Topic 2 L.2 – Exponents

Student Outcomes

* Students discover that is not the same thing as which is.
* Students understand that a base number can be represented with a positive whole number, positive fraction, or positive decimal and that for any number , we define to be the product of factors of . The number is the base, and is called the exponent or power of .

Warm-Up

As you evaluate these expressions, pay attention to how you arrive at your answers.

Discussion (15 minutes)

* How many of you solved the problems by “counting on”? That is, starting with , you counted on more each time .
* If you did not find the answer that way, could you have done so?
	+ *Yes, but it is time-consuming and cumbersome.*
* Addition is a faster way of “counting on.”
* How else could you find the sums using addition?
	+ *Count by , , or .*
* How else could you solve the problems?
	+ *Multiply times ; multiply times ; or multiply times .*
* Multiplication is a faster way to add numbers when the addends are the same.
* When we add five groups of , we use an abbreviation and a different notation, called multiplication.
* If multiplication is a more efficient way to represent addition problems involving the repeated addition of the same addend, do you think there might be a more efficient way to represent the repeated multiplication of the same factor, as in

Allow students to make suggestions; some will recall this from previous lessons.

* We see that when we add five groups of , we write , but when we multiply five copies of , we write . So, multiplication by in the context of addition corresponds exactly to the exponent in the context of multiplication.

Make students aware of the correspondence between addition and multiplication because what they know about *repeated addition* will help them learn exponents as *repeated multiplication* as we go forward.

* The little we write is called an exponent and is written as a *superscript*. The numeral is written only half as tall and half as wide as the, and the bottom of the should be halfway up the number . The top of the can extend a little higher than the top of the zero in . Why do you think we write exponents so carefully?
	+ *It reduces the chance that a reader will confuse with .*

Guided Practice (5 minutes)

Work through Examples 1–5 as a group; supplement with additional examples if needed. Remind students of the definitions of exponential and expanded.

* There is a special name for numbers raised to the second power. When a number is raised to the second power, it is called *squared*.  **Remember that in geometry, squares have the same two dimensions: length and width. For is the area of a square with side length**
* There is also a special name for numbers raised to the third power. When a number is raised to the third power, it is called *cubed*. Remember that in geometry, cubes have the same three dimensions: length, width, and height.
Examples 1–5

Write each expression in exponential form.

|  |  |
| --- | --- |
| 1.
 |  |
| Write each expression in expanded form. |  |
|  |  |

* **The repeated factor is called the *base,* and the exponent is also called the *power*. Say the numbers in examples 1–5 to a partner.**

Check to make sure students read the examples correctly:

Go back to Examples 1–4, and use a calculator to evaluate the expressions.

|  |  |
| --- | --- |
| 1.
 |  |
|  |  |

What is the difference between and ?

 or times ;

Take time to clarify this important distinction.

* The base number can also be written in decimal or fraction form. Try Examples 6, 7, and 8. Use a calculator to evaluate the expressions.

You may use a calculator on problems 6-8

1. Write the expression in expanded form, and then evaluate.

*Note to teacher:*

If students need additional help multiplying fractions, refer to the first four modules of Grade 5.

1. Write the expression in exponential form, and then evaluate.

1. Write the expression in exponential form, and then evaluate.

You may NOT use a calculator on problems 9–10

|  |  |
| --- | --- |
| 1. Write the expression in exponential form, and then evaluate.
 | 1. Write the expression in expanded form, and then evaluate.
 |

Do on whiteboards/orally with students ***if needed***

* What is the value of squared?
* What is the value of squared?
* What is the value of squared?
* What is the value of squared?
* What is the value of cubed?
* What is the value of cubed?
* What is the value of cubed?

Independent Practice

Ask students to fill in the chart, supplying the missing expressions.

INDEPENDENT PRACTICE

1. Fill in the missing expressions for each row. For whole number and decimal bases, use a calculator to find the standard form of the number. For fraction bases, leave your answer as a fraction.

|  |  |  |
| --- | --- | --- |
| Exponential Form | Expanded Form | Standard Form |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

1. Write five cubed in all three forms: exponential form, expanded form, and standard form.

; ;

1. Write *fourteen and seven-tenths squared* in all three forms.

; ;

1. One student thought two to the third power was equal to six. What mistake do you think he made, and how would you help him fix his mistake?

The student multiplied the base, , by the exponent, . This is wrong because the exponent never multiplies the base; the exponent tells how many copies of the base are to be used as factors.

Lesson Summary

Exponential Notation for Whole Number Exponents: Let be a nonzero whole number.  For any number , the expression is the product of factors of , i.e.,

The number is called the *base,* and  is called the *exponent* or *power* of .

When is , “the product of one factor of ” just means , i.e., . Raising any nonzero number to the power of is defined to be , i.e., for all .

HOMEWORK

1. Complete the table by filling in the blank cells. Use a calculator when needed.

|  |  |  |
| --- | --- | --- |
| Exponential Form | Expanded Form | Standard Form |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

1. Why do whole numbers raised to an exponent get greater, while fractions raised to an exponent get smaller?

As whole numbers are multiplied by themselves, products are larger because there are more groups. As fractions of fractions are taken, the product is smaller. A part of a part is less than how much we started with.

1. The powers of that are in the range through are , ,,,,,,, and . Find all the powers of that are in the range through .

,,,,,

1. Find all the powers of in the range through .

, , ,

1. Write an equivalent expression for using only addition.
2. Write an equivalent expression for using only multiplication.
	* 1. Explain what is in this new expression.

 is the factor that will be repeatedly multiplied by itself.

* + 1. Explain what is in this new expression.

 is the number of times will be multiplied.

1. What is the advantage of using exponential notation?

It is a shorthand way of writing a multiplication expression if the factors are all the same.

1. What is the difference between and ? Evaluate both of these expressions when.

means four times , this is the same as . On the other hand, means to the fourth power, or .

When ,

When ,