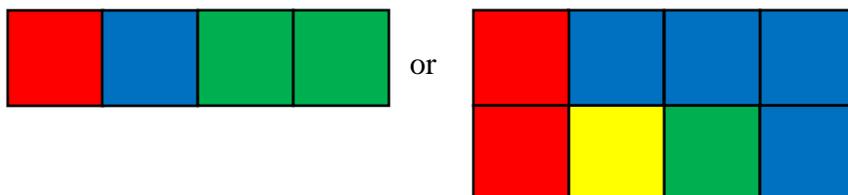
by combining the products of 12 and 24 as well as combining the factors of 1 and 2 the students will produce another multiple of 12, namely $3 \times 12 = 24$

$$\begin{array}{r} 1 \times 12 = 12 \\ + 2 \times 12 = 24 \\ \hline 3 \times 12 = 36 \end{array}$$

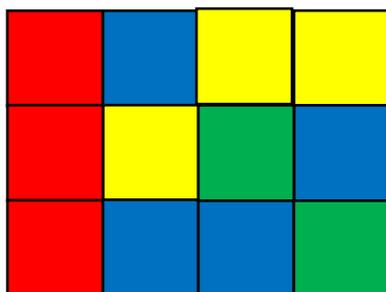
Doubling and halving will also help students build equivalent fractions. (Number Talks, 2010, Sherry Parrish).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: For this activity students are asked to build several rectangles that are colored red over the same proportional area. However, each rectangle has to be a different, namely larger, size. Students will need a number or red color tiles but also they will need other colors as well. They will have to build several rectangles, they can do this with as much creativity as they choose, with the given proportions. For example, the following example can be utilized as a mini lesson or students could discover this on their own; If a child is asked to build a rectangle that is $\frac{1}{4}$ red they could approach it several ways.



or



All three of these rectangles are a different size, however they all contain the same proportion of red tiles. Each is $\frac{1}{4}$ red however a different number of tiles make up that proportion of $\frac{1}{4}$. Students are then asked to write an equivalency statement similar to the one below, as well as record their work on graph paper:

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$$

This is an excellent opportunity to discuss the importance of equivalency as it relates to the same-sized whole. These fractions are proportionally equivalent in relationship to these models, but they do not reference the same sized whole. Are they truly equal?

If available, students can glue die-cut color tiles. Additionally this task could be introduced by reading *The Hershey's Milk Chocolate Bar Fractions Book*, by Jerry Pallotta, illustrated by Rob Bolster or another story about equivalent fractions.

Task Directions:

How can you use Color Tiles to show fractional parts of different rectangles?

- Working with a partner, use Color Tiles to build more than 2 different-sized rectangles to represent each of these situations:

$\frac{5}{6}$ of the tiles are red

$\frac{2}{3}$ of the tiles are red

$\frac{3}{4}$ of the tiles are red

$\frac{2}{5}$ of the tiles are red

- Record your solutions. For each rectangle, write both the total number of tiles and the number of tiles that are red.
- Be ready to talk about how you know your solutions are correct.

FORMATIVE ASSESSMENT QUESTIONS

- How are you keeping your work organized?
- Have you found all more than 2 possible equivalent fractions? How do you know?
- How do you know these two fractions are equivalent?
- How many different illustrations can be created to show equivalent fractions? How do you know?
- How did you go about deciding how many tiles should be in each rectangle?
- How did you figure out how many red tiles to use?
- How did you find a second solution for each situation?
- How do the numbers in the fraction relate to the different tiles you used to create your rectangles?
- Look at the posted solutions. Are there any that you think are not correct? If so, tell why.
- Do you think that there are other solutions beyond those posted? Explain.
- How is it possible that there could be more than one solution for each situation?

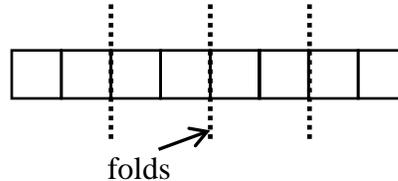
DIFFERENTIATION

Extension

- Students could explore denominators other than those listed in the standard, such as 9, 15, 24, etc.
- Students could experiment with only two colors and begin to make discoveries about what the remaining fraction may be. For example, if $\frac{1}{4}$ of our tiles are red the remaining blue tiles must be $\frac{3}{4}$.
- Students could also make very large rectangles of up to 100 and begin to discover some of the equivalent fraction that work with decimal fraction nicely such as $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{2}$, etc.
- Students could continue far beyond 3 or more rectangles and challenge one another to find as many as possible, which could lead to a discussion of whether there are a finite number of ways to represent a fraction.

Intervention

- Allow students to begin by representing rectangles that are $\frac{1}{2}$ red prior to exploring other fractions.
- Some students may be more successful utilizing virtual manipulatives like those found at http://nlvm.usu.edu/en/nav/frames_asid_203_g_2_t_1.html?from=category_g_2_t_1.html
- Cutting out strips of inch graph paper would also allow students to fold and manipulate the rectangles into $\frac{1}{4}$. For example if you cut out a strip of 8 squares on graph paper and had students fold in half twice they would see that they now have four parts with 2 squares in each part.



TECHNOLOGY

- http://nlvm.usu.edu/en/nav/frames_asid_203_g_2_t_1.html?from=category_g_2_t_1.html
This resource provides virtual fraction bars. It can be used as a mini-lesson for this task or remediation purposes.
- <http://illuminations.nctm.org/ActivityDetail.aspx?ID=80>
This lesson involves creating equivalent fractions by dividing and shading squares or circles. It can be used for additional practice or remediation purposes.
- <http://www.illustrativemathematics.org/standards/k8>
This resource provides additional lessons which can be used for practice or remediation purposes.

Name _____ Date _____

Red Rectangles

How can you use Color Tiles to show fractional parts of different rectangles?

PART 1.

- Working with a partner, use Color Tiles to build more than 2 different-sized rectangles to represent each of these situations:

$\frac{5}{6}$ of the tiles are red

$\frac{2}{3}$ of the tiles are red

$\frac{3}{4}$ of the tiles are red

$\frac{2}{5}$ of the tiles are red

- Record your solutions. For each rectangle, write both the total number of tiles and the number of tiles that are red.
- Be ready to talk about how you know your solutions are correct.

PART 2.

Write several of your equivalent fraction in the spaces provided below:

$$\frac{5}{6} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\frac{2}{3} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\frac{3}{4} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\frac{2}{5} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

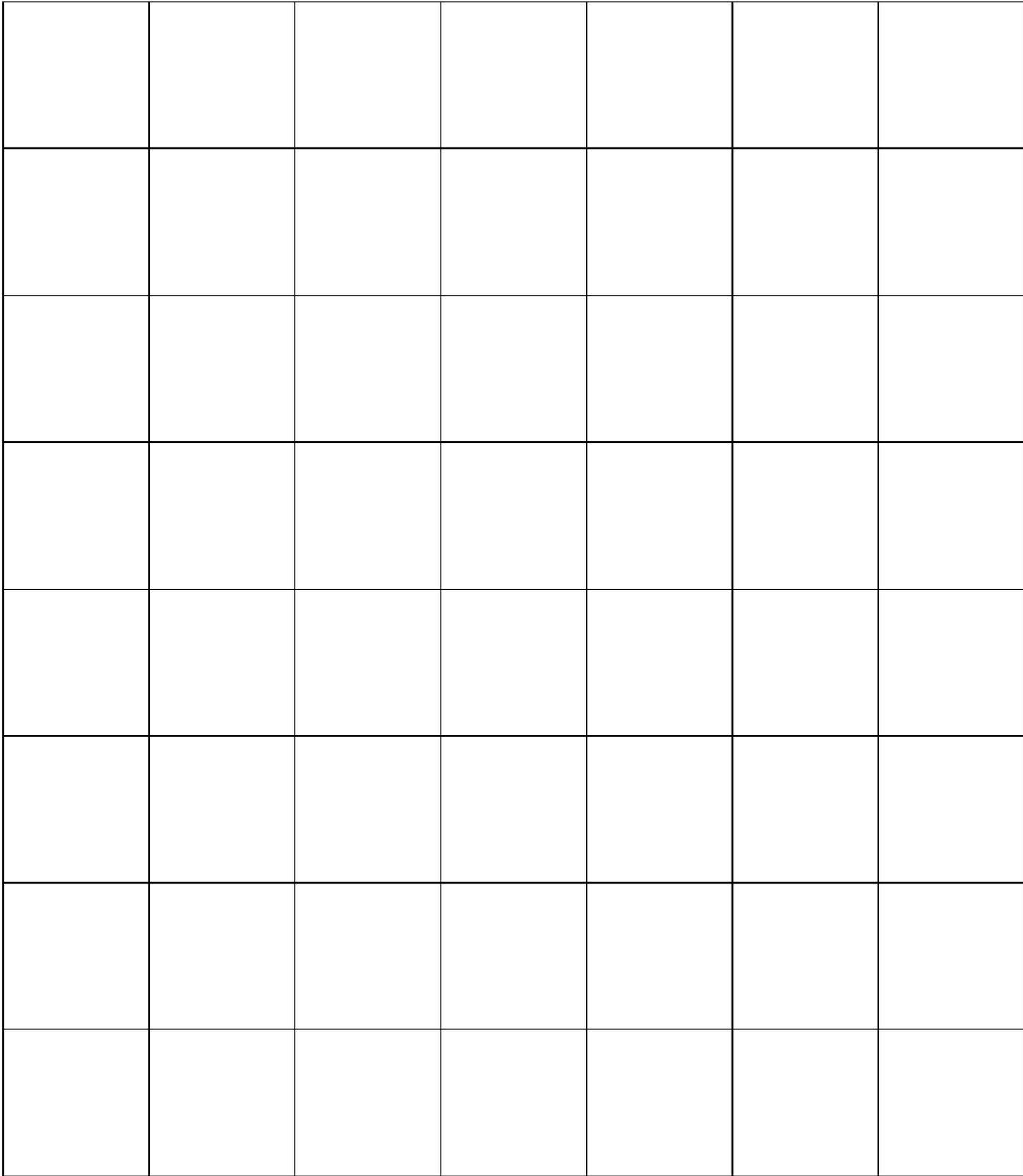
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Describe any patterns you see in the numerators and/or denominators:

Describe some ways that you could create an equivalent fraction:

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1 inch graph paper



Constructing Task: Pattern Block Puzzles

TASK CONTENT: This task could be introduced by reading *The Hershey's Milk Chocolate Bar Fractions Book*, by Jerry Pallotta, illustrated by Rob Bolster or another story about equivalent fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

MCC4.NF.1 Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \times a}{n \times b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

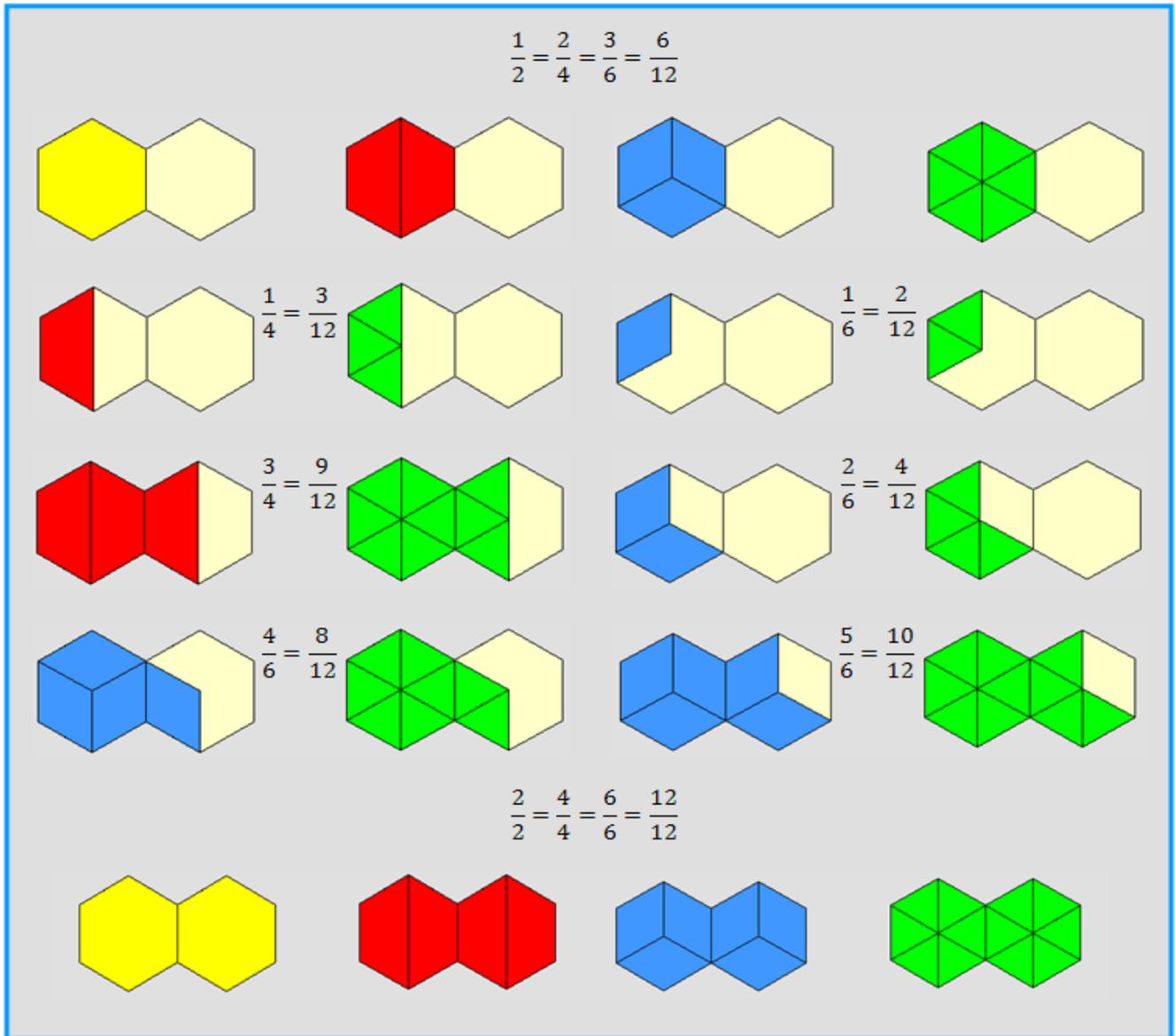
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

Encourage students to organize their thinking to be sure they have found as many ways as possible to represent equivalent fractions. Possible equivalent fractions are shown below.

Before asking students to work on this task, be sure students are able to:

- identify the number of equal pieces needed to cover one whole as the denominator
- show equivalent fractions with the pattern blocks
- record on the student sheet equivalent fractions (either by coloring or gluing die cut pattern blocks)
- write an equation which shows the equivalent fractions



ESSENTIAL QUESTIONS

- How can we find equivalent fractions?
- In what ways can we model equivalent fractions?
- How can we find equivalent fractions and simplify fractions?

MATERIALS

- Pattern blocks
- “Pattern Block Puzzles” student recording sheet
- Die cut paper pattern blocks (or pencils and crayons for recording findings)

- “Pattern Block Puzzles, Version 2” student recording sheet, for intervention students only

GROUPING

Individual or partner

NUMBER TALK

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem “strings” that could be included in a number talks to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

Students can once again utilize the friendly number strategies to begin discussing multiples and specifically combining multiples to arrive at a larger product. For example, students could solve the following string of numbers:

$$1 \times 12 = 12$$

$$2 \times 12 = 24$$

by combining the products of 12 and 24 as well as combining the factors of 1 and 2 the students will produce another multiple of 12, namely $3 \times 12 = 36$

$$\begin{array}{r} 1 \times 12 = 12 \\ + 2 \times 12 = 24 \\ \hline 3 \times 12 = 36 \end{array}$$

Doubling and halving will also help students build equivalent fractions. (Number Talks, 2010, Sherry Parrish).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: A limited set of pattern blocks should be available for the students. It may be helpful to create sets of pattern blocks in plastic bags beforehand for ease in distributing them to the student groups. One possible “limited set” of pattern blocks might have 1 hexagon, 1 trapezoid, 1 rhombus, and 3 triangles. This set would require students to apply their understanding of equivalent fractions as developed in this unit.

If available, students can glue die-cut pattern block pieces. Alternately, students can manipulate the virtual pattern blocks online and easily print and then label their work. One site for pattern blocks is: <http://gingerbooth.com/flash/patblocks/patblocks.php>.

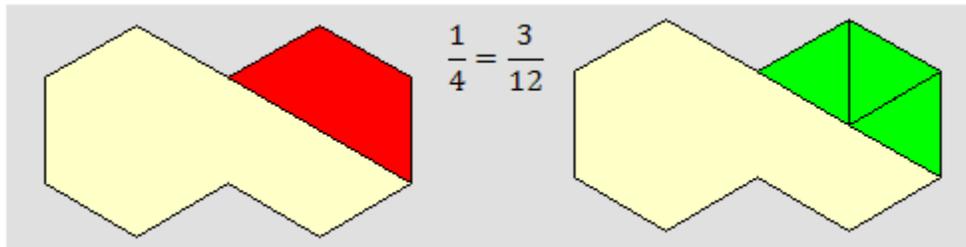
This task could be introduced by reading *The Hershey’s Milk Chocolate Bar Fractions Book*, by Jerry Pallotta, illustrated by Rob Bolster or another story about equivalent fractions.

Task Directions:

Students will follow directions below from the “Pattern Block Puzzle” student recording sheet.

- Obtain a set of pattern blocks.
- Use the pattern blocks to show equivalent fractional amounts.
- Record the equivalent fractions on the “one whole” pairs below.
- Write a number sentence for each equivalent fraction. (See example.)
- How many equivalent fractions can you find?
- Use what you know about factors and multiples to identify two equivalent fractions without using the pattern blocks.
- Which equivalent fractions are the smallest of all of its equivalent fractions? How do you know?

Example:



FORMATIVE ASSESSMENT QUESTIONS

- How are you keeping your work organized?
- Have you found all of the possible equivalent fractions? How do you know?
- How do you know these two fractions are equivalent?
- How many different illustrations can be created to show equivalent fractions? How do you know?

DIFFERENTIATION

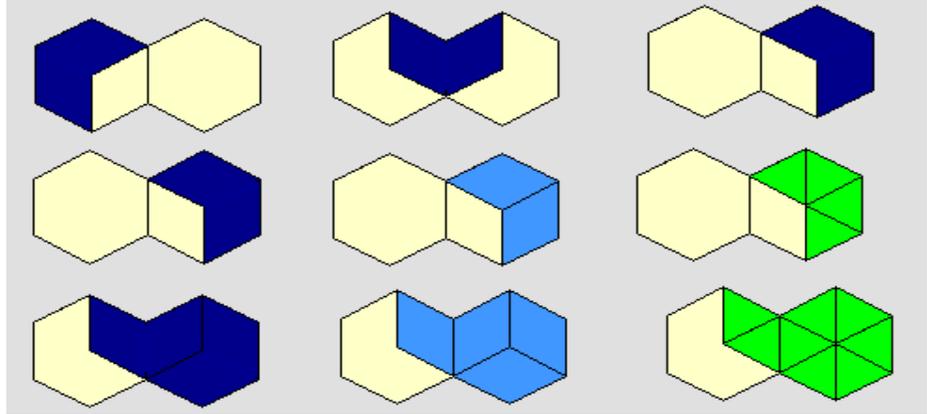
Extension

- Use a set of 8, 10 or 12 unit cubes to represent one whole.
- Use a hexagon and rhombus to represent one whole or use a hexagon and a trapezoid to represent one whole.
- How could you show one-third of this figure? (See diagram to the right.) Students will need to create a “new” pattern block made from two rhombi to show one-third. Students could then be asked to find equivalent fractions for thirds.

Three examples
of one third

$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$$

$$\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$$



- Encourage students to add some of the fractions found in this task. This will give students an opportunity to begin exploring improper fractions and mixed numbers. Be sure pattern block models are used to show the addition problems.

Intervention

- Allow students to use the “Pattern Block Puzzles, Version 2” student recording sheet. In version 2, the whole is represented by the yellow hexagon. Students should be able to find most if not all of the possible equivalent fractions.

TECHNOLOGY

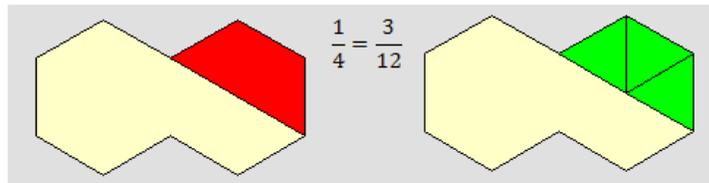
- http://real.doe.k12.ga.us/content/math/destination_math/msc4/msc4/msc4/MSC4/MSC4/Module1/Unit2/Session3/Tutorial.html This tutorial lesson involving equivalent fractions can be used to introduce the task or for remediation purposes.
- http://real.doe.k12.ga.us/content/math/destination_math/msc3/msc3/msc3/MSC3/MSC3/Module3/Unit1/Session3/Tutorial.html This tutorial video involving equivalent fractions can be used to introduce the task or for remediation purposes.
- <http://www.prometheanplanet.com/en-us/Resources/Item/75410/using-fraction-bars> This lesson can be used with an ActivSlate or Smartboard. It can be used as a mini-lesson to introduce the task or for remediation purposes.

Name _____ Date _____

Pattern Block Puzzles

Directions

- Obtain a set of pattern blocks from your teacher.
- Use the pattern blocks to show equivalent fractional amounts.
- Record the equivalent fractions on the “one whole” pairs below.
- Write a number sentence for each equivalent fraction. (See example.)
- How many equivalent fractions can you find?
 - Use what you know about factors and multiples to identify two equivalent fractions without using the pattern blocks.
 - Which equivalent fractions are the smallest of all of its equivalent fractions? How do you know?



Name _____ Date _____

Pattern Block Puzzles

Version 2

Directions

- Obtain a set of pattern blocks.
- Use the pattern blocks to show equivalent fractional amounts.
- Record the equivalent fractions on the “one whole” pairs below.
- Write a number sentence for each equivalent fraction. (See example.)
- How many equivalent fractions can you find?
 - Use what you know about factors and multiples to identify two equivalent fractions without using the pattern blocks.
 - Which equivalent fractions use the smallest unit of all of the equivalent fractions you have found? How do you know?

$$\frac{3}{6} = \frac{1}{2}$$



Constructing Task: Benchmark Fractions

Adapted from Fosnot, C. The Field Trip, Context for Learning Mathematics.

TASK CONTENT: The main focus of this task is on the development of reasonableness of judging the magnitude of fractional amounts.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(nxa)/(nxb)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students were familiarized with benchmark fractions in 2nd grade. They worked to partition circles and rectangles into halves and thirds. In 3rd grade students generated simple equivalent fractions and were required to explain why the fractions were equivalent.

ESSENTIAL QUESTIONS

- How can benchmark fractions be used to compare fractions?

MATERIALS

- Connecting cubes

- Strips of equal length paper
- Fraction Kits

GROUPING

Small group or partner

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed problem “strings” which may be included in number talks to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: For this task, if some students immediately say $17/22$, having constructed the idea of fractions as fair sharing division from previous tasks, acknowledge their thinking but explain you were wondering about how much that would be. In a real situation, no one is going to cut a candy bar into 22 pieces, and you are wondering about how much that amount is so that the fewest possible cuts could be made. Direct students to the guiding questions listed on their student sheet. The main focus of this task is on the development of reasonableness of judging the magnitude of fractional amounts.

Task directions:

Students will follow the directions below from the “Benchmark Fractions” task sheet.

Mr. Toms’ 4th grade class just concluded a unit on the solar system. To celebrate how hard students worked, Mr. Toms decided to purchase Milky Way candy bars for students to share. The grocery store only had 17 of the king size candy bars, but there are 22 students in his class. If Mr. Toms buys the bars, about how much of a bar would each student receive? Is the amount about $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{3}$ or 1 whole? Where could one cut be made that would be a nice approximate?

FORMATIVE ASSESSMENT QUESTIONS

- What do you notice about the unit fractions you have created?
- If you tried dividing, would you have the same results?
- Can you give me an estimated size of the fraction you created with that cut?
- Can you tell me what you are doing with the pieces of the bars as you work with them?
- Does your answer make sense? How do you know?

DIFFERENTIATION

Extension

- Explain how the process for redistributing the pieces differs as the number of candy bars and/or number of students changes?

Intervention.

- Students can cut out their drawings of the bars then cut off pieces and move them to make approximate equivalent amounts.

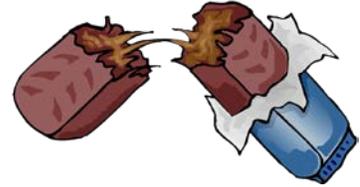
TECHNOLOGY

- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L541> This lesson has students work with fraction strips to compare and order fractions using relationships. It can be used for additional practice, remediation or extending understanding of the concept.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L347> This lesson has students consider the size or value of the same fraction when different wholes compared. It can be used for remediation purposes, additional practice, and/or extending understanding of the concept.

Name _____ Date _____

Benchmark Fractions

Directions



Mr. Toms' 4th grade class just concluded a unit on the solar system. To celebrate how hard students worked, Mr. Toms decided to purchase Milky Way candy bars for students to share. The grocery store only had 17 of the king size candy bars, but there are 22 students in his class. If Mr. Toms buys the bars, about how much of a bar would each student receive? Is the amount about $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{3}$ or 1 whole? Where could one cut be made that would be a nice approximate?

Is the amount about

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{3}$ or 1?

Where could one cut be made that would be a nice approximate?

Practice Task: More or Less

Adapted from Actions on Fractions, Navigating through Number and Operations in Grades 3-5.

TASK CONTENT: In this task, students will practice compare fractions to a given amount.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

STANDARDS FOR MATHEMATICAL PRACTICES TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

BACKGROUND KNOWLEDGE

Students were familiarized with benchmark fractions in 2nd grade. They worked to partition circles and rectangles into halves and thirds. In 3rd grade students generated simple equivalent fractions and were required to explain why the fractions were equivalent.

ESSENTIAL QUESTIONS

- What are benchmark fractions?
- How are benchmark fractions helpful when comparing fractions?

MATERIALS

- More or Less game cards

GROUPING

small group or partner

NUMBER TALKS

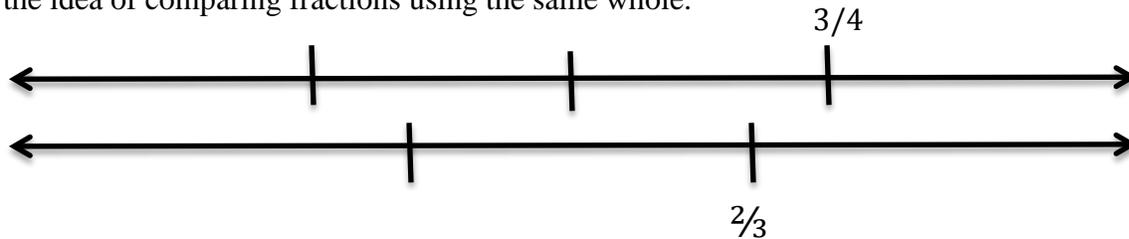
Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem “strings” which may be included in number talks to further develop mental math skills. See Mini-lessons for Operations with

Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard) Also, students can continue to explore area models and partial products to improve their ability to find equivalent fractions and skip county by any number. See page 247 in *Number Talks* (Number Talks, 2010, Sherry Parrish).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: Review the meaning of fraction and the ways in which the students can compare two fractions. Explore the patterns that the students have discovered for determining the larger or smaller fractions. Help students think about fractions in four groups: fractions with the same denominators, fractions with a numerator of 1, fractions with same numerators (other than 1), and fractions with different numerators and different denominators.

Discuss placing fractions on a number line separated into equal parts. It is important to maintain the idea of comparing fractions using the same whole.



Task directions:

Students will follow the directions below from the “More or Less” student sheet.

Round 1: Benchmark fraction is $\frac{3}{4}$

1. Pull a card from the top of the stack.
2. Determine if the fraction is larger or smaller than $\frac{3}{4}$. Explain to your partner(s) why you believe it is larger or smaller.
3. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
4. Pull another card and complete steps 2-3 until all cards have been used.

Round 2: Benchmark fraction $\frac{1}{2}$

1. Pull a card from the top of the stack.
2. Identify an equivalent fraction for the benchmark fraction.
3. Determine if the fraction is larger or smaller than $\frac{1}{2}$. Explain to your partner(s) why you believe it is larger or smaller using a number line.
4. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
5. Pull another card and complete steps 2-4 until all cards have been used.

Round 3: Benchmark fraction $\frac{1}{3}$

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1. Pull a card from the top of the stack.
2. Identify an equivalent fraction for the benchmark fraction.
3. Determine if the fraction is larger or smaller than $\frac{1}{3}$. Explain to your partner(s) why you believe it is larger or smaller a number line.
4. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
5. Pull another card and complete steps 2-4 until all cards have been used.

Round 4:

1. Pull two cards from the top of the stack.
2. Determine which of the fractions is largest. Explain to your partner(s) why you believe it is larger.
3. For the other player(s), think of another way to explain why the fraction is larger and share with the group.
4. Pull two more cards and complete steps 2-3 until all cards have been used.

FORMATIVE ASSESSMENT QUESTIONS

- Can you think of another way to explain the comparison?
- What equivalent fractions can you identify for _____?
- How do you know those fractions are equivalent?
- Which fraction is smaller?
- Which fraction is larger?

DIFFERENTIATION

Extension

- Have students work with fractions greater than 1, such as $\frac{5}{4}$ and $\frac{13}{12}$

Intervention

- Allow students to use manipulatives to help compare the fractions.
- Encourage students use number lines to compare the fractions.

TECHNOLOGY

- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L541> This lesson has students work with fraction strips to compare and order fractions using relationships. It can be used for additional practice, remediation or extending understanding of the concept.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L347> This lesson has students consider the size or value of the same fraction when different wholes compared. It can be used for remediation purposes, additional practice, and/or extending understanding of the concept.

Name _____ Date _____

More or Less

Directions

Round 1: Benchmark fraction is $\frac{3}{4}$

1. Pull a card from the top of the stack.
2. Determine if the fraction is larger or smaller than $\frac{3}{4}$. Using a number line, explain to your partner(s) why you believe it is larger or smaller.
3. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
4. Pull another card and complete steps 2-3 until all cards have been used.

Round 2: Benchmark fraction $\frac{1}{2}$

1. Pull a card from the top of the stack.
2. Identify an equivalent fraction for the benchmark fraction.
3. Determine if the fraction is larger or smaller than $\frac{1}{2}$. Using a number line, explain to your partner(s) why you believe it is larger or smaller.
4. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
5. Pull another card and complete steps 2-4 until all cards have been used.

Round 3: Benchmark fraction $\frac{1}{3}$

1. Pull a card from the top of the stack.
2. Identify an equivalent fraction for the benchmark fraction.
3. Determine if the fraction is larger or smaller than $\frac{1}{3}$. Using a number line, explain to your partner(s) why you believe it is larger or smaller.
4. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
5. Pull another card and complete steps 2-4 until all cards have been used.

Round 4:

1. Pull two cards from the top of the stack.
2. Determine which of the fractions is largest. Explain to your partner(s) why you believe it is larger.
3. For the other player(s), think of another way to explain why the fraction is larger and share with the group.
4. Pull two more cards and complete steps 2-3 until all cards have been used.

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$\frac{6}{7}$	$\frac{3}{4}$	$\frac{5}{8}$
$\frac{7}{10}$	$\frac{3}{5}$	$\frac{5}{12}$
$\frac{5}{6}$	$\frac{3}{7}$	$\frac{1}{2}$
$\frac{3}{6}$	$\frac{1}{4}$	$\frac{1}{3}$

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$\frac{1}{5}$	$\frac{1}{8}$	$\frac{7}{9}$
$\frac{2}{4}$	$\frac{2}{6}$	$\frac{3}{8}$
$\frac{4}{5}$	$\frac{1}{1}$	$\frac{4}{4}$

Practice Task: Closest to 0, $\frac{1}{2}$, or 1

Inspired by Math Solutions Publications Teaching Arithmetic: Lessons for Introducing Fractions by Marilyn Burns

TASK CONTENT: This task was developed to use as a continuation from the *Fraction Kits* task, but this is not a requirement. If students did not construct their own fraction kits, then they should be able to use other manipulatives that will enable them to have visual representations of equivalent fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.1 Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \times a}{n \times b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Constructing the idea that fractions are relationships and that the size or amount of the whole matters is a critical step in understanding fractions. Fair sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. Students have worked with the concept of fair share or partitioning from 2nd grade, with standards which refer to same-sized shares or equal shares. Students should have knowledge of vocabulary terms such as: *numerator and denominator*.

Some common misconceptions, found in *Math Misconceptions*, that children may include:

- Dividing nontraditional shapes into thirds, such as triangles, is the same as dividing a rectangle into thirds. If they are only used to dividing traditional shapes – circles, squares, and rectangles – they begin to think that all shapes are divided similarly.
- Children often do not recognize groups of objects as a whole unit. Instead they will incorrectly identify the objects. For example, there may be 2 cars and 4 trucks in a set of 6 vehicles. The student may mistakenly confuse the set of cars as $\frac{2}{4}$ of the unit instead of $\frac{2}{6}$ or $\frac{1}{3}$ (Bamberger, Oberdorf, & Schultz-Ferrell, 2010).

Therefore, it is important that students are exposed to multiple units of measure, various shapes, and denominators other than halves, thirds, and fourths. Additionally, the denominator used as an expression of the whole is a key concept to express for mastery.

ESSENTIAL QUESTIONS

- How can equivalent fractions be identified?
- How can fractions be ordered on a number line?

MATERIALS

- Fraction kits constructed from Fraction Kits Task
- Number line
- Two number cubes (dice) per group

GROUPING

Small group or partner

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem “strings” which may be included in a number talks to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

Students can once again utilize the friendly number strategies to begin discussing multiples and specifically combining multiples to arrive at a larger product. For example, students could solve the following string of numbers:

$$1 \times 12 = 12$$

$$2 \times 12 = 24$$

by combining the products of 12 and 24 as well as combining the factors of 1 and 2 the students will produce another multiple of 12, namely $3 \times 12 = 24$

$$\begin{array}{r} 1 \times 12 = 12 \\ + 2 \times 12 = 24 \\ \hline \end{array}$$

$$3 \times 12 = 36$$

Doubling and halving will also help students build equivalent fractions. (Number Talks, 2010, Sherry Parrish).

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Comments: Students should have constructed their own fraction kit from the activity, *Fraction Kits*. It is important that you allow time for students to determine where each fraction should be placed on the number line. To enhance the understanding in this task, students may need to construct their own number lines from 0 to 1. This would also be an opportunity to review benchmark fractions by determining where they are placed on the number line. Students may need to stop to get fraction strips to check for equivalence. For students who are unable to determine the appropriate measures, ensure they are exposed to strategies discovered by their peers through classroom discussions.

This task was developed to use as a continuation from the *Fraction Kits* task, but this is not a requirement. If students did not construct their own fraction kits, then they should be able to use other manipulatives that will enable them to have visual representations of equivalent fractions.

Task Directions:

- First player will roll the dice.
- The smaller number will represent the numerator while the larger will be the denominator.
- Students will discuss if the fractions are closer 0, $\frac{1}{2}$, or 1.
- Record the discussion result on the number line with a partner or group members. (Example: Write on the number line where $\frac{3}{4}$ would be located).
- Continue taking turns until each person has had a 10th turn.
- Once all turns have been taken, students should be able to accurately answer assessment questions.

FORMATIVE ASSESSMENT QUESTIONS

- Which fractions are close to $\frac{1}{2}$?
- According to the fraction kit, which fraction is exactly the same as $\frac{2}{4}$?
- How does the relationship between the denominator and the numerator change when it gets closer to 0, $\frac{1}{2}$, or 1?
- How does $\frac{1}{2}$ and 1 compare?
- What are some examples of equivalent fractions you identified during the game?

DIFFERENTIATION

Extension

- Ask students to roll the dice twice for each turn. When they are rolled the first time, they should add the two dice, record it. They should roll them again and record the addition of the two dice again. The smaller number goes on the top and the larger number goes on the bottom. This allows for fractions with larger numerators and denominators (i.e. $9/12$).

Intervention

- Have the students use the whole piece from the fraction kit as the number line. When they roll their fraction, for example $3/4$, they should collect the pieces from the fraction kit, line them up under the whole unit, and then record their fraction using this as a marker. This may have to be modeled before they work independently.

TECHNOLOGY

- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L541> This lesson has students work with fraction strips to compare and order fractions using relationships. It can be used for additional practice, remediation or extending understanding of the concept.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L347> This lesson has students consider the size or value of the same fraction when different wholes compared. It can be used for remediation purposes, additional practice, and/or extending understanding of the concept.

Performance Task: Their Fair Share

Adapted from Fosnot, C. The Field Trip, Context for Learning Mathematics.

TASK CONTENT: In this task, students construct the idea that fractions are relationships and that the size or amount of the whole matters. The fair sharing context also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF1 Explain why a fraction a/b is equivalent to a fraction $(nxa)/(nxb)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

STANDARDS FOR MATHEMATICAL PRACTICES EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Constructing the idea that fractions are relationships and that the size or amount of the whole matters is a critical step in understanding fractions. Fair sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. Students have worked with the concept of fair share or partitioning from 2nd grade, with standards which refer to same-sized shares or equal shares.

ESSENTIAL QUESTIONS

- How can we use fair sharing to determine equivalent fractions?
- How do we know fractional parts are equivalent?

MATERIALS

- Connecting cubes
- Equal length strips of paper/index cards
- Chart paper

- Fraction Kits from previous task (optional)

GROUPING

Small group or partner

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem “strings” which may be included in a number talks to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: The numbers in this story have been chosen purposefully. Given the chosen numbers, students are catapulted into proportional reasoning. It is important that you facilitate discussions regarding their thinking at this point allowing them the opportunity to figure out how much each person in each group received.

When students begin to compare and/or add pieces together, some students may attempt to represent subs with the connecting cubes, but may not make equal size subs. If you see students using various sizes, be sure to point this out by asking if one group received bigger subs. Stay grounded in the context to help students realize the meaning of what they are doing. You want students to derive a common length sub in order to compare and/or add the fractional amounts. This idea is important for the construction of common denominators.

Make all materials available for use. *Once students realize the need for a common length sub, you may offer them index cards to represent the subs. This becomes a powerful tool for understanding the fractional parts in relation to the same-sized whole.*

Task directions:

Students will follow the directions below from the “Their Fair Share” student sheet.

A fourth- grade class traveled on a field trip in four separate vehicles. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows:

- The first group had 3 people and shared 2 subs equally.
- The second group had 4 people and shared 3 subs equally.
- The third group had 9 people and shared 6 subs equally.
- The last group had 6 people and shared 4 subs equally.

When they returned from the field trip, the children began to argue that the portion of the sandwiches they received was not fair, that some children got more to eat than others. Were they right? Or did everyone get the same amount?

Create a poster of the ideas and strategies you used to solve this problem. Be sure your poster is concise and clear presentations of the important ideas and strategies you want to present.

FORMATIVE ASSESSMENT QUESTIONS

- What initial cut did you make to your subs? Why?
- Did you use all parts of the subs? Were each parts equivalent?
- (For students using connecting cubes) Were all subs the same size? What amount of cubes could you use to show all of the subs were the same size?

DIFFERENTIATION

Extension

- Encourage students to try a different strategy to determine if they will arrive at the same fractional parts.

Intervention

- Have students use strips of equal length paper to represent the submarine sandwiches.
- Encourage students to use connecting cubes. Have them begin with 12 cubes to get them thinking about how to show common length subs.

TECHNOLOGY

- http://real.doe.k12.ga.us/content/math/destination_math/msc3/msc3/msc3/MSC3/MS C3/Module3/Unit1/Session3/Tutorial.html This tutorial involving equivalent fractions can be used to introduce the concept or for remediation purposes.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L543> This lesson has students investigate the length model to find equivalent fractions. It can be used for additional practice, remediation or extending understanding of the concept.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L338> This lesson involving finding equivalent fractions using eggs can be used for additional practice, remediation or extending understanding of the concept.

Name _____ Date _____

Their Fair Share

Directions

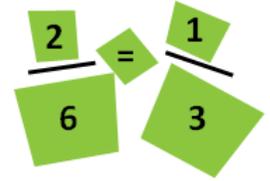


A fourth- grade class traveled on a field trip in four separate vehicles. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows:

- The first group had 3 people and shared 2 subs equally.
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Create a poster of the ideas and strategies you used to solve this problem. Be sure your poster is concise and clear presentations of the important ideas and strategies you want to present.



Practice Task: Equivalent Fractions

TASK CONTENT: This task allows students to explore the relationship between equivalent fractions and write equations for equivalent fractions using the product and quotient of a fraction equivalent to one.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Constructing the idea that fractions are relations and that the size or amount is relative to the whole is a critical step in understanding fractions. Fair sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. Students have worked with the concept of fair share or partitioning since 2nd grade, with standards which refer to same-sized shares or equal shares. Students should have knowledge of vocabulary terms such as: *numerator and denominator*.

Some common misconceptions, found in *Math Misconceptions*, that children have include:

- Dividing nontraditional shapes into thirds, such as triangles, is the same as dividing a rectangle into thirds. If they are only used to dividing traditional shapes – circles, squares, and rectangles – they begin to think that all shapes are divided similarly.
- Children often do not recognize groups of objects as a whole unit. Instead they will incorrectly identify the objects. For example, there may be 2 cars and 4 trucks in a set of 6 vehicles. The student may mistakenly confuse the set of cars as $\frac{2}{4}$ of the unit instead of $\frac{2}{6}$ or $\frac{1}{3}$ (Bamberger, Oberdorf, & Schultz-Ferrell, 2010).

Therefore, it is important that students are exposed to multiple units of measure, various shapes, and denominators other than halves, thirds, and fourths. Additionally, the denominator used as an expression of the whole is a key concept to express for mastery.

ESSENTIAL QUESTIONS

- What happens to the value of a fraction when the numerator and denominator are multiplied or divided by the same number?
- How are equivalent fractions related?

MATERIALS

- “Equivalent Fractions, $\frac{2}{3}$,” student recording sheet
- “Equivalent Fractions, $\frac{3}{4}$,” student recording sheet

GROUPING

Individual or partner

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem “strings” that could be included in a number talk to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

Students can utilize the friendly number strategies to begin discussing multiples and specifically combining multiples to arrive at a larger product. For example, students could solve the following string of numbers:

$$1 \times 12 = 12$$

$$2 \times 12 = 24$$

by combining the products of 12 and 24 as well as combining the factors of 1 and 2 the students will produce another multiple of 12, namely $3 \times 12 = 24$

$$1 \times 12 = 12$$

$$\begin{array}{r} + \quad 2 \times 12 = 24 \\ \quad 3 \times 12 = 36 \end{array}$$

Doubling and halving is also a helpful strategy in building equivalent fractions. (Number Talks, 2010, Sherry Parrish).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: This task allows students to explore the relationship between equivalent fractions and write equations for equivalent fractions using the product and quotient of a fraction equivalent to one.

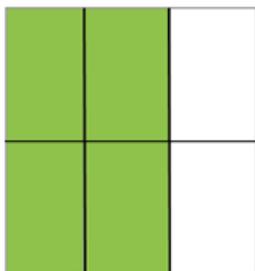
This task is adapted from Van De Walle, J. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.) Boston: Pearson Education, Inc. See the section on “Equivalent-Fraction Concepts.” (This task is adapted from an activity titled “Slicing Squares” on p. 311.)

Give students the opportunity to explore equivalent fractions with this task. Encourage students to look for patterns, both in the models as well as in the numerical representations. Equivalent fractions can be thought of as different names for a fraction.

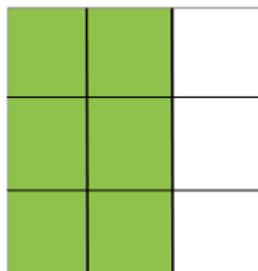
Once students have written equivalent fractions and are able to show that the fraction was multiplied by a fraction equivalent to 1, then begin the discussion about using the inverse operation. Ask students how they can simplify a fraction by dividing it by a fraction equivalent to 1. See the examples below.

$$\frac{2}{3} = \frac{2}{3} \times \frac{4}{4} = \frac{8}{12}; \frac{8}{12} = \frac{8}{12} \div \frac{4}{4} = \frac{2}{3} \qquad \frac{3}{4} = \frac{3}{4} \times \frac{2}{2} = \frac{6}{8}; \frac{6}{8} = \frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$$

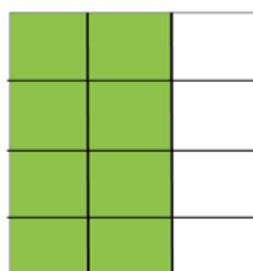
Possible solutions to the “Equivalent Fractions²,” student recording sheet are shown below:



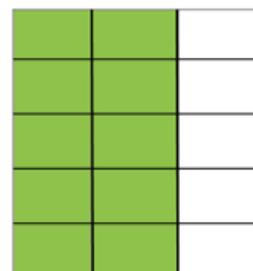
$$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$$



$$\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$$

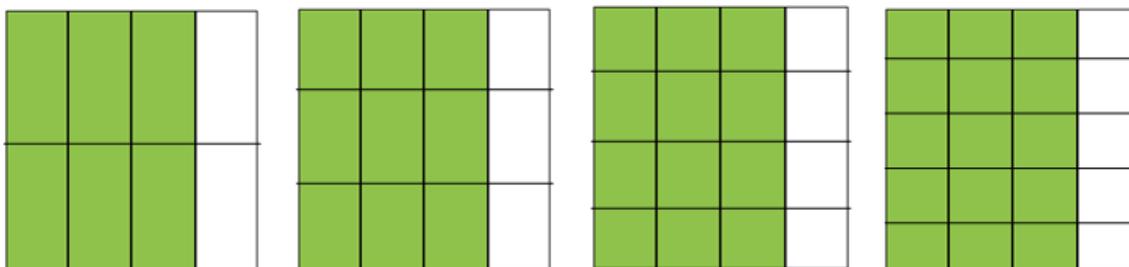


$$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$$



$$\frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$$

Possible solutions to the “Equivalent Fractions $\frac{3}{4}$ ” student recording sheet are shown below.



$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$$

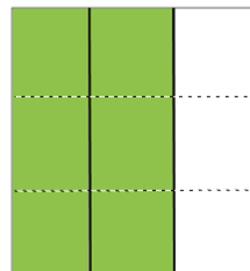
$$\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$$

$$\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$$

Task Directions

Students will follow the directions below from the “Equivalent Fractions $\frac{2}{3}$ ” student recording sheet.

Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each model. See the example below.



Students will follow the directions below from the “Equivalent Fractions $\frac{3}{4}$ ” student recording sheet.

I cut each piece into equal 3 parts, making 9 pieces.
 $\frac{2}{3} = \frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$

Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square.

FORMATIVE ASSESSMENT QUESTIONS

- Into how many parts did you slice each piece?
- What is a fraction that is equivalent to one? (If the student sliced each piece into three parts, they need to multiply the fraction by $\frac{3}{3}$.)
- How could you use equivalent fractions to simplify this fraction (i.e. $\frac{6}{9}$)?

DIFFERENTIATION

Extension

- Invite students to play the Fraction Game. <http://illuminations.nctm.org/ActivityDetail.aspx?ID=18>

Georgia Department of Education
Common Core Georgia Performance Standards Framework
Fourth Grade Mathematics Unit 3

This game requires students to recognize equivalent fractions. Students should think about scenarios from the game that could be presented to the class so that students can discuss strategies and choices available to them.

Intervention

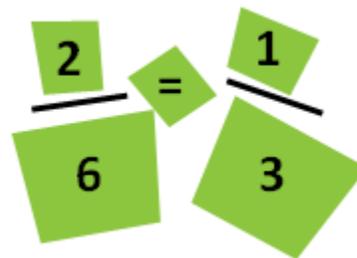
- Some students may benefit from having a table of equivalent fractions. Ask students to complete the table by multiplying the numerator and denominator by the same number.

TECHNOLOGY

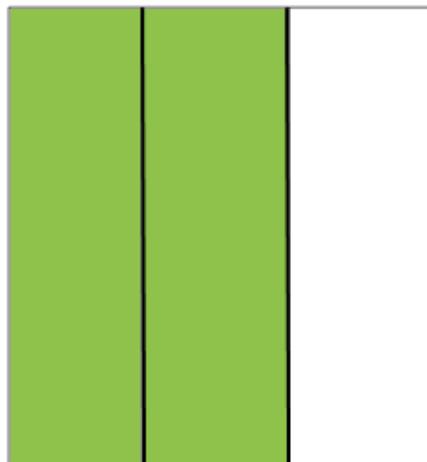
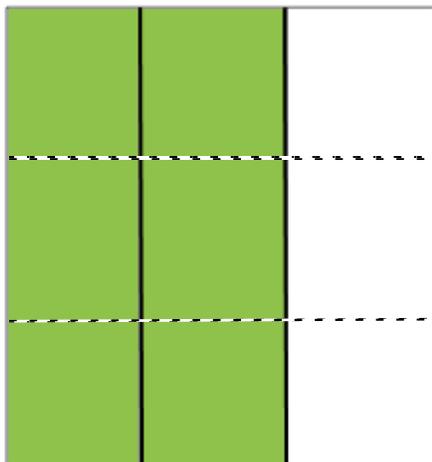
- <http://www.prometheanplanet.com/en-us/Resources/Item/75410/using-fraction-bars> This lesson can be used with an ActivSlate or Smartboard. It can be used as a mini-lesson to introduce the task or for remediation purposes.
- <http://learnzillion.com/lessons?utf8=%E2%9C%93&filters%5Bsubject%5D=math&query=&filters%5Bgrade%5D%5B%5D=4&filters%5Bdomain%5D=NF%3A+Number+and+Operations-Fractions&filters%5Bstandard%5D=4.NF.1%3A+Explain+why+a+fraction+a%2Fb+is+equival...>
- <http://learnzillion.com/lessons?utf8=%E2%9C%93&filters%5Bsubject%5D=math&query=&filters%5Bgrade%5D%5B%5D=4&filters%5Bdomain%5D=NF%3A+Number+and+Operations-Fractions&filters%5Bstandard%5D=4.NF.2%3A+Compare+two+fractions+with+differen...>
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Name _____ Date _____

Equivalent Fractions – $\frac{2}{3}$

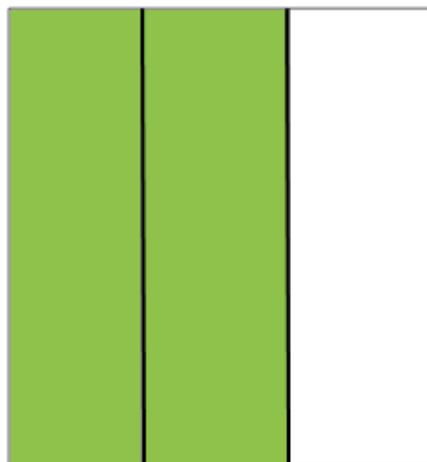
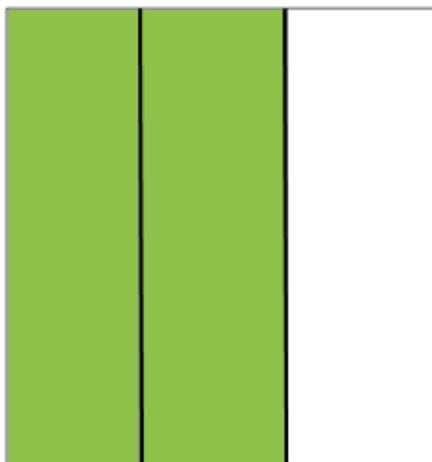


Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square. See the example below.



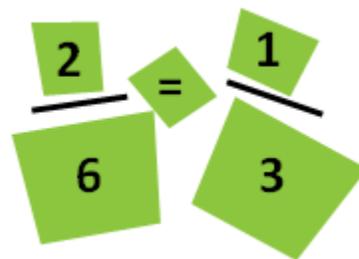
I cut each piece into equal 3 parts,
making 9 pieces.

$$\frac{2}{3} = \frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$$

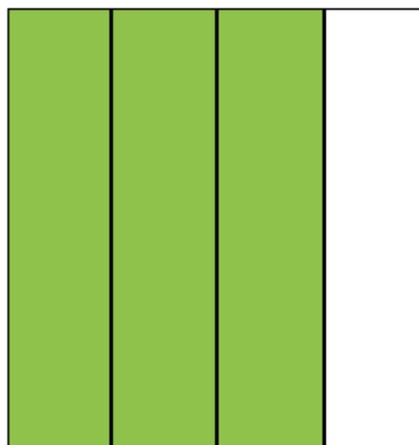
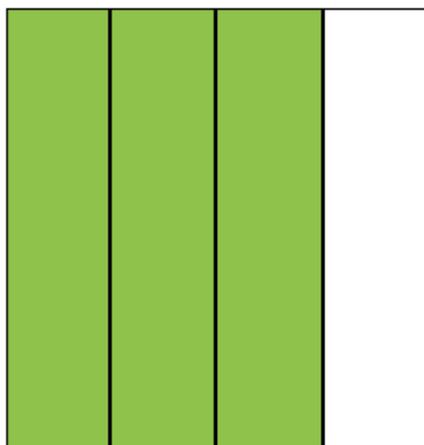
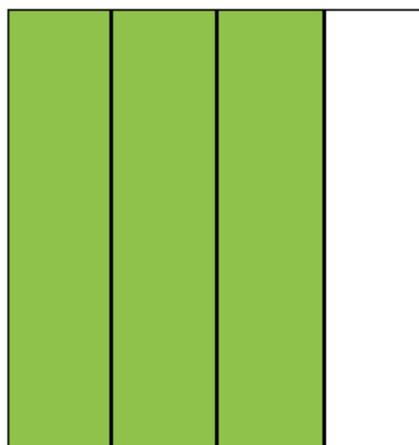
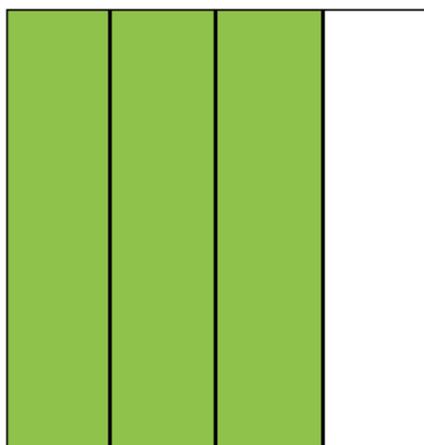


Name _____ Date _____

Equivalent Fractions – $\frac{3}{4}$

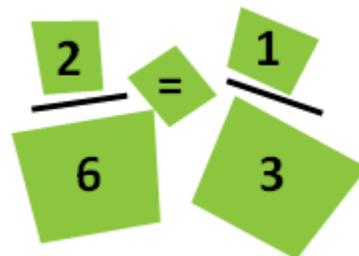


Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square.



Name _____ Date _____

Equivalent Fractions



Find fractions equivalent to the fractions in the table below. Record the equivalent fractions in the white boxes.

$\frac{1}{2}$						$\frac{3}{4}$		
$\frac{1}{3}$						$\frac{2}{5}$		
$\frac{1}{4}$						$\frac{3}{5}$		
$\frac{1}{5}$						$\frac{4}{5}$		
$\frac{1}{6}$	$\frac{2}{12}$					$\frac{4}{6}$		
$\frac{2}{3}$						$\frac{5}{6}$		

Practice Task: Making Fractions

Adapted from Actions on Fractions, Navigating through Number and Operations in Grades 3-5.

TASK CONTENT: Students play a game to practice comparing fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

BACKGROUND KNOWLEDGE

Students were familiarized with benchmark fractions in 2nd grade. They worked to partition circles and rectangles into halves and thirds. In 3rd grade students generated simple equivalent fractions and were required to explain why the fractions were equivalent.

ESSENTIAL QUESTIONS

- How can you compare and order fractions?
- How are benchmark fractions helpful when comparing fractions?
- How do I compare fractions with unlike denominators?

MATERIALS

- Playing cards with all face cards removed

GROUPING

Partner

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed problem “strings” which may be

included in number talks to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: Students should follow the rules discussed on the “Making Fractions” student sheet.

How to Play:

- A player deals four cards to themselves and four to their partner.
- Both players make the largest fraction that they can by choosing a numerator and a denominator from the numbers on their cards.
- Compare your fractions. The one with the larger fraction wins all the cards and becomes the new dealer for the next round.
- If the fractions are equal, the round ends in a tie, and the players keep their cards.
- The game continues in this manner until the players have used all cards.
- Each player counts their cards and the one with more cards wins.

FORMATIVE ASSESSMENT QUESTIONS

- Can you think of another way to explain the comparison?
- What equivalent fractions can you identify for _____?
- How do you know those fractions are equivalent?
- Which fraction is smaller?
- Which fraction is larger?

DIFFERENTIATION

Extension

- Change the rules of the game by having players create the smaller fraction.
- Increase the group size to require students to compare and order more than two fractions.
- Instruct students to only use proper fractions.

Intervention

- Allow students to create their fractions using manipulatives.
- Encourage students to use benchmark fractions to help compare created fractions.
- Encourage students to compare fractions created on number lines.

TECHNOLOGY

- <http://www.prometheanplanet.com/en-us/Resources/Item/75410/using-fraction-bars> This lesson can be used with an ActivSlate or Smartboard. It can be used as a mini-lesson to introduce the task or for remediation purposes.
- <http://learnzillion.com/lessons?utf8=%E2%9C%93&filters%5Bsubject%5D=math&query=&filters%5Bgrade%5D%5B%5D=4&filters%5Bdomain%5D=NF%3A+Number+and+Operations-Fractions&filters%5Bstandard%5D=4.NF.2%3A+Compare+two+fractions+with+differen+...>

Name _____ Date _____

Making Fractions

Directions

How to Play:

- A player deals four cards to themselves and four to their partner.
- Both players make the largest fraction that they can by choosing a numerator and a denominator from the numbers on their cards.
- Compare your fractions. The one with the larger fraction wins all the cards and becomes the new dealer for the next round.
- If the fractions are equal, the round ends in a tie, and the players keep their cards.
- The game continues in this manner until the players have used all cards.
- Each player counts their cards and the one with more cards wins.

Performance Task: Write About Fractions

Adapted from Fosnot, C. The Field Trip, Context for Learning Mathematics.

TASK CONTENT: In this task, students use understanding of equivalent fractions compare fractions with different denominators.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF1 Explain why a fraction a/b is equivalent to a fraction $(nxa)/(nxb)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Many students develop misconceptions about adding fractions, thinking the numerators can be added and denominators can be added. For example, they might think that

$$\frac{1}{2} + \frac{1}{3} \text{ is equivalent to } \frac{2}{5}$$

ESSENTIAL QUESTIONS

- How can you compare fraction quantities without adding fractions?
- How do you know fractions are equivalent?
- What can you do to decide whether your answer is reasonable?

MATERIALS

- Connecting cubes
- Strips of equal length paper
- Write About Fractions task sheet
- Fraction Kits

GROUPING

Individual

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem “strings” which may be included in number talks to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: Students have had some opportunities to work with comparing unit fractions with different denominators in context. By exploring this procedure in context and examining the results, students are supported to develop deep understandings about fractions, thereby avoiding common misconceptions.

Task directions:

Students will follow the directions below from the “Write About Fractions” task sheet.

Write a convincing argument for the following statement:

$$\frac{1}{2} + \frac{1}{3} \text{ does not equal } \frac{2}{5}$$

FORMATIVE ASSESSMENT QUESTIONS

- What do you notice about the unit fractions you have created?
- How can drawing a model help you answer this question?
- What strategies can you use to help write about this topic?
- Can you give me an estimated size of the fraction you created with that cut?
- Does your answer make sense? How do you know?

DIFFERENTIATION

Extension

- In addition to this task, student can identify a fraction that is equal to $\frac{1}{2} + \frac{1}{3}$.

Intervention

- Provide fraction strips the students can manipulate to determine their response.
- Encourage students to use words, numbers and/or pictures in their explanation.

TECHNOLOGY

- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L543> This lesson has students investigate the length model to find equivalent fractions. It can be used for additional practice, remediation or extending understanding of the concept.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L338> This lesson involving finding equivalent fractions using eggs can be used for additional practice, remediation or extending understanding of the concept.

Name _____ Date _____

Write About Fractions

Write a convincing argument for the following statement:

$$\frac{1}{2} + \frac{1}{3} \text{ does not equal } \frac{2}{5}$$

Culminating Task: Pattern Block Puzzles Revisited

TASK CONTENT: For this activity students are asked to cover up shapes that were made with pattern blocks. The shapes are “grayed out” so students will be forced to determine how best to cover them.

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

MCC4.NF.1 Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \times a}{n \times b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

BACKGROUND KNOWLEDGE

Encourage students to organize their thinking to be sure they have found as many ways as possible to represent equivalent fractions. Possible equivalent fractions are shown below. Before asking students to work on this task, be sure students are able to:

- recognize how pattern blocks can represent fractions
- find equivalent fractions for pattern blocks if a hexagon is assigned the value of one whole
- record on the student sheet equivalent fractions either by coloring or gluing die cut color tiles
- write an equation which shows the equivalent fractions
- can identify where a fraction would be found on a number line
- be familiar with the concept that fractions are unique values that can be greater than or less than the value of one whole

ESSENTIAL QUESTIONS

- How can we find equivalent fractions?
- In what ways can we model equivalent fractions?
- How can we find equivalent fractions and simplify fractions?
- How can identifying factors and multiples of denominators help to identify equivalent fractions?
- How can the denominator tell me if a fraction is in the smallest equivalent fraction?
- How do we represent a fraction that is greater than one?
- How do we locate fractions on a number line?

MATERIALS

- Pattern blocks
- “Pattern Block Puzzles” student recording sheet
- Die cut pattern blocks or pencils and crayons for recording findings
- Isometric graph paper

GROUPING

Individual or partner

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem “strings” that could be included in a number talk to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

Students can utilize the friendly number strategies to begin discussing multiples and specifically combining multiples to arrive at a larger product. For example, students could solve the following string of numbers:

$$1 \times 12 = 12$$

$$2 \times 12 = 24$$

by combining the products of 12 and 24 as well as combining the factors of 1 and 2 the students will produce another multiple of 12, namely $3 \times 12 = 36$

$$\begin{array}{r} 1 \times 12 = 12 \\ + 2 \times 12 = 24 \\ \hline 3 \times 12 = 36 \end{array}$$

Doubling and halving is also a helpful strategy in building equivalent fractions. (Number Talks, 2010, Sherry Parrish).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: For this activity students are asked to cover up shapes that were made with pattern blocks. The shapes are “grayed out” so students will be forced to determine how best to cover them. There are several ways to accomplish each puzzle. The first puzzle is a larger hexagon and it is possible to cover this shape with all trapezoids, all rhombi, all triangle or a combination of trapezoid, rhombi, triangles and hexagons. Through discovery students will realize that hexagon alone will not cover up the larger hexagon puzzle. The second puzzle can be covered up by all trapezoids, all triangles or a combination of hexagons, trapezoids, rhombi and triangles. The final puzzle can be covered up with all rhombi or all triangles or a combination of hexagons, trapezoids, rhombi and triangles.

As a mini lesson introduce the yellow hexagon and have students explore all the other ways you can make a hexagon. Through this activity students will discover that a hexagon can be made with a combination of other pattern blocks or it can be made with 2 trapezoids, 3 rhombi or 6 triangles. Through discussion the class can now assign a value to all the pattern block in relation to the hexagon, namely a trapezoid = $\frac{1}{2}$, a rhombus = $\frac{1}{3}$, and a triangle = $\frac{1}{6}$.

In many ways this task is similar to the “Tangram Task” but whereas that task only allowed students to explore the fraction of $\frac{1}{2}$ to explore repeated addition and multiplication of fractions, this activity allows students to continue exploring operations with fractions while also finding equivalent fractions, as well as creating their own puzzles!

Task Directions:

Cover the shape below with pattern blocks any way you’d like.
Now cover the shape using only one color of pattern blocks. Record your information below:
If a hexagon has a value of one whole, what is the value of the pattern block you chose?

How many pattern blocks did you use to cover this shape? _____

Write the area of the shape below as a fraction of a hexagon? _____

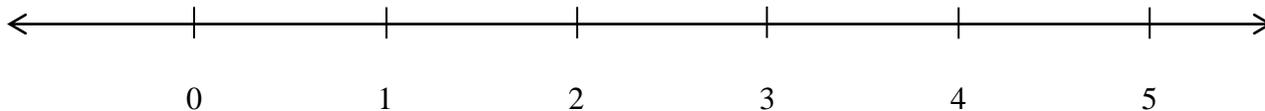
Is there another way you could cover this shape using only one color of pattern blocks?

If a hexagon has a value of one whole, what is the value of the pattern block you chose?

How many pattern blocks did you use to cover this shape? _____

Write the area of the shape below as a fraction of a hexagon? _____

Locate that fraction on the number line below:



FORMATIVE ASSESSMENT QUESTIONS

- How are you keeping your work organized?
- Have you found all the possible ways to cover up each puzzle?
- Is there another way to cover up that puzzle?
- How do you know these two fractions are equivalent?
- How many triangles cover up a hexagon, a rhombus, a trapezoid?
- Do any shapes seem to work with all puzzles?
- Do some shapes not work with certain puzzles? If so, why?
- Where would that fraction be on a number line?
- What do you notice about numerators and denominators in relation to the whole number one?
- How did you find a second solution for each situation?
- How do the numbers in the fraction relate to the different number or pattern blocks you used to cover and create each puzzle?
- Look at the posted solutions. Are there any that you think are not correct? If so, tell why.
- Do you think that there are other solutions beyond those posted? Explain.
- How is it possible that there could be more than one solution for each situation?

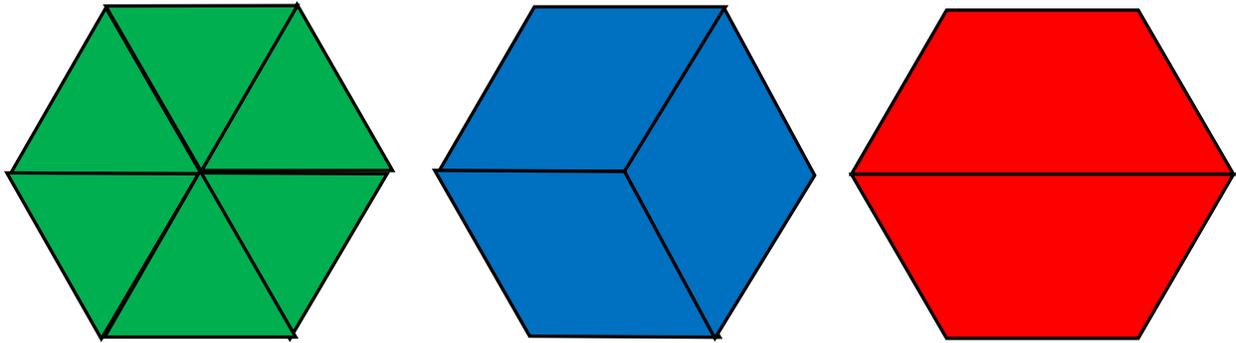
DIFFERENTIATION

Extension

- Once students have completed the task above, this lesson can be extended in several ways
- You can introduce the idea that the value of one can always change, such as 1 dozen doughnuts or eggs can be another way to think of a whole. Students can put two hexagons together and consider this their new whole. This will then change the value of all the shapes, namely a hexagon = $\frac{1}{2}$, a trapezoid = $\frac{1}{4}$, and a rhombus = $\frac{1}{6}$, and a triangle would equal $\frac{1}{12}$. Students could then solve each puzzle again, as the one they created and see if they can find a pattern between their previous solutions and their new ones.
- Students could also exchange the puzzles they created and solve one another's puzzle.
- Students could outline just the perimeter of a puzzle of their creation and challenge classmates to solve it and find both equivalent fractions as well as area.
- Students could also make very large rectangles puzzles and begin to find the patterns that exist between improper fractions and mixed numbers.

Intervention

- Allow students to record equivalent fractions using pre-cut pattern blocks.
- Allow students to utilize isometric graph paper or dot paper to see the relationships that exist between the various pattern blocks. Students may even need to color them and use this as a key.



- Some students may be more successful utilizing isometric geoboards like those found at http://nlvm.usu.edu/en/nav/frames_asid_129_g_2_t_3.html?open=activities&from=category_g_2_t_3.html
- There are several websites that offer virtual pattern blocks such as http://nlvm.usu.edu/en/nav/frames_asid_170_g_2_t_3.html?open=activities&from=category_g_2_t_3.html and <http://illuminations.nctm.org/ActivityDetail.aspx?ID=27>
- Some students will likely struggle to find an improper fraction or mixed number on a number line. Those students could utilize fraction tiles or they could also take advantage of online manipulatives such as http://nlvm.usu.edu/en/nav/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html, which allows students to create their own fraction number lines.

TECHNOLOGY

Please refer to the links under interventions for technology links.

Name _____ Date _____

Pattern Block Puzzles

Cover the shape below with pattern blocks any way you'd like.

Now cover the shape using only one color of pattern blocks. Record your information below:

If a hexagon has a value of one whole, what is the value of the pattern block you chose?

How many pattern blocks did you use to cover this shape? _____

Write the area of the shape below as a fraction of a hexagon? _____

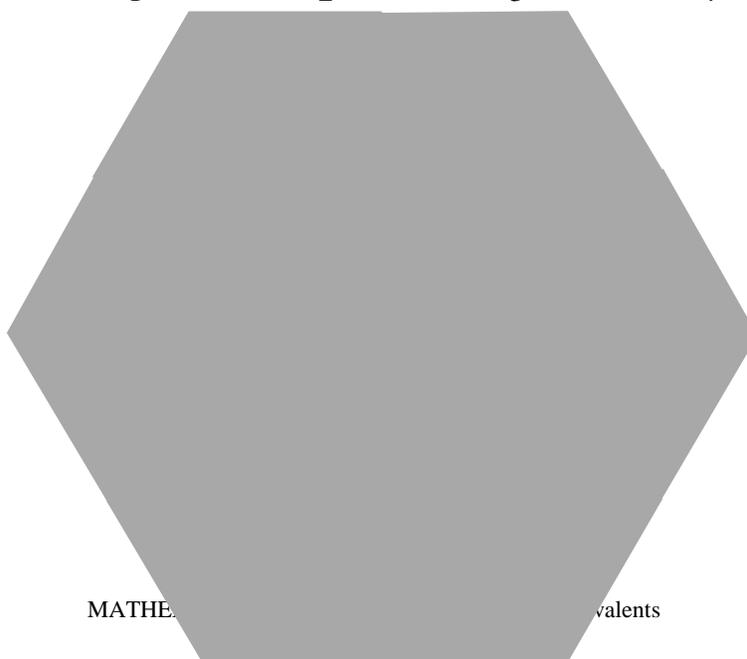
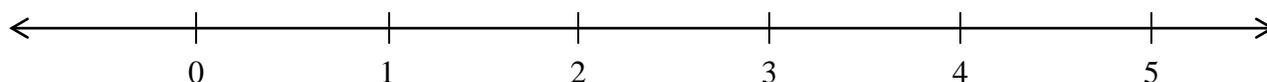
Is there another way you could cover this shape using only one color of pattern blocks?

If a hexagon has a value of one whole, what is the value of the pattern block you chose?

How many pattern blocks did you use to cover this shape? _____

Write the area of the shape below as a fraction of a hexagon? _____

Locate that fraction on the number line below:



MATHE... valents

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Cover the shape below with pattern blocks any way you'd like.

Now cover the shape using only one color of pattern blocks. Record your information below:

If a hexagon has a value of one whole, what is the value of the pattern block you chose?

How many pattern blocks did you use to cover this shape? _____

Write the area of the shape below as a fraction of a hexagon? _____

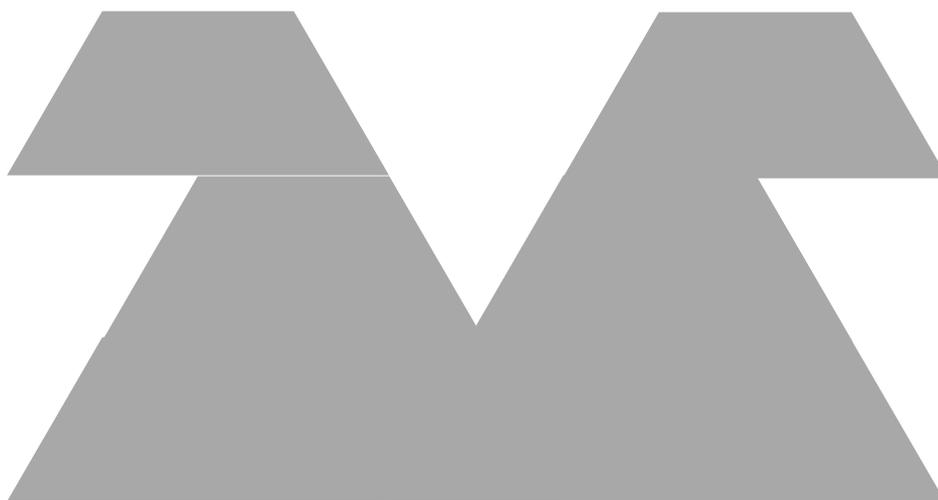
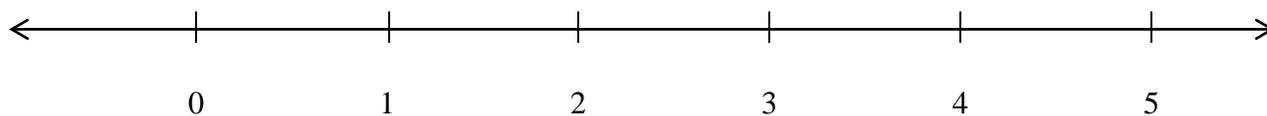
Is there another way you could cover this shape using only one color of pattern blocks?

If a hexagon has a value of one whole, what is the value of the pattern block you chose?

How many pattern blocks did you use to cover this shape? _____

Write the area of the shape below as a fraction of a hexagon? _____

Locate that fraction on the number line below:



Georgia Department of Education
Common Core Georgia Performance Standards Framework
Fourth Grade Mathematics Unit 3

Create your own shape below with pattern blocks.

Now cover the shape using only one color of pattern blocks. Record your information below:

If a hexagon has a value of one whole, what is the value of the pattern block you chose?

How many pattern blocks did you use to cover this shape? _____

Write the area of the shape below as a fraction of a hexagon? _____

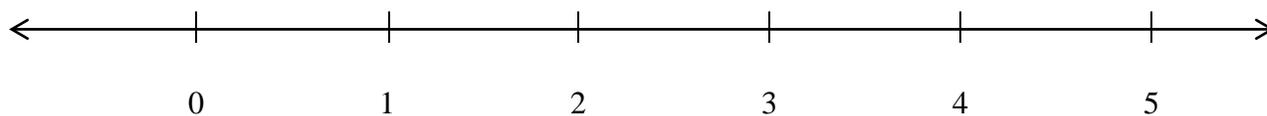
Is there another way you could cover your shape using only one color of pattern blocks?

If a hexagon has a value of one whole, what is the value of the pattern block you chose?

How many pattern blocks did you use to cover this shape? _____

Write the area of the shape below as a fraction of a hexagon? _____

Locate that fraction on the number line below:



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PART 2

Create your own shape below with pattern blocks.

Now cover the shape using only one color of pattern blocks. Record your information below:

If a hexagon has a value of one whole, what is the value of the pattern block you chose?

How many pattern blocks did you use to cover this shape? _____

Write the area of the shape below as a fraction of a hexagon? _____

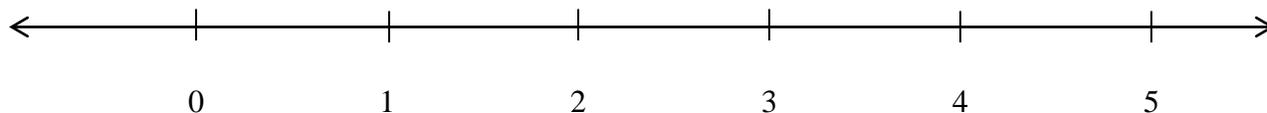
Is there another way you could cover your shape using only one color of pattern blocks?

If a hexagon has a value of one whole, what is the value of the pattern block you chose?

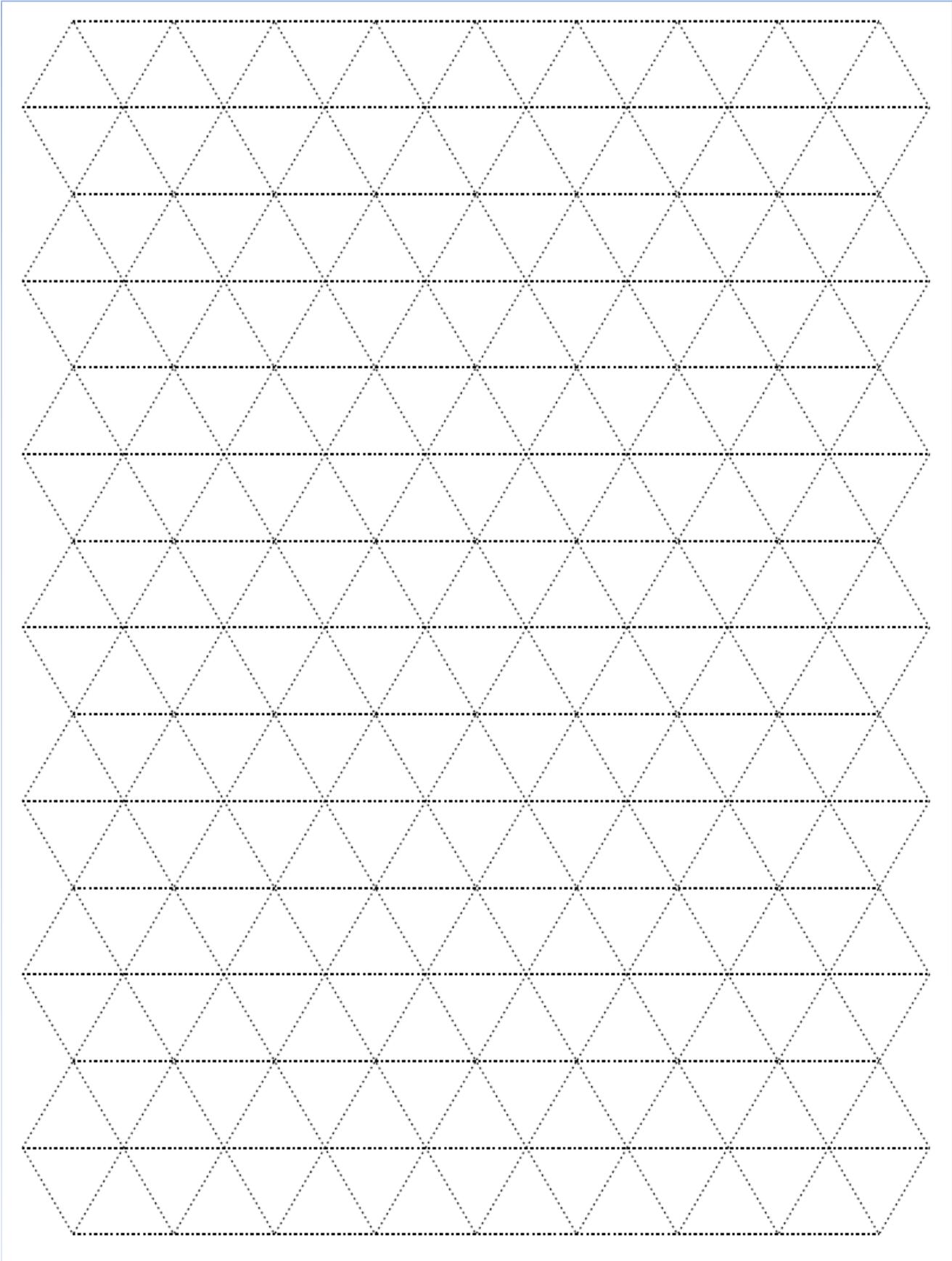
How many pattern blocks did you use to cover this shape? _____

Write the area of the shape below as a fraction of a hexagon? _____

Locate that fraction on the number line below:



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