

Using Virtual Manipulatives to Investigate Patterns and Generate Rules in Algebra

This department features children's hands-on and minds-on explorations in mathematics and presents teachers with open-ended investigations to enhance mathematics instruction. These tasks invoke problem solving and reasoning, require communication skills, and connect various mathematical concepts and principles. The ideas presented here have been tested in classroom settings.

A mathematical investigation—

- has multidimensional content;
- is open-ended, with several acceptable solutions;
- is an exploration requiring a full period or longer to complete;
- is centered on a theme or event; and
- is often embedded in a focus or driving question.

In addition, a mathematical investigation involves processes that include—

- researching outside sources;
- collecting data;
- collaborating with peers; and
- using multiple strategies to reach conclusions.

This department presents a scripted sequence and set of directions for a mathematical investigation for the purpose of communicating what happened in *this* classroom. NCTM's Standards (NCTM 2000) encourage teachers and students to explore multiple approaches and representations when engaging in mathematical activities. The investigation will come alive through students' problem-solving decisions and strategies in the reader's own classroom.

As a result of their exploration, students will incorporate their reasoning and proof skills as they evaluate their strategies. The use of multiple

approaches creates the richness that is so engaging in an investigation. It also helps students find new ways of looking at things and understand different ways of thinking about a problem.

Materials

The following items are needed for lesson one:

- Virtual hundreds boards and calculator, available online at standards.nctm.org/document/eexamples/chap4/4.5/index.htm
- Recording Sheet #1: Investigating Algebra—Number Patterns

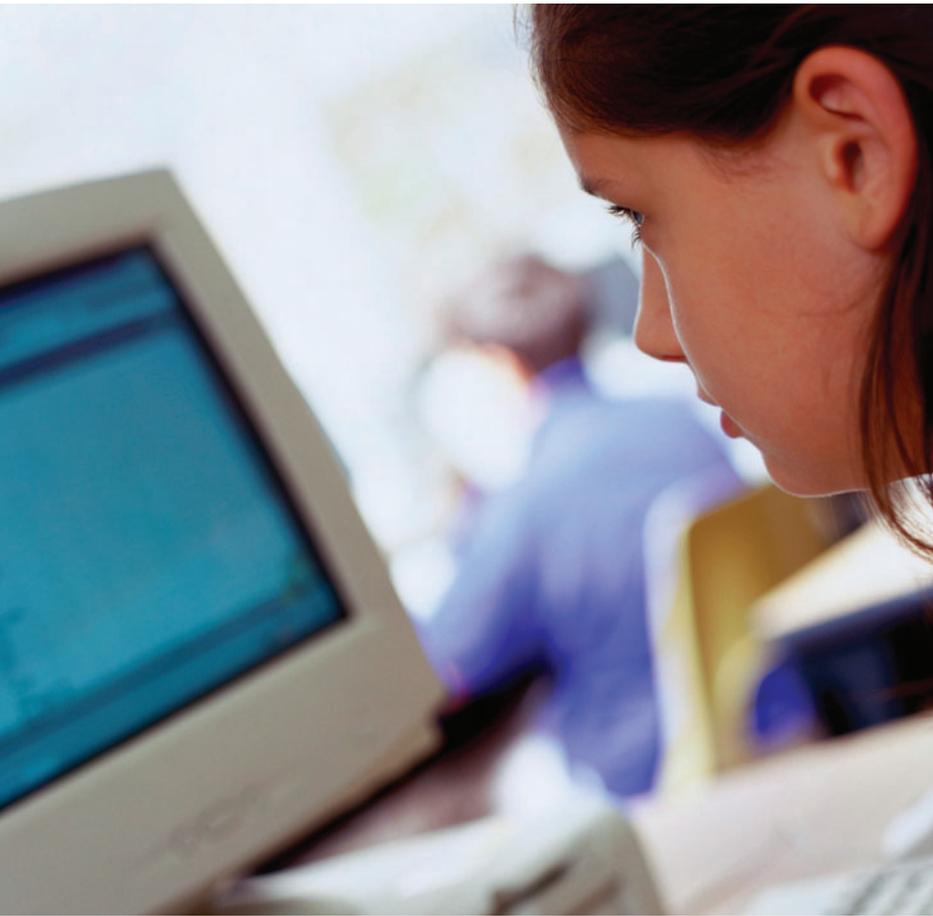
The following items are needed for lesson two:

- Virtual pattern blocks, available online at standards.nctm.org/document/eexamples/chap4/4.1/index.htm, www.arcytech.org/java, and matti.usu.edu/nlvm
- Recording Sheet #2: Investigating Algebra—Shape Patterns

The following items are needed for lesson three:

- Virtual pattern blocks, available online at standards.nctm.org/document/eexamples/chap4/4.1/index.htm, www.arcytech.org/java, and matti.usu.edu/nlvm
- Recording Sheet #3: Investigating Algebra—Generalizing Rules

Edited by Patricia S. Moyer-Packenham, pmoyer@gmu.edu, George Mason University, Fairfax, VA 22030. This section is designed for teachers who wish to give students new insights into familiar topics in grades K–6. This material can be reproduced by classroom teachers for use with their own students without requesting permission from the National Council of Teachers of Mathematics. Readers are encouraged to send manuscripts appropriate for this section to "Investigations," NCTM, 1906 Association Dr., Reston, VA 20191-1502; or send electronic submissions to tcm@nctm.org.



Lesson One: Exploring Number Patterns

These lessons are described using computer technology and virtual manipulatives. The activities can also be adapted using the physical versions of the virtual manipulatives featured. Before each lesson, teachers should spend some time familiarizing themselves with the virtual-manipulative tools and their functions. Students can work on each lesson individually or in pairs.

At the beginning of the investigation, ask the students if they have ever noticed patterns in numbers. On a large piece of chart paper, record examples of the different number patterns that students provide. Tell students that they will be investigating a variety of numerical patterns using a virtual calculator and virtual hundreds board. Introduce students to the virtual hundreds board applet on NCTM's Web site. Allow students a few minutes to explore the virtual hundreds board and ask them to share some of the functions they have discovered about the virtual tool.

Provide a guided experience with the virtual hun-

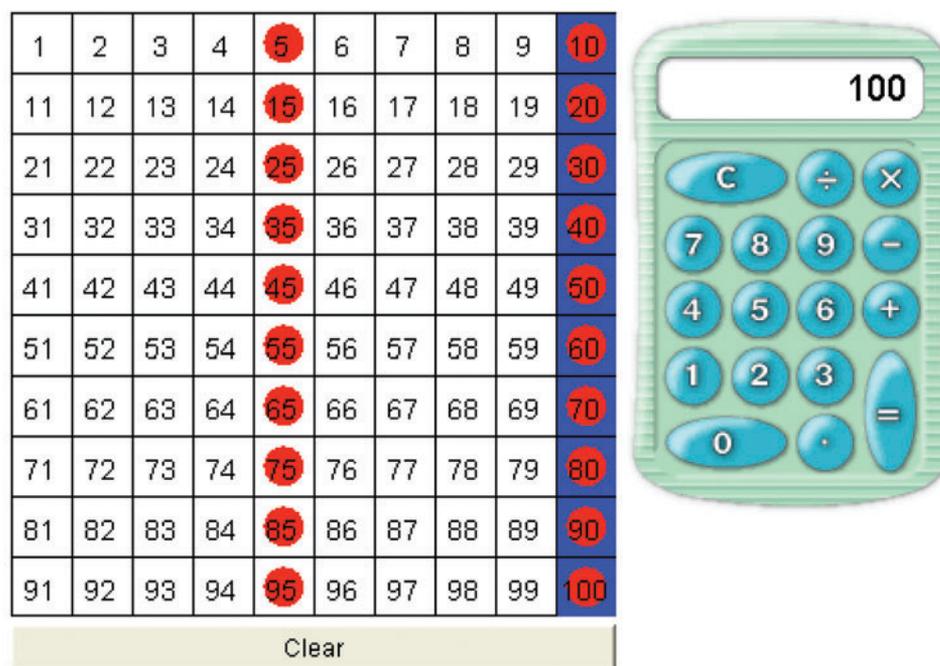
reds board by exploring several number-pattern tasks. Point out to students that they will be looking for *visual patterns* and *numeric patterns*, and help them understand the difference between these two terms. In the first task, instruct students to use the virtual calculator to show all the multiples of 10 on the virtual hundreds board. (To show all the multiples of 10, students should use the following key sequence on the virtual calculator: $10 + 10 = = =$, and so on, until all the multiples of 10 are displayed.) The multiples of 10 will be highlighted on the hundreds board using blue markers. After students have highlighted the multiples, ask them to describe the pattern that is made on the hundreds board by the multiples of 10. Make sure that students have the opportunity to share descriptions of both visual patterns and numeric patterns for the multiples. Encourage students to think of a rule for describing the multiples of 10, such as "If a number has a zero in the ones place, it is divisible by 10."

Next, guide students to find the multiples of both 10 and 5, then display them simultaneously on the virtual hundreds board. To do this task, students use the key strokes for finding the multiples of 10; that is, they push the following buttons on the virtual calculator: $10 + 10 = = =$, and so on, until all the multiples of 10 are displayed. Then, without clearing the hundreds board, follow the procedure for finding the multiples of 5; that is, push the following buttons on the virtual calculator: $5 + 5 = = =$, and so on, until all the multiples of 5 are displayed. This will provide students with the multiples of 10 highlighted in blue and the multiples of 5 highlighted in red (see **fig. 1**). Students can use both highlighted sets to compare the multiples of 10 with the multiples of 5. Ask students what patterns they noticed. Encourage them to generate a rule comparing the relationship between the multiples of 10 and 5. Highlighting each of the numbers on the virtual hundreds board allows students to look at the numbers as a group and find common characteristics among them.

After this guided introduction, students can work individually or in pairs to investigate and describe other number patterns that can be created using the virtual calculator and hundreds board. The recording sheet "Investigating Algebra—Number Patterns" provides an investigation of the multiples of several numbers and the multiples of pairs of numbers. As students investigate each of these number patterns, the recording sheet asks them to describe the pattern and to generate a rule of their own.

Figure 1

The virtual hundreds board highlights multiples of 10 in blue and multiples of 5 in red.



After students have time to independently investigate the number patterns, bring the group together for a whole-class discussion. Ask students to share the descriptions they wrote for each pattern and articulate their rules for the multiples and pairs of multiples. Recording the terminology that students use to describe their patterns and rules on the chalkboard or a wall chart is beneficial. This helps promote the use of accurate mathematical vocabulary in describing the patterns. Compare students' descriptions of the patterns and look for the unique ways in which they describe their patterns. Encourage students to look for numerical relationships and ways to state those relationships using numbers and comparison words.

Lesson Two: Exploring Growing Shape Patterns

Begin the second lesson by asking students to describe shape patterns that they have noticed in their world. Allow time for students to look around the classroom for shape patterns in their environment. Introduce students to the virtual pattern

blocks applet on the NCTM, NLVM, or Arcytech Web sites. Allow students a few minutes to explore the virtual pattern blocks and ask them to share some of the functions they have discovered. Students should notice that they can rotate these virtual blocks and change their color. They will also find that the virtual applet allows them to create an unlimited number of blocks.

Show students an example of a growing shape pattern using the virtual pattern blocks. A *growing pattern* is a sequence of shapes or numbers that increases in a regular way. Ask students to use their virtual blocks to copy and extend the pattern (see **fig. 2**). Explain to students that, in addition to the visual aspects of the growing shape pattern, these patterns also have a numeric component. You may need to introduce some new terminology to the students in using these growing patterns. For example, each picture in the pattern can be referred to as a *figure* or *term*. Students must examine the relationship between the first, second, third, and fourth figures or terms to determine the number and shape of the blocks that should be included in the fifth figure or term. The number of blocks used to create

Investigating Algebra—Number Patterns

Recording Sheet #1

Patterns in Multiples

1. Use the virtual hundreds board to show the multiples of 10.

What patterns do you see? _____

My rule for recognizing multiples of 10: _____

2. Use the virtual hundreds board to show the multiples of 3.

What patterns do you see? _____

My rule for recognizing multiples of 3: _____

3. Use the virtual hundreds board to show the multiples of 9.

What patterns do you see? _____

My rule for recognizing multiples of 9: _____

Patterns in Pairs of Multiples

4. Use the virtual hundreds board to show the multiples of 5 and 10.

What patterns do you notice about the multiples of these two numbers? _____

My rule for the relationship between the multiples of 5 and 10: _____

5. Use the virtual hundreds board to show the multiples of 4 and 8.

What patterns do you notice about the multiples of these two numbers? _____

My rule for the relationship between the multiples of 4 and 8: _____

6. Use the virtual hundreds board to show the multiples of 3 and 5.

What patterns do you notice about the multiples of these two numbers? _____

My rule for the relationship between the multiples of 3 and 5: _____

each figure can be recorded in a table; this number is called a *figurate number*.

Using the virtual pattern blocks, give students one or two additional examples of growing shape patterns to copy and extend. After they are able to copy and extend the patterns using the virtual blocks, show them how to record the number of the figure (or term) and the number of tiles (or figurate number) in a table format. On a large sheet of chart paper, create a table such as the one in **figure 3**, and using the pattern of tiles in **figure 2**, record terms 1, 2, 3, and 4 in the table. Next, record the number of tiles needed to create each term. In the example in **figure 2**, the number of tiles used in each term is 1, 3, 5, and 7, respectively. Ask students to use the first few terms in the sequence to conjecture how many tiles will be in the fifth, sixth, seventh, and eighth terms of the pattern.

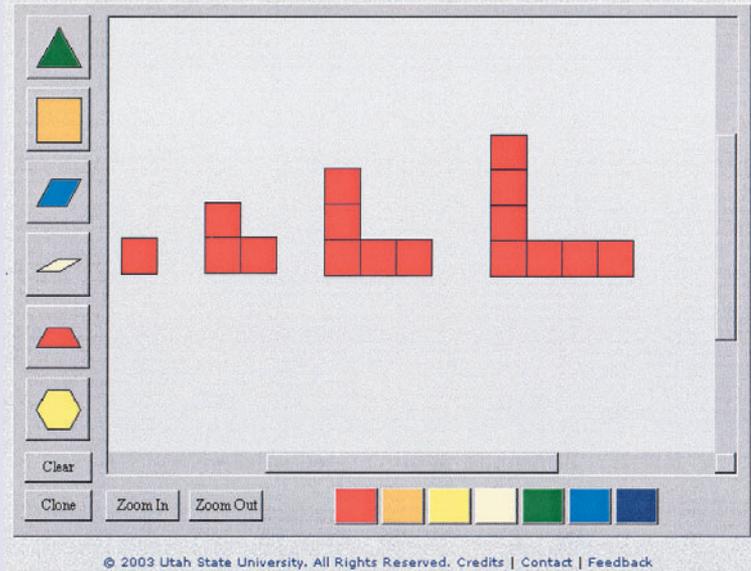
After this guided introduction, students can use the virtual pattern blocks to create their own growing shape patterns. Students should design the growing shape pattern using the virtual pattern blocks. Next, they can print a picture of the growing pattern from the computer screen (using the “screen shot” or “print” computer commands, depending on the Web site) or draw a picture of their pattern on the recording sheet. The recording sheet “Investigating Algebra—Shape Patterns” provides examples of growing shape patterns and space for students to record their work. There is room on the recording sheet for students to show a visual image of their growing patterns and to complete a table that indicates the terms and number of tiles used in their pattern. At the end of this class session, ask students to share some of the growing patterns they created and to describe to their peers the numerical pattern they found as they counted the number of blocks in each figure in their sequences.

Lesson Three: Generalizing Rules for Patterns

At the beginning of the third lesson, make connections to lesson two by revisiting some of the students’

Figure 2

Growing shapes pattern created using virtual pattern blocks



findings from the previous investigation. Use the virtual pattern blocks to show an example of a growing shape pattern. Ask students to verbally describe the visual aspects, as well as the numeric components, of the pattern. Use an example of a growing pattern generated from the student work in lesson two and review with the class how to record the numeric components of the pattern in a table.

Explain to students that this lesson will focus on generalizing rules for numeric patterns. Using a copy of the table on Recording Sheet #2, show students how to examine the pattern in the sequence of the term numbers, the pattern in the sequence of the number of blocks, and the relationship between each term number and its number of blocks. For example, in the growing pattern in **figure 2**, students may recognize that the term numbers (1, 2, 3, 4, and so on) are the counting numbers. They may

Figure 3

Recording the numeric pattern in the table

Term	1	2	3	4	5	6	7	8
Number of Blocks								

Investigating Algebra—Shape Patterns

Recording Sheet #2

Patterns in Figures

1. Use the virtual pattern blocks to copy the growing pattern.

What patterns do you see? _____

Record the numeric pattern in the table.

Term	1	2	3	4	5	6	7	8
Number of Blocks								

2. Use the virtual pattern blocks to copy the growing pattern.

What patterns do you see? _____

Record the numeric pattern in the table.

Term	1	2	3	4	5	6	7	8
Number of Blocks								

3. Use the virtual pattern blocks to create your own growing pattern. Display the pattern below.

What patterns do you see? _____

Record the numeric pattern in the table.

Term	1	2	3	4	5	6	7	8
Number of Blocks								

also recognize that the pattern in the number of blocks (1, 3, 5, 7, and so on) is the odd numbers. That they see the relationship between each term number and its number of blocks is also important; that is, Term 1 = 1 block, Term 2 = 3 blocks, Term 3 = 5 blocks, Term 4 = 7 blocks, and so on. Encourage students to generate a rule for this relationship. Students might say, “Term 1 is equal, Term 2 is one more, Term 3 is two more, Term 4 is three more, and so on.” Help students find different ways to articulate the relationships that they see in the sequence. These strategies will encourage students to look for relationships among numbers and will allow them to see connections among different number patterns.

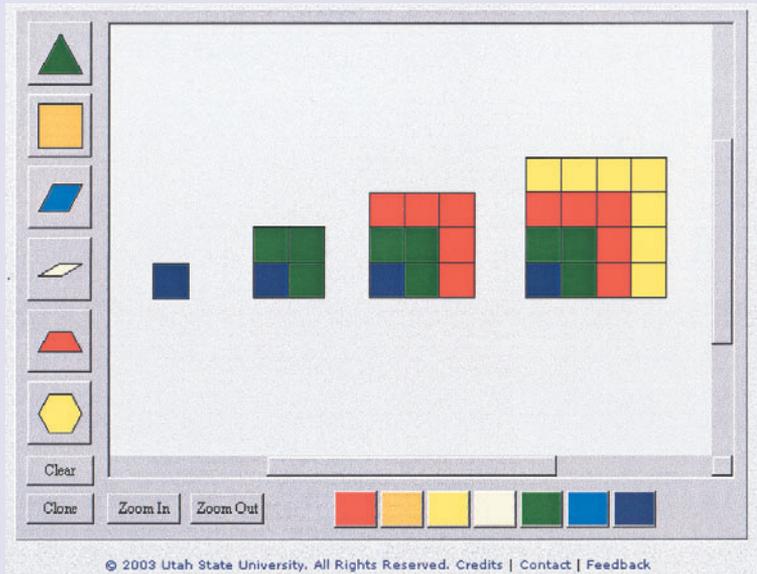
Help students understand that a rule for the pattern is a way of explaining the number of blocks that is added to each term of the pattern. Use the virtual pattern blocks to provide one additional example of a growing shape pattern for students to copy, extend, and record in a table (see **fig. 4**). Give the students time to work individually, and then with a partner articulate and generate a rule for the pattern. Next, have students write down the rule for the pattern in their own words. Ask several students to share their rules with the class, and be sure to point out when your students use mathematical words accurately. This will develop students’ skills in communicating their mathematical thinking clearly to others and encourage their use of new mathematics vocabulary. Ask students to share their thinking as they generate their own rules for finding the number of tiles in different patterns.

After this group exploration, students should generate their own growing shape patterns using the virtual pattern blocks. After constructing the patterns, just as they did in lesson two, students can print pictures of their patterns from the computer screen or draw their patterns on the recording sheet. The recording sheet “Investigating Algebra—Generalizing Rules” provides room for students to record the pattern visually, construct a table for the pattern, and generate a rule for the numerical sequence. Students should use the table on the recording sheet to help them make conjectures about a rule for their growing patterns.

Encourage students to test their own rules during their independent investigation. Students should use the rules they have generated to determine how many blocks will be in the twelfth term of each of their growing patterns. After they have determined this, they can use the virtual pattern blocks to build the figures and check their conjecture. This experimen-

Figure 4

Growing shape patterns can easily be extended using virtual pattern blocks.



tion may allow some students to refine their rules to make them more accurate. At the end of the lesson, allow students to show their examples using the computer screen or their printed copies or drawings. Ask each student to explain the rule that he or she used to generalize about the number of blocks in each term of the pattern sequence. Challenge other students to see if they can use the rule to determine the number of blocks that would be in the fifteenth term of their peers’ patterns.

Conclusion

These lessons allow students to make connections among various representations for understanding algebraic concepts. They emphasize mathematical communication by encouraging students to verbalize and write about mathematical relationships. Many underlying mathematical ideas important for developing algebraic thinking are part of these experiences. Helping students recognize and articulate these ideas is an important step in beginning to understand algebra.

Reference

Cuevas, Gilbert J., and Karol Yeatts. *Navigating through Algebra in Grades 3–5*. Reston, Va.: National Council of Teachers of Mathematics, 2001. ▲

Investigating Algebra—Generalizing Rules Recording Sheet #3

Rules for Sequences of Numbers

1. Use the virtual blocks to create your own growing pattern. Display the pattern below.

Record the numeric pattern in the table.

Term	1	2	3	4	5	6	7	8
Number of Blocks								

My rule for creating this pattern: _____

Use your rule to answer the following question: How many blocks will be in the 12th term?

Use the virtual blocks to build the 12th term and check your answer. What did you find?

2. Use the virtual blocks to create your own growing pattern. Display the pattern below.

Record the numeric pattern in the table.

Term	1	2	3	4	5	6	7	8
Number of Blocks								

My rule for creating this pattern: _____

Use your rule to answer the following question: How many blocks will be in the 12th term?

Use the virtual blocks to build the 12th term and check your answer. What did you find?
