Title: An Introduction To Solving And Graphing Inequalities

Brief Overview:

This is a three-day unit on solving one-step and two-step inequalities. The lessons include identifying and graphing various inequalities. Students are expected to have previous knowledge of inequality symbols and solving one-step and two-step equations.

NCTM Content Standard/National Science Education Standard:

Number and Number Relationships
- Standard 7.1.B.2.a – Write equations and inequalities to represent relationships

Grade/Level:

Pre-Algebra/Algebra

Duration/Length:

Three 70 – 90 minute lessons

Student Outcomes:

Students will be able to:
- graph inequalities on a number line
- create the appropriate inequality expression from a provided graph
- solve one-step inequalities and graph the solutions
- solve two-step inequalities and graph the solutions
- convert verbal expressions into numeric inequalities and solve

Materials and Resources:

- Circular pieces of paper with a diameter of approximately 2 inches
- Transparencies of number lines to demonstrate and model class exercises
- Student worksheets of practice problems
- Teacher overhead set of number line and large inequality arrows
- Student sets of inequality arrows
- Activity dice
- 3 x 5 cards
- Crossword puzzles
- Worksheets and transparencies
    - Master Copy of Inequality Arrows
    - Model Problems – Graphing Inequalities Transparency Resource
    - Graphing Inequalties

      o Model Problems – Writing Inequalities Transparency Resource
      o Writing Inequalities
      o Graphing and Writing Inequalities Exit Ticket
      o Student Application 1a – Graphing Inequalities
      o Student Application 1b – Crossword Puzzle
      o Solving Inequalities Exit Ticket
      o I Have, Who Has cards
      o Flash cards with inequality terms
      o Solving Inequalities with Multiple Operations
      o Translating Verbal Inequalities

Development/Procedures:

Lesson 1

Preassessment – In their notebooks, students will draw a number line from -10 to $+10$, including ½ increments. Assess student recall of integer location on the number line by having them place a point at several integers, such as $-5, 7.5, 3, -1.5$. Display the inequality symbols ($<, >, \leq, \geq$) and have the students define what each means.

Launch – Using circular pieces of construction paper, have students write their favorite number on the paper. Post a large number line written on register tape on the front chalk board. Instruct the students to come to the number line on the chalkboard and place their favorite number where it belongs. Lead a class discussion using the students' data to review concepts of exact points, greater than and less than, ($>,<$) and greater/less than or equal to ($\leq, \geq$). Ask questions such as, "How many students have favorite numbers greater than -2?"

Teacher Facilitation – Use a transparency of "Model Problems – Graphing Inequalities Transparency Resource", and distribute a copy to the students. Demonstrate how to graph the four types of inequality expressions ($<, >, \leq, \geq$). Emphasize that having the variable on the left side of the inequality shows the direction of the arrow and shading on the number line to be the same direction of the symbol in the inequality. Explain and illustrate the use of open and closed circles in their solution depending on the type of inequality.

Student Application – Students will be given two arrows and a number line using "Master Copy of Inequality Arrows". One arrow has an attached closed circle on one end; the other has an attached open circle on its end. Provide several examples for the students to graph using their materials. Some volunteers will be asked to come

to the front of the class to do the same examples on the board number line.

Students will then complete the worksheet "Graphing Inequalities" independently. Go over the problems with the students afterwards.

Teacher Facilitation – Teacher will use overhead "Model Problems – Writing Inequalities Transparency Resource" to model how to write the related inequality expression for each graph. All four types of inequality symbols will be included in the examples shown.

Student Application – Students will complete "Writing Inequalities" with graphed inequalities provided. Volunteers will be asked to do some in front of the class while others work with help from teacher or classmates. Students will then complete the worksheet independently. Invite students to go over the problems as a class afterwards.

Embedded Assessment – Distribute "Graphing and Writing Inequalities Exit Ticket", which includes three problems for students to complete before leaving class.

Reteaching/Extension –
  ▪ For those who have not completely understood the lesson, review what is needed.
  ▪ For those who have understood the lesson, take them to the next step in development of the concept.

Lesson 2
Preassessment – Using un-graded exit tickets from previous day, students work in small groups to compare answers, share ideas on how they solved exercises, and identify common errors. Then review homework

Launch – Present the scenario, "You are going to a carnival that charges $3 for each ride." Ask students to calculate how many times they can go on their favorite rides if they have $40 to spend. Verbally discuss student results and have students share their strategies to arrive at their solutions.

Teacher Facilitation I – Lead a review of solving one-step equations. After a few examples, none involving multiplication or division by a negative value, substitute inequality symbols in those equations to demonstrate the similarity of solving an equation and

an inequality. Discuss how the inequality changes the meaning of the solution. After finding and graphing the solutions, identify a few representative values to use in the original inequalities to check and demonstrate the accuracy of the solution. Choose values that lie within and without of the given range for the variable to emphasize the solution set.

Student Application Ia - Assign a worksheet with several one-step inequalities to solve none involving multiplication or division by a negative value. The first few will be done together, checking for understanding. Require students to choose three values to check the accuracy of the solution set. Then, students will complete the rest of the worksheet independently.

Student Application Ib - Inequality Crossword

Teacher Facilitation II – Provide a problem on the board or overhead that involves a one-step inequality necessitating solution using multiplication or division by a negative value, such as $-6x < 18$. Ask students to solve for x and graph the solution. Use the solution to explain why there is the need to reverse the direction.*)*

Student Application II – Build An Inequality

Students are divided into groups of 4. Each group is provided with four dice, an inequality card, and 3 x 5 cards. One of the dice has a math operation on each side; two have a positive or negative number on each side, and the fourth has a letter of the alphabet on each side (available online). In their groups, students roll the four dice, choose an inequality card, and then build their inequality with the results. For example, if the dice ended up with the numbers +8 and -4, the letter 'k', and the math operation 'addition,' the students can build the inequality 8 + k = -4, or -4 + k = 8. Students then write the problem on a two blank 3 x 5 cards. They solve the inequality on one of the cards which they keep, and they leave the second card 'unsolved.' Groups then exchange unsolved cards with other groups and each group solves the new card they received.

Embedded Assessment – Assign "Solving Inequalities Exit Ticket" which presents three problems for students to complete before leaving class.

Reteaching/Extension –
- For those who have not completely understood the lesson, review what is needed. Provide students who are struggling more practice problems (see worksheet).

- For those who have understood the lesson, have them form small groups to play a concept reinforcing game, such *as I Have, Who Has?,* using one-step inequalities and verbal expressions.

Lesson 3

Pre-assessment – Students will complete 4 problems using **all** operations of 1-step inequalities. Negative coefficients will be included.

Launch – Divide the class into pairs. Give each pair a bag of inequality flashcards, created from the master found in the resource pages. These flashcards review the inequality terms. Students will place the cards into different categories according to what they feel is appropriate, providing a perfect opportunity to have a class discussion to review the terms. Discuss as a class the different categorization the students made and their reasoning behind it. The vocabulary words from this activity are included in this lesson.

Teacher Facilitation – Review two-step equations with the following examples: $5p + 3 = 18$, $3n - 4 = 14$, $4m + 9 = -3$, $\frac{b}{2} + 3 = 9$, and $\frac{x}{4} - 2 = 3$. Write these examples on sentence strips, and post them on the front board. Invite students to the board to solve the two-step equations. If necessary, use the following examples as extra practice: $5c + 7 = 28$, $2x + 5 = 19$, $\frac{z}{2} - 5 = 3$, $3y - 8 = 16$, $5d + 2 = -33$. Cover the equality symbol on each example with an inequality symbol written on a small piece of sentence strip. Use this to emphasize that when an inequality sign is substituted in for the equal sign, the problem is still solved the same way. Explain how the inequality changes the meaning of the answer. Provide additional two-step inequalities for students to solve in which the teacher and class to work on together. Students will come to the overhead to place their answers.

Student Application – Assign the worksheet, "Solving Inequalities with Multiple Operations" as an independent assessment on two-step inequality problems.

Teacher Facilitation II – Teacher will give examples of inequalities written as verbal expressions for the class to solve. Teacher will call on students to answer each example and write their answers on the overhead.

Examples:
1) Twice the number *r* increased by 15 is no more than –17.
2) 8 subtracted from 3 times a number *c* is less than 31.
3) The product of 20 and a number *x* is fewer than 53.
4) The quotient of 100 and 5 is greater than 18.

Student Application – Assign the worksheet, "Translating Verbal Inequalities." Students will solve a verbal expression worksheet. Go over the answers with the class once they complete their assignment. Teacher may use a transparency or have kids participate by going to the chalkboard to write their answers.*)*

Embedded Assessment – Hand every student a 3 x 5 colored index card. The teacher will read problems aloud, giving students time to answer. Instruct students that they are to define the variable, write an inequality and solve each problem on the index card. Collect the cards as they walk out the door. This is a quick and easy way to assess the students and provide feedback in a timely fashion.

Problems to read:
1) The sum of a number and negative six is greater than 9.
2) Negative five times a number is less than 20
3) The cost of a $20 flashlight and a package of batteries is less than $39.

Reteaching/Extension –
▪ For those who have not completely understood the lesson, review what is needed. Provide students who are struggling more practice problems. Make sure the supplementary worksheets are created for different learning levels.
▪ For those who have understood the lesson, have them form small groups to play *Pass The Problem*
Pass The Problem: Divide students into groups of 4 (or 5); each person in the group receives a different problem to start. Step 1: Do the first step of the problem on your paper. Pass your problem to the person on your right.

Step 2: Do the 2nd step of the new problem you've just received. Pass this paper again to the right.

Step 3: Do the 3rd step of the problem on the new paper you've just received. Again pass the paper to the right.

Step 4: Complete the solution.

Step 5: Graph the inequality solution on a line (optional).

Example: $3x - 8 \leq 16$
Step 1:   +8   +8
Step 2: $3x \leq 24$
Step 3   ÷ 3   ÷ 3
Step 4: $x \leq 8$
Step 5: Graph the solution (optional)

Summative Assessment:

Using your own local or state standards and testing formats, create a variety of questions that include student selected response (multiple choice/SRs), student produced responses (grid-ins/SPRs), and student constructed responses (BCRs/ECRs).

Authors:

Lindsay Robinson   Mark Schilling
Meade Middle School   Stevensville Middle School
Anne Arundel County, MD   Queen Anne's County, MD

Teacher Transparency Resource
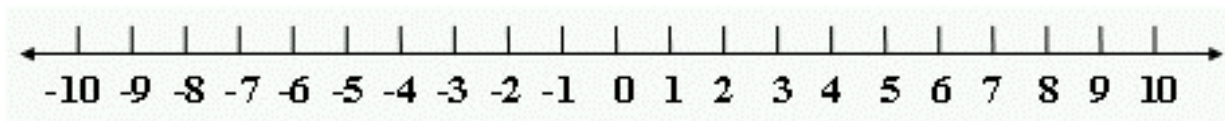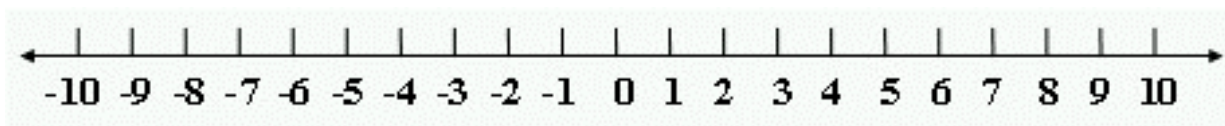
Master Copy of Inequality Arrows
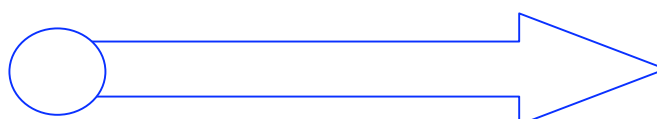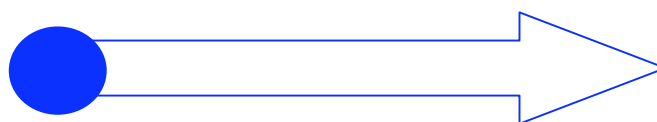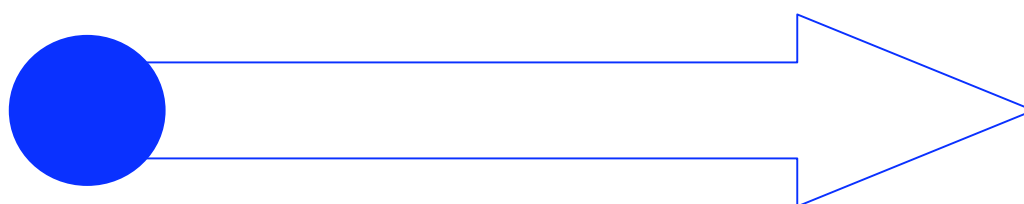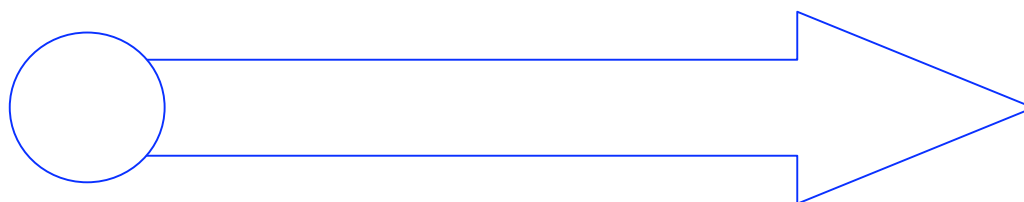
$x \leq 0$
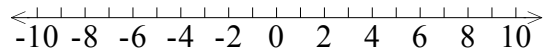
-10 -8 -6 -4 -2 0 2 4 6 8 10

$x > -1$

-10 -8 -6 -4 -2 0 2 4 6 8 10

$x \geq -5$

-10 -8 -6 -4 -2 0 2 4 6 8 10

$x < 5$

-10 -8 -6 -4 -2 0 2 4 6 8 10

Graphing Inequalities

Name: _____
Date: _____

Directions: Graph the following inequalities.

1. $f > 7$

⟵ | | | | | | | | | | | ⟶
-10 -8 -6 -4 -2 0 2 4 6 8 10

2. $m < 4$

⟵ | | | | | | | | | | | ⟶
-10 -8 -6 -4 -2 0 2 4 6 8 10

3. $a \geq -1$

⟵ | | | | | | | | | | | ⟶
-10 -8 -6 -4 -2 0 2 4 6 8 10

4. $z \leq 5$

⟵ | | | | | | | | | | | ⟶
-10 -8 -6 -4 -2 0 2 4 6 8 10

5. $6 \geq p$

⟵ | | | | | | | | | | | ⟶
-10 -8 -6 -4 -2 0 2 4 6 8 10

6. $c > -4$

⟵ | | | | | | | | | | | ⟶
-10 -8 -6 -4 -2 0 2 4 6 8 10

7. $k < -2$

⟵ | | | | | | | | | | | ⟶
-10 -8 -6 -4 -2 0 2 4 6 8 10

8. $x \geq 0$

⟵ | | | | | | | | | | | ⟶
-10 -8 -6 -4 -2 0 2 4 6 8 10

9. $y < 9$

⟵ | | | | | | | | | | | ⟶
-10 -8 -6 -4 -2 0 2 4 6 8 10

10. $-7 \leq q$

⟵ | | | | | | | | | | | ⟶
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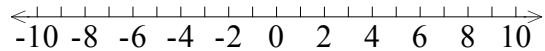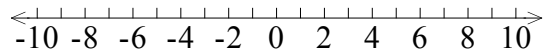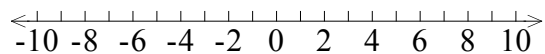
Graphing Inequalities

Name: __ANSWER KEY___
Date: _____

Directions: Graph the following inequalities.

1. $f > 7$
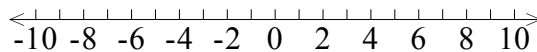
2. $m < 4$
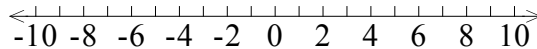
3. $a \geq -1$
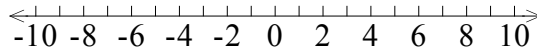
4. $z \leq 5$
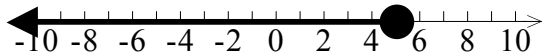
5. $6 \geq p$
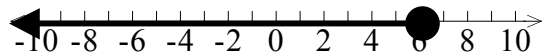
6. $c > -4$
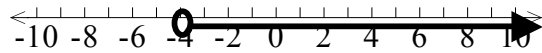
7. $k < -2$
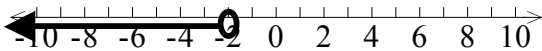
8. $x \geq 0$
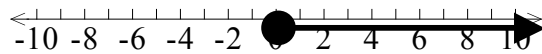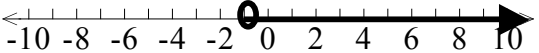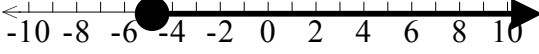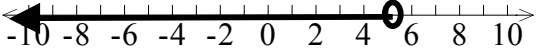
9. $y < 9$
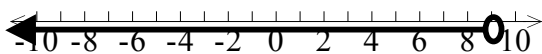
10. $-7 \leq q$
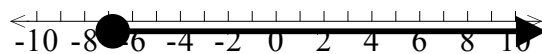
_____	A number line from -10 to 10 with a closed circle at 0 and an arrow pointing left.
_____	A number line from -10 to 10 with an open circle at -1 and an arrow pointing right.
_____	A number line from -10 to 10 with a closed circle at -5 and an arrow pointing right.
_____	A number line from -10 to 10 with an open circle at 5 and an arrow pointing left.

Writing Inequalities

Name_____

Date_____

Directions: Write the inequalities which represent the given graph.

1. _____
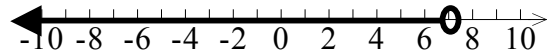
2. _____
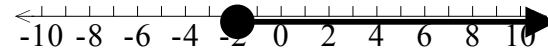
3. _____
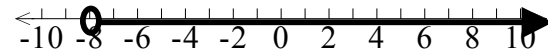
4. _____
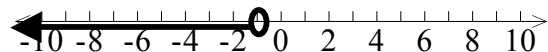
5. _____
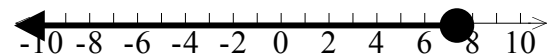
6. _____
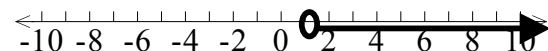
7. _____
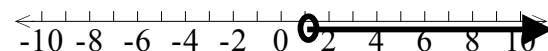
8. _____
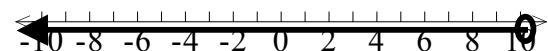
9. _____
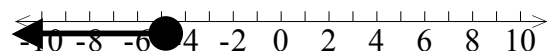
10. _____
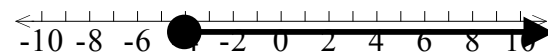
Writing Inequalities

Name____ANSWER KEY_____
Date_____

Directions: Write the inequalities which represent the given graph.

1. $x < 7$
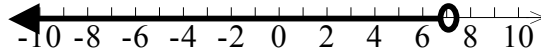
2. $x \geq -2$
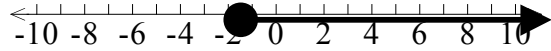
3. $x > -8$

4. $x < -1$
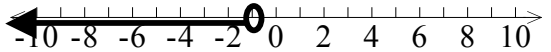
5. $x \leq 7$
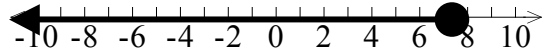
6. $x > 1$

7. $x > -1$

8. $x < 10$

9. $x \leq -5$
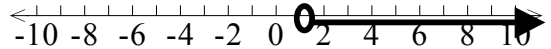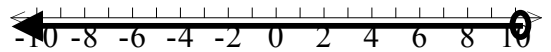
10. $x \geq -4$

14

Graphing and Writing Inequalities
Exit Ticket

Name: _____
Date: _____

Graph the inequality.

    1. $k \geq 2$

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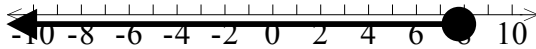
    2. $4 > s$

<-10 -8 -6 -4 -2 0 2 4 6 8 10->

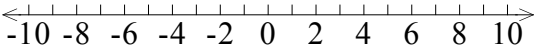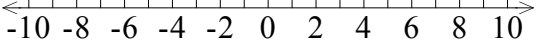Write the inequality represented by the graph given.

    3. _____

<-10 -8 -6 -4 -2 0 2 4 6 8 10->

Graphing and Writing Inequalities
Exit Ticket

Graph the inequality.

1. $k \geq 2$

2. $4 > s$

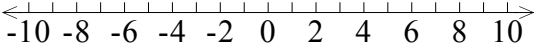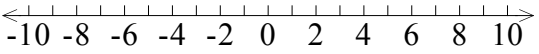Write the inequality represented by the graph given.

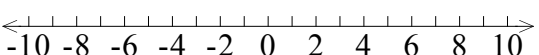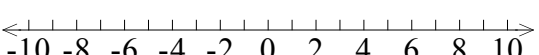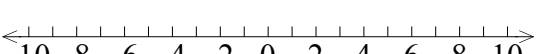3. __$x \leq 8$_____
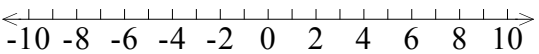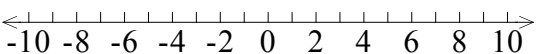
Student Application 1a: Graphing Inequalities   Name: _____

Date: _____

Directions: Solve and graph the following inequalities.

1. y -3 ≤ -10	<-10 -8 -6 -4 -2 0 2 4 6 8 10->
2. n + 4 ≥ 9	<-10 -8 -6 -4 -2 0 2 4 6 8 10->
3. 8d < 24	<-10 -8 -6 -4 -2 0 2 4 6 8 10->
4. 17 + r > 20	<-10 -8 -6 -4 -2 0 2 4 6 8 10->
5. c – 12 ≥ -13	<-10 -8 -6 -4 -2 0 2 4 6 8 10->
6. 5a < 20	<-10 -8 -6 -4 -2 0 2 4 6 8 10->
7. -63 + w ≥ -70	<-10 -8 -6 -4 -2 0 2 4 6 8 10->
8. $\frac{1}{3}q < 2$	<-10 -8 -6 -4 -2 0 2 4 6 8 10->
9. 11a < 44	<-10 -8 -6 -4 -2 0 2 4 6 8 10->
10. 14 + r > 20	<-10 -8 -6 -4 -2 0 2 4 6 8 10->

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Student Application 1a: Graphing Inequalities

Name: __ANSWER KEY___
Date: _____
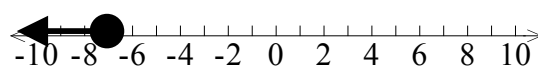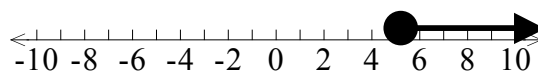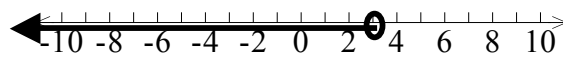
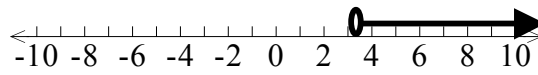Directions: Solve and graph the following inequalities.
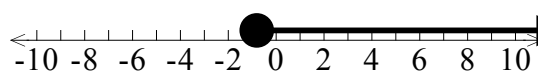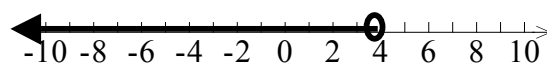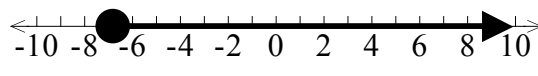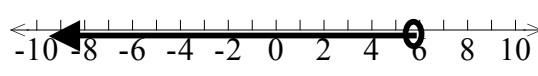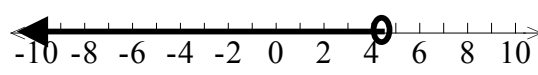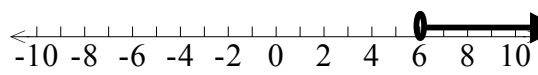
1. y -3 ≤ -10	
2. n + 4 ≥ 9	
3. 8d < 24	
4. 17 + r > 20	
5. c – 12 ≥ -13	
6. 5a < 20	
7. -63 + w ≥ -70	
8. $\frac{1}{3}q < 2$	
9. 11a < 44	
10. 14 + r > 20	

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Inequalities

Complete the crossword puzzle.
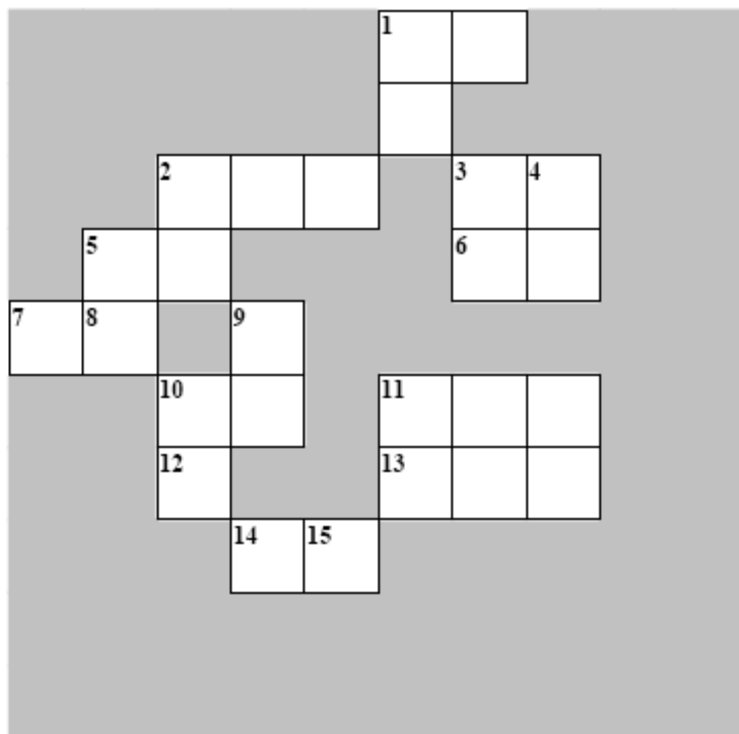
Across

1   100 - x < 41
2   x + 9 < 120
3   x + 4 < 20
6   x - 4 > 20
7   2/5x <18
8   2x > 10
10   x - 5 < 5
11   x -45 > 189
12   x + 8 < 15
13   x + 1 < 210
14   3x> 99

Down

1   2x < 100
2   1/2 m <7
3   2x < 24
4   x/8 > 8
5   33 + x < 88
9   5x > 100
10   20 < x + 3
11   x + 6 < 28
15   8d < 24

Inequalities

Complete the crossword puzzle.

					1 5	9
					0	
	2 1	1	1		3 1	4 6
	5 5	4			6 2	4
7 4	8 5		9 2			
	10 1	0		11 2	3	4
	12 7			13 2	0	9
		14 3	15 3			

20
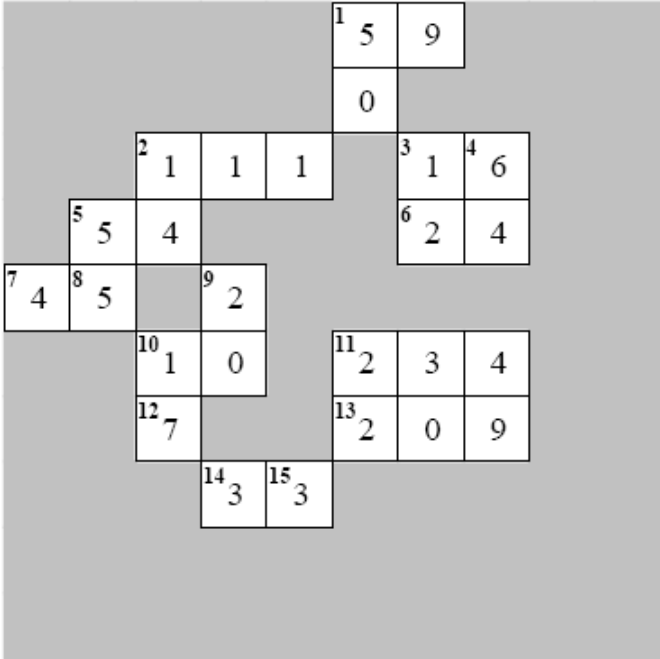
Solving Inequalities   Name: _____
Exit Ticket   Date: _____

Solve.

    1. $b - 8 \geq -4$ 2. $17 + t < 14$

Solve and graph the solution.

    3. $-3v + 7 > 22$

-10 -8 -6 -4 -2 0 2 4 6 8 10

Solving Inequalities
Exit Ticket

Name: __ANSWER KEY___
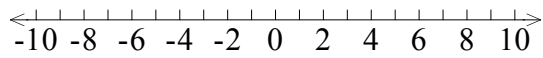Date: _____

Solve.

1. $b - 8 \geq -4$
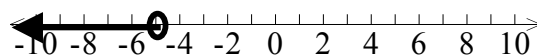
    Answer: $b \geq 4$

2. $17 + t < 14$

    Answer: $t < -3$

Solve and graph the solution.

3. $-3v + 7 > 22$ Answer: $v < -5$

I Have, Who Has Cards

I have: **x > -7** Who Has? **x + 7 < 13**	I have: **x < -1** Who Has? **12 X > -24**
I have: **x < 6** Who Has? **x + 2 > 4**	I have: **x ≥ 10** Who Has? **x – 3 < -4**
I have: **x > 2** Who Has? **6x ≤ -36**	I have: **x < 12** Who Has? **2x ≥ 20**
I have: **x ≤ -6** Who Has? **x – 4 > -3**	I have: **x > 1** Who Has? **x – 5 < 7**

I have: **x > -2** Who Has? **x + 5 < 0**	I have: **x < 0** Who Has? **7 X > -56**
I have: **x < -5** Who Has? **x + 1 > 4**	I have: **x ≥ 4** Who Has? **x – 4 < -4**
I have: **x > 3** Who Has? **12x ≤ -36**	I have: **x < 15** Who Has? **5x ≥ 20**
I have: **x ≤ -3** Who Has? **x – 9 > -2**	I have: **x > 7** Who Has? **x – 5 < 10**

I have: **x > -8** Who Has? **x + 11 < 0**	I have: **x < 14** Who Has? **8 X > -56**
I have: **x < -11** Who Has? **x + 15 > 4**	I have: **x ≥ 8** Who Has? **x – 15 < -1**
I have: **x > -11** Who Has? **9x ≤ -81**	I have: **x < 30** Who Has? **5x ≥ 40**
I have: **x ≤ -9** Who Has? **x – 3 > 13**	I have: **x > 16** Who Has? **x – 20 < 10**

Flash Cards
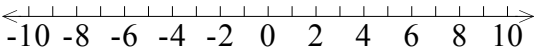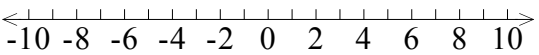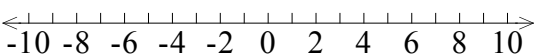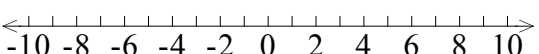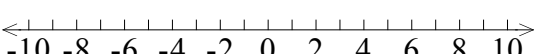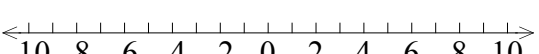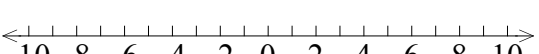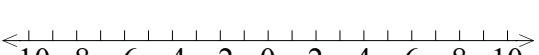
less than	fewer than	greater than	more than	at most
no more than	less than or equal to	at least	no less than	greater than or equal to
exceeds	up to			

Lesson 3: Graphing Inequalities

Name: _____

Date: _____

Directions: Solve and graph the following inequalities.

1. -4 a < 40	
2. 4 a < -40	
3. -4 a < -40	
4. -3a > 15	
5. 3a > -15	
6. -3a > -15	
7. $-\frac{1}{2}p \geq 4$	
8. $\frac{1}{2}p \geq -4$	
9. $-\frac{1}{2}p \geq -4$	

Lesson 3: Graphing Inequalities

Name: __ANSWER KEY___
Date: _____

Directions: Solve and graph the following inequalities.

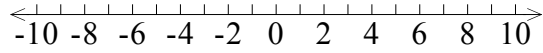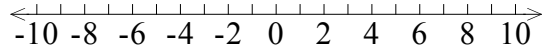|	
1. -4 a < 40	A number line from -10 to 10. Open circle at -10, shaded to the right with arrow.
2. 4 a < -40	A number line from -10 to 10. Open circle at -10, shaded to the left with arrow.
3. -4 a < -40	A number line from -10 to 10. Open circle at 0, shaded to the right with arrow.
4. -3a > 15	A number line from -10 to 10. Open circle at -5, shaded to the left with arrow.
5. 3a > -15	A number line from -10 to 10. Open circle at -5, shaded to the right with arrow.
6. -3a > -15	A number line from -10 to 10. Open circle at 5, shaded to the left with arrow.
7. $-\dfrac{1}{2}p \geq 4$	A number line from -10 to 10. Closed circle at -8, shaded to the left with arrow.
8. $\dfrac{1}{2}p \geq -4$	A number line from -10 to 10. Closed circle at 8, shaded to the right with arrow.
9. $-\dfrac{1}{2}p \geq -4$	A number line from -10 to 10. Closed circle at 8, shaded to the left with arrow.

Solving Inequalities with Multiple Operations   Name: _____

Date: _____

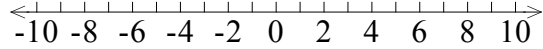Directions: Solve each two-step inequality and graph the solution.
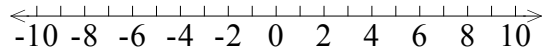
1. $6a - 3 > 21$

$\xleftarrow{\hspace{1cm}}$ -10 -8 -6 -4 -2 0 2 4 6 8 10 $\xrightarrow{\hspace{1cm}}$

2. $5 > 4m - 7$

$\xleftarrow{\hspace{1cm}}$ -10 -8 -6 -4 -2 0 2 4 6 8 10 $\xrightarrow{\hspace{1cm}}$

3. $-2x - 10 \geq 4$

$\xleftarrow{\hspace{1cm}}$ -10 -8 -6 -4 -2 0 2 4 6 8 10 $\xrightarrow{\hspace{1cm}}$

4. $-15 > -3x - 45$

$\xleftarrow{\hspace{1cm}}$ -10 -8 -6 -4 -2 0 2 4 6 8 10 $\xrightarrow{\hspace{1cm}}$

5. $-\dfrac{t}{3} + 1 \leq 3$

$\xleftarrow{\hspace{1cm}}$ -10 -8 -6 -4 -2 0 2 4 6 8 10 $\xrightarrow{\hspace{1cm}}$

6. $-9n + 1 > 91$

$\xleftarrow{\hspace{1cm}}$ -10 -8 -6 -4 -2 0 2 4 6 8 10 $\xrightarrow{\hspace{1cm}}$

7. $9 \leq 6y - 15$

$\xleftarrow{\hspace{1cm}}$ -10 -8 -6 -4 -2 0 2 4 6 8 10 $\xrightarrow{\hspace{1cm}}$

8. $2 \geq 2x - 8$

$\xleftarrow{\hspace{1cm}}$ -10 -8 -6 -4 -2 0 2 4 6 8 10 $\xrightarrow{\hspace{1cm}}$

9. $6e + 1 < 4$

$\xleftarrow{\hspace{1cm}}$ -10 -8 -6 -4 -2 0 2 4 6 8 10 $\xrightarrow{\hspace{1cm}}$

10. $-14 + 2x > -8$

$\xleftarrow{\hspace{1cm}}$ -10 -8 -6 -4 -2 0 2 4 6 8 10 $\xrightarrow{\hspace{1cm}}$
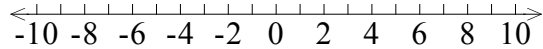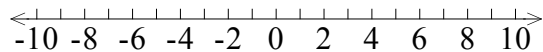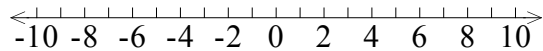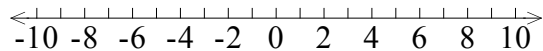
Solving Inequalities with Multiple Operations   Name: _ANSWER KEY____
                                                  Date: _____

Directions: Solve each two-step inequality and graph the solution.

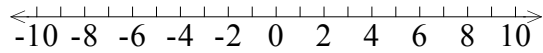1. $6a - 3 > 21$
    Answer: $a > 4$

2. $5 > 4m - 7$
    Answer: $m < 3$

3. $-2x - 10 \geq 4$
    Answer: $x \leq -7$

4. $-15 > -3x - 45$
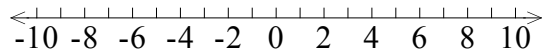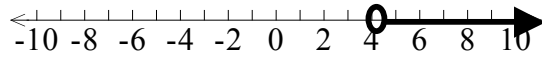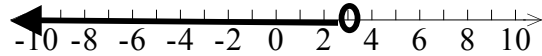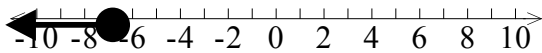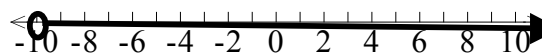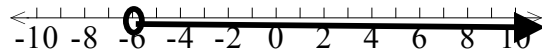   Answer: $x \geq -10$
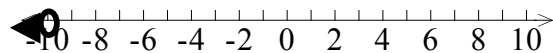
5. $-\dfrac{t}{3} + 1 \leq 3$
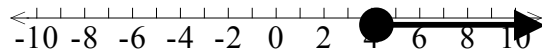   Answer: $t \geq -6$

6. $-9n + 1 > 91$
    Answer: $n < -10$

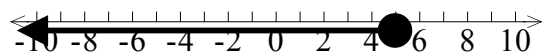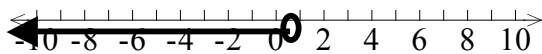7. $9 \leq 6y - 15$
    Answer: $y \geq 4$

8. $2 \geq 2x - 8$
    Answer: $x \leq 5$

9. $6e + 1 < 4$
    Answer: $e < 0.5$

10. $-14 + 2x > -8$
    Answer: $x > 3$

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Translating Verbal Inequalities   Name _____
                                         Date_____

Write each verbal expression as an inequality. Then solve the equation.

1. A number decreased by 7 is at most 13.

2. A number plus 6 is greater than 1.

3. A sum of a number and 14 is at least 28.

4. A number decreased by 5 is less than 32.

5. Twenty is no greater than the sum of a number and –4.

6. Twice a number is more than the sum of that number and 8.

7. Which statement is modeled by $y + 6 \geq 5$?
    A. The sum of a number and six is at least five.
    B. The sum of a number and six is greater than five.
    C. The sum of a number and six is at most five.
    D. The sum of a number and six is no greater than 5.

Name __ANSWER KEY_____

Date_____
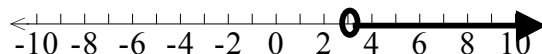
Write each verbal expression as an inequality. Then solve the equation.

1. A number decreased by 7 is at most 13.

    $x - 7 \leq 13$

    $\quad x \leq 20$

2. A number plus 6 is greater than 1.

    $x + 6 > 1$

    $\quad x > -5$

3. A sum of a number and 14 is at least 28.

    $x + 14 \geq 28$

    $\quad x \geq 14$

4. A number decreased by 5 is less than 32.

    $x - 5 < 32$

    $\quad x < 37$

5. Twenty is no greater than the sum of a number and –4.

    $20 < (x + -4)$

    $24 < x$

    $\quad x > 24$

6. Twice a number is more than the sum of that number and 8.

    $2x > x + 8$

    $\quad x > 8$

7. Which statement is modeled by $y + 6 \geq 5$?
    | A. The sum of a number and six is at least five. |
    B. The sum of a number and six is greater than five.
    C. The sum of a number and six is at most five.
    D. The sum of a number and six is no greater than 5.