3rd Grade Unit 1: Extending Addition and Subtraction Understanding

Teacher Notes

This activity is adapted from a NCTM classroom activity found in *Teaching Children Mathematics*. The full article is attached. Reading through the article will help facilitate tasks 4 & 5.

The school district wants to have a Family Math Night in the fall. It would be something like Family Reading Night. Families would come to school to learn about the great things going on in math classes. Families could experience the fun of math together.

The school leaders want to call the night "Experience the Magic of Mathematics." Each grade is in charge of designing family activities. The activities should match the theme of the night and relate to their grade level. Some third grade teachers say that third graders could show Sudoku puzzles since they are working with addition and subtraction in class.

Task 1: Look below to learn how to complete various levels of Sudoku puzzles. Which math practices are you using when learning and working on Sudoku puzzles?

Fill the grid with the numbers1 to 4 in such a way that each number appears only once in each row, column and region (a 2 by 2 block).

2	3	1	
3		2	

		2	
	4		
	3	1	

2			4
			3
1			

	1		
	2	4	
		3	

You can find more printable Sudoko puzzles on this website if you want to continue the fun.
http://www.mathinenglish.com/Sudoku.php

Adapted from *Teaching Children Mathematics* Vol. 22. No. 7 pg. 405 "Reverse and Add to 100 Explorations in Place Value: A collaborative number quest challenges third graders to strengthen their understanding of patterns, multi-digit addition, and number operations.

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Fill the grid with the numbers1 to 4 in such a way that each number appears only once in each row, column and region (a 2 by 2 block).

1	2	4	3
2	3	1	4
4	1	3	2
3	4	2	1

3	1	2	4
2	4	3	1
4	3	1	2
1	2	4	3

3	2	4	1
2	3	1	4
4	1	2	3
1	4	3	2

3	1	2	4
1	2	4	3
4	3	1	2
2	4	3	1

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Task 2: Some third graders say the Sudoku puzzles are not too magical. In response, some teachers suggest that the third grade activity be Magic Squares. Look below to learn how to complete various magic squares. Which math practices are you using when learning and working on Magic Squares?

In a Magic Square, the numbers in each row, column, and diagonal have to same sum – the magic sum. Complete this original form of Magic Square with a magic sum of 15 using whole numbers from 1 to 9.

Here are some more Magic Squares to try. (You may need to use multi-digit numbers to complete these.)

Magic Number: 18

9		
		8
5		3

Magic Number: 24

11		7
	8	
	10	

Magic Number: 45

	21	
3		27
		12

You can find more printable magic squares on this website if you want to continue the fun.
http://www.timvandevall.com/math/magic-squares-worksheets-3x3-4x4-puzzles/

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2	9	4
7	5	3
6	1	8

Here are some more Magic Squares to try. (You may need to use multi-digit numbers to complete these.)

Magic Number: 18

9	2	7
4	6	8
5	10	3

Magic Number: 24

11	6	7
4	8	12
9	10	5

Magic Number: 45

18	21	6
3	15	27
24	9	12

You can find more printable magic squares on this website if you want to continue the fun.
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Task 3: The Magic Squares are cool, but some third graders say that in second grade they learned all about adding single digits and adding with more than two addends. The third graders wonder if the Family Math Night activity should be a little more challenging. Since in math class third graders are working with multi-digit addition and subtraction, one third grader suggests, "We could have people Reverse and Add. Here is how you do it. Write a two-digit number. Reverse the ones and tens digits to create a second two-digit number, and find the sum of the two numbers."

Experiment with the Reverse and Add algorithm or procedure. See if you can uncover any patterns. What do you notice? Which math practices are you using when working to find a Reverse and Add sum?

- Record your wonderings
- Record what you notice as you experiment with this procedure
- List the mathematical practices you have been using in this task

Teacher talk: What patterns do you see? Facilitate and collect conjectures. Have students discuss noticings…encourage students to find counter examples to their conjectures.

Conjectures and patterns

In the paragraphs that follow, we explore patterns that students (and teachers) uncovered as they worked on the Reverse and Add to 100 task. Furthermore, we share ways in which the task encouraged the third graders to think deeply about place value.

The Near 50 conjecture

Working in small groups, many students selected 50 as their initial addend. This was a reasonable first step, because 50 is half of the desired sum, 100. However, the third graders quickly discovered that 50 yields a surprisingly small final sum (see **fig. 1**). At this point, students persevered in problem solving (SMP 1), experimenting with other two-digit starting values (see **fig. 2**). As students generated calculations, such as those illustrated in **figure 2**, we asked them to comment on patterns they observed. Many students noted, "All the numbers in the answer are the same" and "All digits in the sum are the same." We refer to this observation as the All Digits Are the Same conjecture.

FIGURE 1

This was a popular first attempt when students explored Reverse and Add to 100.

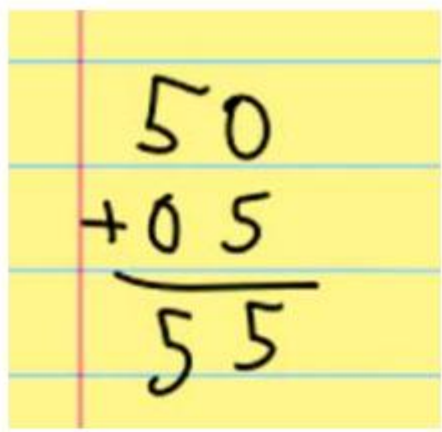
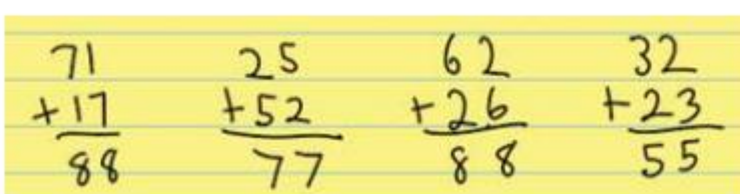


FIGURE 2

When students discovered the small final sum that 50 yields, they experimented with other two-digit starting values. A pattern soon emerged.



Adapted from *Teaching Children Mathematics* Vol. 22. No. 7 pg. 405 "Reverse and Add to 100 Explorations in Place Value: A collaborative number quest challenges third graders to strengthen their understanding of patterns, multi-digit addition, and number operations.

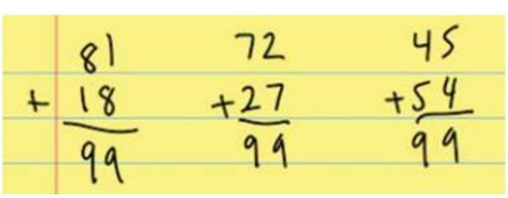
The All Digits Are the Same conjecture

Students of all ages are quick to draw mathematical conclusions from only a handful of examples (Healy and Hoyes 1998; Furinghetti, Olivero, and Paulo 2001). Recognizing that students had overgeneralized when they made the All Digits Are the Same conjecture, we asked them to look for one or more counter-examples to their initial hypothesis, using their thinking to further instruction (NCTM 2014). Students enjoyed this challenge because they knew that a single counterexample would prove that the All Digits Are the Same conjecture was false. Our charge required students to construct viable mathematical arguments, critique their own reasoning and the reasoning of others, and attend to precision (SMP 3 and SMP 6). Finding a counterexample also encouraged productive struggle among students as they deeply explored mathematics (NCTM 2014).

Several third graders noticed that the addends in each of the examples in **figure 2** included one digit that was relatively "large" (e.g., 6, 7, 8) and a second digit that was relatively "small" (e.g., 1, 2, 3). In their quest for a counterexample, these students explored examples that did not fit this pattern (see **fig. 3**). When students attempted the procedure themselves, adding 74 + 47, some of them struggled with regrouping. The Reverse and Add to 100 task offered students an opportunity to practice adding within 1000, using strategies based on place value (Common Core content standard 3.NBT.A.2). As students worked, we encouraged them to consider ones and tens separately. For instance, in the problem 74 + 47, 4 ones plus 7 ones equals 11 ones; 7 tens plus 4 tens equals 11 tens (or 110). Because the digits in the sum 121 are not the same (i.e., 1 and 2 are different), the sum is a counterexample. The All Digits Are the Same conjecture turned out to be false! Many students were surprised that their conjecture was not true. This fueled their desire to find other patterns and conjectures that held in general.

FIGURE 4

The class successfully generated possible sums equaling 99. When pushed to think deeply and reason about emerging patterns, students were able to formulate the Summing Digits conjecture.



The Summing Digits conjecture

After several third graders had generated 99 as a sum (see **fig. 4**), we sought to promote deeper reasoning and problem solving (NCTM 2014) by asking students to find all possible ways to generate 99. Several made use of the mathematical structure they had noticed in their earlier work (SMP 7) to formulate the Summing Digits conjecture:

> When the sum of the digits in the first addend is less than 10 (e.g., 81, 8 + 1 = 9), then the Reverse and Add algorithm yields a two-digit sum with all digits the same, namely, the sum of the digits of either addend.

For instance, applying the Reverse and Add algorithm to 81 yields a sum of 81 + 18 = 99; each digit, 9, is the sum of 8 and 1 (8 + 1 = 9). Using this observation and basic addition facts, students uncovered ten ways to generate 99 (see **fig. 5**).

FIGURE 3

This is one possible counterexample to the All Digits Are the Same conjecture.

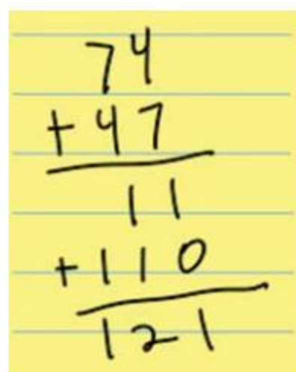
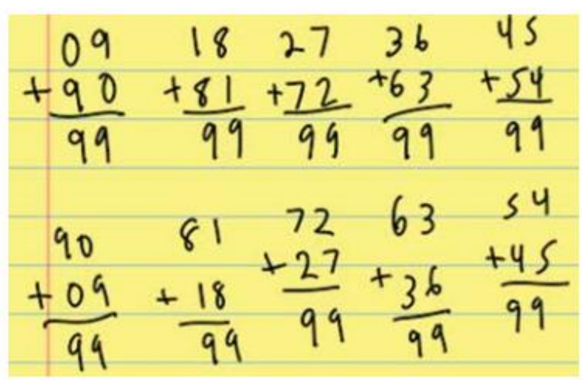


FIGURE 5

Using basic addition facts and their observation that applying the Reverse and Add algorithm to 81 yields 81 + 18 = 99, students uncovered ten ways to generate 99.



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Task 4: A student from another class wonders if they could make a sum of 100 using Reverse and Add? Continue working with the Reverse and Add algorithm to see if you can find two numbers with the same digits in different positions that have a sum of 100. Which math practices are you using when searching a 100 sum?

- Record all of your guesses
- Record your wonderings
- Record what you notice as you experiment with this procedure
- Can you find a pair with digits reversed that has a sum of 100? List all the pairs.
- Can you find a pair with digits reversed that has a sum close to 100? List all the pairs.
- List the mathematical practices you have been using in this task

Teacher talk: Can anyone get to 100? If not, what is the closest number to 100? How many ways can students make the number closes to 100? Hand out table on next page to further facilitate the conversation. * The attached article has possible extension activities.

Why 100 cannot be reached

As class came to a close for the day, none of the third graders (or their teachers) had been able to generate a sum of 100. For homework, we asked them to either find an initial two-digit number that would generate 100 or else explain why such a sum was impossible.

As we prepared for the subsequent class session, we generated a table of all possible two-digit addends and sums resulting from the Reverse and Add algorithm (see **table 1**). Students who enjoyed the challenge created similar tables in their homework. As **table 1** reveals, 100 cannot be generated using the Reverse and Add algorithm; 99 is the sum closest to 100. The a + b columns also confirm that *all* sums are multiples of 11. *Why might this be the case?* Although the third graders were unfamiliar with multiplication facts for 11, we asked them to further analyze the sums and explore multiples of 11 using multidigit addition strategies that are present in the grade 3 standards.

Addends are listed in columns *a* and *b*. Sums listed in boldface, are under column headings *a* + *b*.

a	*b*	*a + b*	*a*	*b*	*a + b*	*a*	*b*	*a + b*	*a*	*b*	*a + b*
10	1	**11**	33	33	**66**	56	65	**121**	79	97	**176**
11	11	**22**	34	43	**77**	57	75	**132**	80	8	**88**
12	21	**33**	35	53	**88**	58	85	**143**	81	18	**99**
13	31	**44**	36	63	**99**	59	95	**154**	82	28	**110**
14	41	**55**	37	73	**110**	60	6	**66**	83	38	**121**
15	51	**66**	38	83	**121**	61	16	**77**	84	48	**132**
16	61	**77**	39	93	**132**	62	26	**88**	85	58	**143**
17	71	**88**	40	4	**44**	63	36	**99**	86	68	**154**
18	81	**99**	41	14	**55**	64	46	**110**	87	78	**165**
19	91	**110**	42	24	**66**	65	56	**121**	88	88	**176**
20	2	**22**	43	34	**77**	66	66	**132**	89	98	**187**
21	12	**33**	44	44	**88**	67	76	**143**	90	9	**99**
22	22	**44**	45	54	**99**	68	86	**154**	91	19	**110**
23	32	**55**	46	64	**110**	69	96	**165**	92	29	**121**
24	42	**66**	47	74	**121**	70	7	**77**	93	39	**132**
25	52	**77**	48	84	**132**	71	17	**88**	94	49	**143**
26	62	**88**	49	94	**143**	72	27	**99**	95	59	**154**
27	72	**99**	50	5	**55**	73	37	**110**	96	69	**165**
28	82	**110**	51	15	**66**	74	47	**121**	97	79	**176**
29	92	**121**	52	25	**77**	75	57	**132**	98	89	**187**
30	3	**33**	53	35	**88**	76	67	**143**	99	99	**198**
31	13	**44**	54	45	**99**	77	77	**154**			
32	23	**55**	55	55	**110**	78	87	**165**			

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Task 5: Thinking about the options explored in this RWE, write a proposal that tells which activity third graders should do for Family Math Night. Make sure you explain your thinking fully so that others will agree with your idea.

Be sure to include:

- The specific activity
- How to do the activity
- The math practices used throughout the activity
- How the activity matches the theme Magic of Mathematics
- How the activity matches what third graders are learning and are able to do

Meeting	• Student explains his/her thinking so that others can understand • Student explains the specific activity completely • Student explains how to do the activity • Student explains the math practices used throughout the activity • Student explains how the activity matches the theme, Magic of Mathematics • Student explains how the activity matches what third graders are learning and are able to do
Developing	Student meets 4 of the 6 require criteria
Beginning	Student meets less than 4 of the required criteria Student should rework task 5 after more instruction
Notes	

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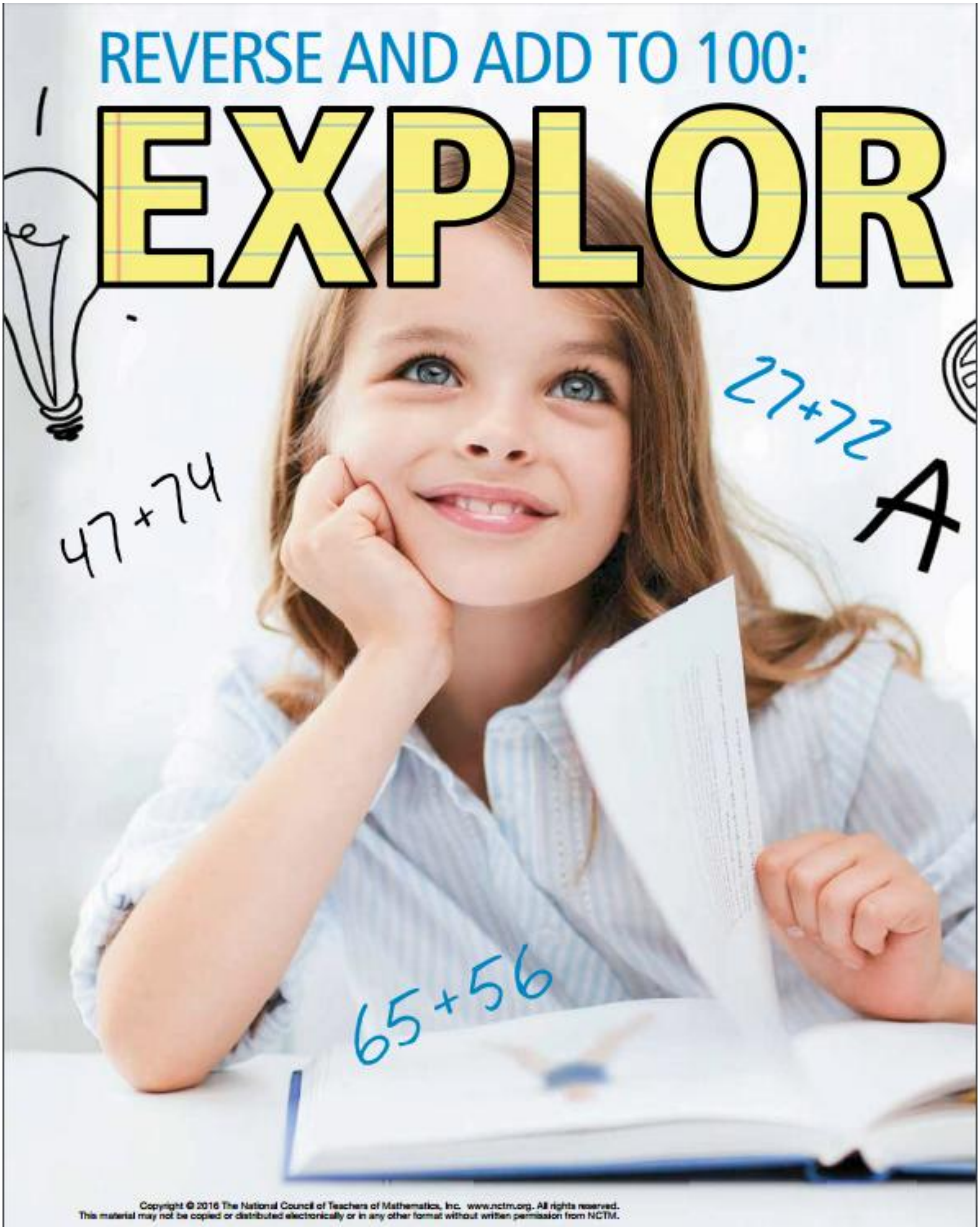
REVERSE AND ADD TO 100:
EXPLOR
27+72
47+74
A
65+56
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ATIONS
IN PLACE VALUE

A collaborative number quest challenges third graders to strengthen their understanding of patterns, multidigit addition, and number operations.

Michael Todd Edwards,
James Quinlan, and Jeremy F. Strayer

During the past few years, we (the authors) have become interested in engaging students as creators, rather than passive recipients, of mathematics. Several of us have incorporated student problem posing as a regular instructional feature in our classrooms. When we offer our students the opportunity to construct their own problems, particularly during the course of an entire school year, they create many novel tasks. Student-created tasks not only serve as teaching tools for our students but also are excellent formative assessments for us, providing insight into students' mathematical conceptions and interests.

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In the pages that follow, we describe classroom interactions surrounding the creation and implementation of the Reverse and Add to 100 task, developed collaboratively with third graders to strengthen their understanding of place value and multidigit addition. Our explorations of Reverse and Add to 100 promoted student reasoning and problem solving, fostered meaningful mathematical discourse, elicited and made use of student thinking during instruction, and supported productive struggle. NCTM encourages teachers to use such research-supported teaching practices because they promote deep mathematical learning (2014). Aligned with the Common Core's Standards for Mathematical Practice (SMP), the task also encouraged third-grade students to *make sense of problems and persevere in solving them* (SMP 1), *construct viable arguments and critique the reasoning of others* (SMP 3), *attend to precision* (SMP 6), and *look for and make use of structure* (SMP 7) (CCSSI 2010, pp. 6–7).

FIGURE 1

This was a popular first attempt when students explored Reverse and Add to 100.

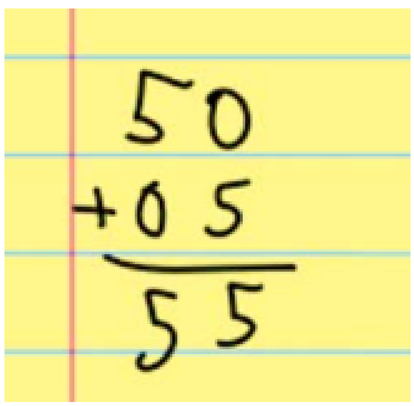
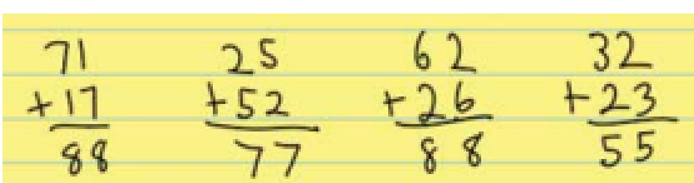


FIGURE 2

When students discovered the small final sum that 50 yields, they experimented with other two-digit starting values. A pattern soon emerged.



Introducing the Reverse and Add to 100 task

During our recent visit to a local elementary school, a third grader shared with us an interesting algorithm for two-digit numbers. We refer to it as the Reverse and Add procedure:

> Write a two-digit number. Reverse the ones and tens digits to create a second two-digit number. Find the sum of the two numbers.

The student shared the Reverse and Add algorithm with classmates, and a palpable buzz filled the room. Students experimented with the algorithm immediately, uncovering a number of interesting patterns (SMP 1). After a "quick minute" of exploration, a second student, likely influenced by recent work with a Close to 100 investigation (TERC 2008), challenged everyone with the following task:

> Using Reverse and Add, produce a sum as close to 100 as possible.

At the time, none of us (the authors, classroom teacher, or students) knew if 100 was attainable.

Conjectures and patterns

In the paragraphs that follow, we explore patterns that students (and teachers) uncovered as they worked on the Reverse and Add to 100 task. Furthermore, we share ways in which the task encouraged the third graders to think deeply about place value.

The Near 50 conjecture

Working in small groups, many students selected 50 as their initial addend. This was a reasonable first step, because 50 is half of the desired sum, 100. However, the third graders quickly discovered that 50 yields a surprisingly small final sum (see fig. 1). At this point, students persevered in problem solving (SMP 1), experimenting with other two-digit starting values (see fig. 2). As students generated calculations, such as those illustrated in **figure 2**, we asked them to comment on patterns they observed. Many students noted, "All the numbers in the answer are the same"

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Several third graders noticed that the addends in each of the examples in **figure** 2 included one digit that was relatively "large" (e.g., 6, 7, 8) and a second digit that was relatively "small" (e.g., 1, 2, 3). In their quest for a counterexample, these students explored examples that did not fit this pattern (see **fig.** 3). When students attempted the procedure themselves, adding 74 + 47, some of them struggled with regrouping. The Reverse and Add to 100 task offered students an opportunity to practice adding within 1000, using strategies based on place value (Common Core content standard 3.NBT.A.2). As students worked, we encouraged them to consider ones and tens separately. For instance, in the problem 74 + 47, 4 ones plus 7 ones equals 11 ones; 7 tens plus 4 tens equals 11 tens (or 110). Because the digits in the sum 121 are not the same (i.e., 1 and 2 are different), the sum is a counterexample. The All Digits Are the Same conjecture turned out to be false! Many students were surprised that their conjecture was not true. This fueled their desire to find other patterns and conjectures that held in general.

FIGURE 3

This is one possible counterexample to the All Digits Are the Same conjecture.

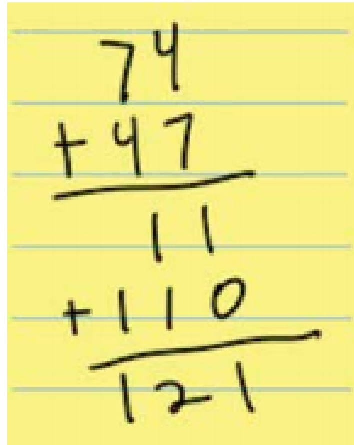
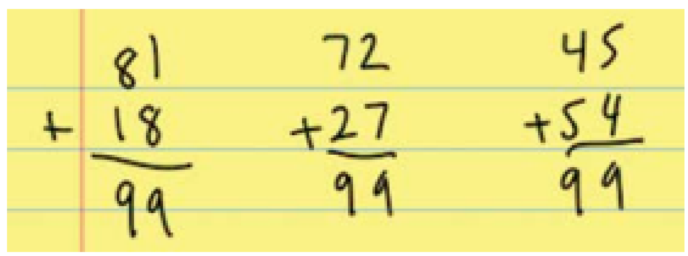


FIGURE 4

The class successfully generated possible sums equaling 99. When pushed to think deeply and reason about emerging patterns, students were able to formulate the Summing Digits conjecture.



The Summing Digits conjecture

After several third graders had generated 99 as a sum (see **fig.** 4), we sought to promote deeper reasoning and problem solving (NCTM 2014) by asking students to find all possible ways to generate 99. Several made use of the mathematical structure they had noticed in their earlier work (SMP 7) to formulate the Summing Digits conjecture:

> When the sum of the digits in the first addend is less than 10 (e.g., 81, 8 + 1 = 9), then the Reverse and Add algorithm yields a two-digit sum with all digits the same, namely, the sum of the digits of either addend.

For instance, applying the Reverse and Add algorithm to 81 yields a sum of 81 + 18 = 99; each digit, 9, is the sum of 8 and 1 (8 + 1 = 9). Using this

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FIGURE 5

Using basic addition facts and their observation that applying the Reverse and Add algorithm to 81 yields 81 + 18 = 99, students uncovered ten ways to generate 99.

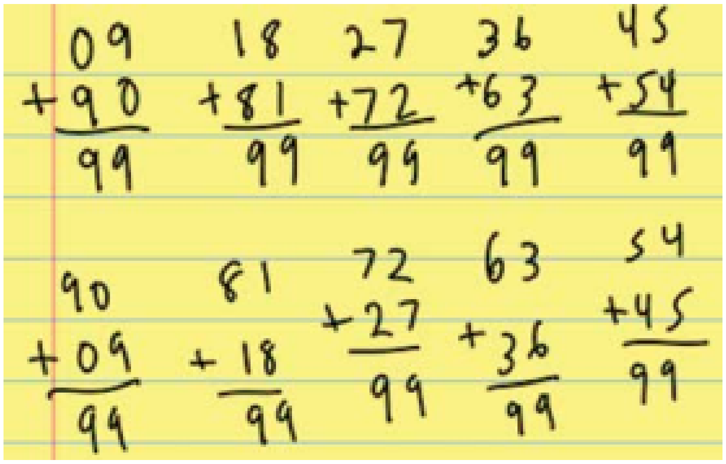

FIGURE 6

Although an algebraic proof is beyond the scope of the third-grade curriculum, it offers the insight that one cannot generate 100 using the Reverse and Add algorithm and that all sums generated with two-digit addends are multiples of 11.

Proof

Let *mn* denote the first addend of the Reverse and Add algorithm. Note that because *m* is in the tens place and *n* is in the ones place, $mn = 10m + n$. We reverse the digits in the tens and ones places in *mn* to construct the second addend, *nm*. By a similar argument, $nm = 10n + m$. Next, we calculate the sum $mn + nm = (10m + n) + (10n + m)$. Simplified, the sum equals $11m + 11n = 11(m + n)$. The simplified sum indicates that the result of the Reverse and Add algorithm is a multiple of 11 for any arbitrary two-digit addend. Because 100 is not a multiple of 11, 100 is not constructible using the algorithm.

observation and basic addition facts, students uncovered ten ways to generate 99 (see fig. 5).

Leading zeros

Uncovering all possible ways to make 99 produced interesting whole-group conversations that supported the third graders' "place value understanding and (use of) properties of operations to perform multi-digit arithmetic" (CCSSI 2010, p. 22). One place-value conversation arose from the fact that students had never written addends with leading zeros (e.g., 09 rather than 9). Deciding whether 09 was a legitimate number prompted spirited debates. Ultimately, we agreed that such numbers as 07 and 7 were equal, because each had 7 ones and 0 of any other placeholder (e.g., 0 tens, 0 hundreds, etc.). The classroom teacher explained that we leave off leading zeros as placeholders because they do not add any information about the number, and they take more work to write. Another discussion arose after students realized that the pairs of examples (see fig. 5) differed only by the order of their addends (e.g., 27 + 72 and 72 + 27). Some students wanted to count each as a separate example; others did not. After some discussion, we decided to count such examples only once because they had the same set of addends.

Why 100 cannot be reached

As class came to a close for the day, none of the third graders (or their teachers) had been able to generate a sum of 100. For homework, we asked them to either find an initial two-digit number that would generate 100 or else explain why such a sum was impossible.

As we prepared for the subsequent class session, we generated a table of all possible two-digit addends and sums resulting from the Reverse and Add algorithm (see **table 1**). Students who enjoyed the challenge created similar tables in their homework. As **table 1** reveals, 100 cannot be generated using the Reverse and Add algorithm; 99 is the sum closest to 100. The a + b columns also confirm that *all* sums are multiples of 11. *Why might this be the case?* Although the third graders were unfamiliar with multiplication facts for 11, we asked them to further analyze the sums and explore multiples of 11 using multidigit addition strategies that are present in the grade 3 standards.

TABLE 1

Addends are listed in columns a and b. Sums, listed in boldface, are under column headings *a* + *b*. Students who enjoyed the homework challenge (to find an initial two-digit number that would generate 100 or else explain why such a sum is impossible) created similar tables to this one that the authors created.

a	*b*	*a* + *b*	*a*	*b*	*a* + *b*	*a*	*b*	*a* + *b*	*a*	*b*	*a* + *b*
10	1	**11**	33	33	**66**	56	65	**121**	79	97	**176**
11	11	**22**	34	43	**77**	57	75	**132**	80	8	**88**
12	21	**33**	35	53	**88**	58	85	**143**	81	18	**99**
13	31	**44**	36	63	**99**	59	95	**154**	82	28	**110**
14	41	**55**	37	73	**110**	60	6	**66**	83	38	**121**
15	51	**66**	38	83	**121**	61	16	**77**	84	48	**132**
16	61	**77**	39	93	**132**	62	26	**88**	85	58	**143**
17	71	**88**	40	4	**44**	63	36	**99**	86	68	**154**
18	81	**99**	41	14	**55**	64	46	**110**	87	78	**165**
19	91	**110**	42	24	**66**	65	56	**121**	88	88	**176**
20	2	**22**	43	34	**77**	66	66	**132**	89	98	**187**
21	12	**33**	44	44	**88**	67	76	**143**	90	9	**99**
22	22	**44**	45	54	**99**	68	86	**154**	91	19	**110**
23	32	**55**	46	64	**110**	69	96	**165**	92	29	**121**
24	42	**66**	47	74	**121**	70	7	**77**	93	39	**132**
25	52	**77**	48	84	**132**	71	17	**88**	94	49	**143**
26	62	**88**	49	94	**143**	72	27	**99**	95	59	**154**
27	72	**99**	50	5	**55**	73	37	**110**	96	69	**165**
28	82	**110**	51	15	**66**	74	47	**121**	97	79	**176**
29	92	**121**	52	25	**77**	75	57	**132**	98	89	**187**
30	3	**33**	53	35	**88**	76	67	**143**	99	99	**198**
31	13	**44**	54	45	**99**	77	77	**154**			
32	23	**55**	55	55	**110**	78	87	**165**			

Algebraic proof

Generating a table of all possible sums provides conclusive evidence that one cannot generate 100 using the Reverse and Add algorithm. The table also confirms that all sums generated with two-digit addends are multiples of 11. Unfortunately, constructing the table is tedious for many students and fails to offer any insight as to why either result holds. Although an algebraic proof is beyond the scope of the third-grade curriculum, it provides such insight. The argument is accessible to anyone with an understanding of first-year algebra. As such, we shared it with the third-grade teachers (see **fig. 6**).

The Reverse and Add to 100 task can be extended for advanced third graders and students in higher grades.

Possible extensions

The Reverse and Add to 100 task can be extended for advanced third graders and students in higher grades. For instance, in modified versions of the task, students and teachers could explore subtraction, multiplication, or division rather than addition (e.g., Reverse and Subtract, Reverse and Multiply, or Reverse and Divide). Alternatively, increasing the number of digits in the Reverse and Add algorithm (e.g., 745 + 547) offers a rich context for further student conjecturing, providing students of varied backgrounds and interests with enhanced opportunities for problem posing. Moreover, they give young students a vehicle for engaging in mathematical sense making and proof.

Discussion and conclusions

In the preceding pages, we explored a task authored by third graders. The resulting student-created problem, the Reverse and Add to 100 task, gave students of various ability levels opportunities to practice two-digit addition skills. Moreover, the task offered students a vehicle for generating and testing their own hypotheses as they engaged in several of the Common Core's Standards for Mathematical Practice (SMP) (CCSSI 2010). Using the Reverse and Add algorithm challenged students to find as many possible ways to generate 99 as they could and to determine whether a sum of 100 was possible. Ultimately, using a table of all possible addends and sums, they were able to verify that 100 is not constructible. By listening to students and allowing them time to create and share their own mathematical thinking, we facilitated instruction that supported a deep learning of mathematics (NCTM 2014).

Common Core Connections

SMP 1 SMP 6
SMP 3 SMP 7

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