**Who Put The Tang In Tangram?

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Adapted from a lesson on the Utah Education Network www.uen.org
Tangram clip art from http://www.who.int/world-health-day/previous/2005/infomaterials/en/

In this hands-on task, students determine the area of tangram pieces without using formulas. Students also develop and use formulas to determine the area of squares, rectangles, triangles, and parallelograms.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.G.1 Find area of right triangles, other triangles, quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Given a two dimensional figure students will solve for the area by composing and decomposing.
3. Construct viable arguments and critique the reasoning of others. Students will be able to review solutions to justify (verbally and written) why the solutions are reasonable.
4. Model with mathematics. Use hands on/virtual manipulatives (tangrams) using every day two-dimensional and three-dimensional shapes.
5. Use appropriate tools strategically. Students will use a tangrams and dot paper to solve for area.
6. Attend to precision. Students will use appropriate measurement units and correct terminology to justify reasonable solutions.
7. Look for and make use of structure. Students compose and decompose two-dimensional figures to find areas.
8. Look for and express regularity in repeated reasoning. Students will explain why decompose and composed figures to find of irregular polygons.

ESSENTIAL QUESTIONS

- How can shapes be composed to create new shapes?
- How can a shape be decomposed into smaller shapes?
- How do we figure the area of a shape without a formula for that shape?

MATERIALS

- *The Warlord's Puzzle* by Virginia Walton Pilegard, or similar book about tangrams
- *Grandfather Tang* by Ann Tompert, or a similar book about tangrams
- "Who Put the Tang in Tangrams? Finding Areas" student recording sheet (2 pages)
- "Who Put the Tang in Tangrams? Deriving Formula I" student recording sheet (2 pages)
- "Who Put the Tang in Tangrams? Deriving Formula II" student recording sheet
- Tangram sets
- Geoboards, Rubber bands
- Geoboard Recording Paper
- 9 x 12 art paper

TASK COMMENTS

Geogebra.org is a free interactive website that allows users to create regular and irregular polygons to find area. Teachers and students can use this to compose and decompose shapes to solve real-world problems.

As an introduction to this task, the book *The Warlord's Puzzle* by Virginia Walton Pilegard or similar book about tangrams can be read to the students. After the story, guide students to create their own tangram pieces through paper folding. Directions with illustrations can be found below and at the following web site: http://mathforum.org/trscavo/tangrams/construct.html.

- Fold a 9 x 12 piece of art paper to form a square. Cut off the extra piece at the bottom and discard.
- Cut the square in half on the diagonal fold to form two triangles.
- Take one of the triangles and fold it in half to form two smaller congruent triangles. Cut along the fold.
- Take the other large triangle and make a small pinch crease in the middle of the baseline (longest side) to identify the center. Take the apex of the triangle (the vertex opposite the longest side) and fold it to touch the center of the baseline. This forms a trapezoid.
- Cut along the fold line. This gives you a trapezoid and a small triangle.
- Fold the trapezoid in half (two congruent shapes) and cut along the fold line.
- Take one half of the trapezoid and fold the pointed end to form a small square. Cut along the fold. This will give you a small square and a small triangle.
- Take the remaining half of the trapezoid. Fold one of the corners of the square end to form a small triangle and a parallelogram. Cut along the fold.

As you deconstruct the square, discuss the relationships between the pieces. Once complete with a full set (One small square, two small congruent triangles, two large congruent triangles, a medium size triangle and a parallelogram), ask students to experiment with the shapes to create new figures.

To start this task, give each student a set of plastic Tangrams to use for this task (they are easier to trace than the paper ones). Ask students to find the two small congruent triangles and review the definition of congruent: same size, same shape. Put them together to make a square. Ask students what the area of this shape would be and ask them to explain how they know. ***(Because the square formed with the two small triangles is the congruent to the tangram square, its area must be the same 1 u^2.)*** Next, ask students to take just one of the small triangles. Ask students what its area would be and ask them to explain how they know. Remember to relate it to the square. ***(The area of each small triangle is half of the area of the square, so its area is $\frac{1}{2}u^2$.)*** Give students the time to try to determine how to make the shapes before students share their work. This processing time for thinking and experimenting will help students develop their spatial reasoning.

BACKGROUND KNOWLEDGE

Students will need to approach this task with the following prerequisite knowledge:

- Experience with common plane figures and the identification of their sides and angles.
- Familiarity with how to use a geoboard and transfer shapes on the geoboard to geoboard paper.
- Knowledge of area and congruence.
- Understanding of the area of a rectangle and its formula.

Students will be figuring the area of each of the tangram pieces by comparing them to the small square. The small square will be "one square unit". Therefore, the length and width are both one unit because 1 x 1 = 1.

Questions/Prompts for Formative Student Assessment

- What is the area of this shape? How do you know?
- What shapes have the same area as the area of this shape?
- What shapes did you use to create a figure congruent to this figure? What are the areas of those shapes used?
- What could you add to find the area of this shape?

Questions for Teacher Reflection

- Which students are able to find a relationship of each tangram piece to the area of the square?
- Which students need to completely cover a figure with tangram pieces in order to find its area?
- Which students are able to use the relationships between tangram pieces to find the area of figures?
- Did students recognize that the area of a figure can be found by finding the area of pieces of the figure and then adding them together?

DIFFERENTIATION

Extension

- There are different ways to create many of the shapes on the "Who Put the Tang in Tangram? Finding Areas" student recording sheet. Allow students to explore these shapes to see if they can find different ways to create them using the tangrams.
- Using a Geoboard and Geoboard recording paper (available at http://www.wiley.com/college/reys/0470403063/appendixc/masters/geoboard_recording.html). Ask students to create and record as many different parallelograms as they can that have an area of $\frac{1}{2}$, 1, or 2 square units. Ask student to identify and give the measure of a base and height for each parallelogram.

Intervention

- Do not have students with fine motor control difficulties trace the shapes; just let them manipulate the tangrams. Also, students may be given a copy of the tangram puzzle so they just have to cut out the shapes, not fold to make the shapes.
- It might be helpful to give some students two sets of tangrams in different colors so they can more easily see the relationships between the shapes.

TECHNOLOGY CONNECTIONS

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=108 Interactive triangle and parallelogram applets. Allows students to explore the relationship between the base and height in any type of triangle or parallelogram. (Do not use the trapezoid feature.) This is a great resource help clarify the misconception about height of a triangle.
- http://www.geogebra.org/en/upload/files/english/Victoria/TriangleArea.html Interactive triangle, students can find the area given the base and height.
- http://www.geogebra.org/en/upload/files/english/Knote/Area/Parallelogram2.html Interactive parallelogram, students can find the area given the base and height, as well as move a triangular piece to create a rectangle.
- http://www.geogebra.org/en/upload/files/english/Knote/Area/parallelograms.html Interactive parallelogram, students can find the area given the base and height.

Sample solutions for the "Who Put the Tang in Tangram? Finding Areas" student recording sheet are shown below.

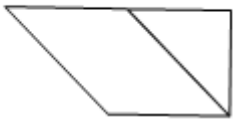
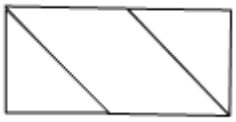
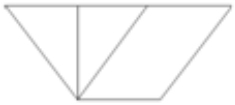
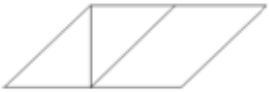
Figure	Show your work	Area of Figure (in square units)
Small Triangle 	Two small triangles can be arranged to make a square congruent to the square with an area of 1 u^2.	The triangle is half of the square so it must have an area of $\frac{1}{2}$ u^2.
Medium Triangle 	Two small triangles can be arranged as shown to make a triangle congruent to the medium triangle.	Since the area of each small triangle is $\frac{1}{2}$ u^2, the area of the medium triangle must be 1 u^2.
Large Triangle (Use the square) 	The square and two small triangles can be arranged as shown to make a triangle congruent to the large triangle.	Since the area of each small triangle is $\frac{1}{2}$ u^2 and the area of the square is 1 u^2, the area of the large triangle must be $2\ u^2\left(\frac{1}{2}\ u^2 + \frac{1}{2}\ u^2 + 1\ u^2\right)$.
Parallelogram 	Two small triangles can be arranged as shown to make a parallelogram congruent to the given parallelogram.	Since the area of each small triangle is $\frac{1}{2}$ u^2, the area of the parallelogram must be 1 u^2.
Trapezoid 	A small triangle and the parallelogram can be combined as shown to create the trapezoid.	Since the area of a small triangle is $\frac{1}{2}$ u^2 and the area of the parallelogram 1 u^2, the area of the large triangle must be $1\frac{1}{2}u^2$ $\left(\frac{1}{2}\ u^2 + 1\ u^2\right)$.
Two small and one medium triangle 	Two small and one medium triangle can be combined as shown to make this square.	Since the area of each small triangle is $\frac{1}{2}$ u^2 and the area of the medium triangle is 1 u^2, the area of the triangle must be 2 $u^2\left(\frac{1}{2}\ u^2 + \frac{1}{2}\ u^2 + 1\ u^2\right)$.
Rectangle 	Two small triangles and the parallelogram can be combined as shown to create this rectangle.	Since the area of each small triangle is $\frac{1}{2}$ u^2 and the area of the parallelogram is 1 u^2, the area of the rectangle must be $2\ u^2\left(\frac{1}{2}\ u^2 + \frac{1}{2}\ u^2 + 1\ u^2\right)$.

Figure **Sketch it below**	**Show your work**	**Area of Figure** **(in square units)**
Triangle congruent to a large triangle (Do not use the square) 	Two small triangles and the medium triangle can be arranged to make a triangle congruent to the large triangle.	Since the area of each small triangle is $\frac{1}{2}$ u^2 and the area of the medium triangle is 1 u^2, the area of the large triangle must be 2 u^2 $\left(\frac{1}{2}\ u^2 + \frac{1}{2}\ u^2 + 1\ u^2\right)$.
Trapezoid (Different from the one page 1) 	One possible trapezoid can be made using two small triangles and the parallelogram.	Since the area of each small triangle is $\frac{1}{2}$ u^2 and the area of the parallelogram is 1 u^2, the area of the trapezoid must be 2 u^2 $\left(\frac{1}{2}\ u^2 + \frac{1}{2}\ u^2 + 1\ u^2\right)$.
Parallelogram (Different from the one on page 1) 	One possible parallelogram can be made as shown using two small triangles and the parallelogram.	Since the area of each small triangle is $\frac{1}{2}$ u^2 and the area of the parallelogram is 1 u^2, the area of the created parallelogram must be 2 u^2 $\left(\frac{1}{2}\ u^2 + \frac{1}{2}\ u^2 + 1\ u^2\right)$. .
Pentagon 	One possible pentagon can be made as shown using two small triangles, the square and the medium triangle.	Since the area of each small triangle is $\frac{1}{2}$ u^2, the area of the square is 1 u^2, and the area of the medium triangle is 1 u^2, the area of the pentagon must be 3 u^2 $\left(\frac{1}{2}\ u^2 + \frac{1}{2}\ u^2 + 1\ u^2 + 1\ u^2\right)$.
Square using all 7 pieces 	One possible square can be made as shown all seven pieces.	Since the area of each small triangle is $\frac{1}{2}$ u^2, the area of the square is 1 u^2, the area of the medium triangle is 1 u^2, the area of the parallelogram is 1 u^2, and the area each of the large triangles is 2 u^2, the area of the square must be 8 u^2 $\left(\frac{1}{2}\ u^2 + \frac{1}{2}\ u^2 + 1\ u^2 + 1\ u^2 + 1\ u^2 + 2\ u^2 + 2\ u^2\right)$.

Name _____ Date _____

Who Put the Tang in Tangram?

Find the area of the following figures.

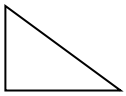
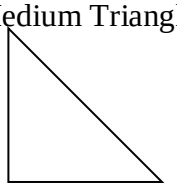
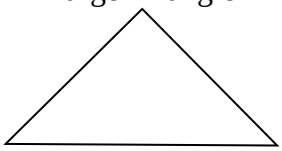
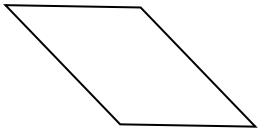
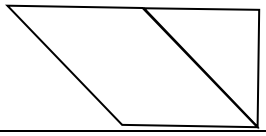
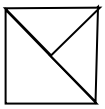
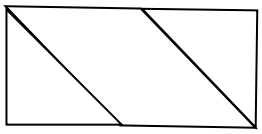
Figure	Show your work	Area of Figure (in square units)
Small Triangle 		
Medium Triangle 		
Large Triangle 		
Parallelogram 		
Trapezoid 		
Two small and one medium triangles 		
Rectangle 		

Figure **Sketch it below**	**Show your work**	**Area of Figure** **(in square units)**
Triangle congruent to a large triangle (Do not use the square)		
Trapezoid (Different from the one page 1)		
Parellelogram (Different from the one on page 1)		
Pentagon		
Square using all 7 pieces		